Acoustics and instability of high-speed boundary layers

Sergey A. Gaponov, Boris V. Smorodsky

Abstract − Present paper is devoted to those acoustical problems which are also related to the problem of the turbulence onset in high-speed boundary layers. Basic equations governing linear development of both hydrodynamic and acoustic waves in high-speed boundary layers are presented. Results of the theoretical investigation on external acoustic wave influence on the flat plate boundary layer are described. The intensity of disturbances excited by an acoustic wave inside the boundary layer has been studied. It has been found that interaction of sound with the boundary layer lead to an increase of the amplitude of disturbances inside the boundary layer and this increase depends on the angle of incidence. It has been established, that maximal amplification of fluctuations inside the boundary layer takes place for the external acoustic wave with the wave-vector orientation parallel to the model surface which is called streamwise acoustic wave. It has been shown that in this case the amplitude of fluctuations inside the boundary layer may exceed the amplitude of the external acoustic field in many times. Besides in the paper nonlinear generation of increasing perturbations by external acoustics is studied.

Keywords — compressible boundary layers, acoustic field, laminar-turbulent transition, hydrodynamic stability.

I. INTRODUCTION

The questions on the interaction of a supersonic boundary layer with acoustic waves which are considered in the present paper were raised mainly in connection with the problem on the turbulence formation. At present the most complex problem on the prediction of the transition position in the boundary layer flows is related to the receptivity of these flows to the external effects. It appears that this problem was discussed in detail for the first time in [1], and till now many works were carried out on it. However this problem has been studied more thoroughly both experimentally and theoretically for the subsonic flows. A review of the early works of the influence acoustic field effect on the transition from a laminar supersonic boundary layer to the turbulent has been given in [2]. The problems on the supersonic flow aeroacoustics were studied mainly within the framework of the investigations on the conditions for the onset of auto-oscillations and sound generation by the supersonic shear flows in the jets and mixing layers [3, 4]. The advanced approach based on the idea of the possibility of the mutual influence of the acoustic and hydrodynamic waves has been demonstrated in [5]. A relation between the acoustics problems and stability was shown also in [6].

The first attempts at the investigation of the sound waves and supersonic boundary layer interaction on the basis of the stability theory were undertaken in [7, 8]. The problem on the excitation of unstable waves by sound was considered in [9]. The interaction of sound with a supersonic boundary layer was studied experimentally in [10] where the main results of the theory [8] were confirmed. The experiments of [11] using a controlled acoustic field have shown that the boundary layer receptivity to the acoustic disturbances depends on the location of the interaction region. It was found in particular that the intensity of the hydrodynamic waves generated by the sound reaches its maximum values when the interaction region is located near the leading edge of the model, the lower branch of the neutral stability curve and the "sonic" branch of the neutral stability. The agreement between the theoretical conclusions and the experiments on the acoustic excitation of unstable waves is discussed in [12]. The generation of sound waves by a transitional boundary layer has been revealed experimentally in [13].

We analyze the state of the art of the linear theory of the supersonic boundary layer receptivity to the acoustic disturbances on the basis materials obtained in the present work and on the available bibliographic data. The main attention is paid to the interaction of a longitudinal (streamwise) sound wave with the boundary layer.

II. GENERAL LINEAR EQUATIONS

Following [14] consider the interaction of a monochromatic wave with a boundary layer. The problem is solved in the approximation of the parallel flow [15] in the boundary layer which implies the independence of the main stream parameters on the longitudinal coordinate. To reduce the governing equations to a dimensionless form one introduces the reference length (the Blasius scale) $\delta=\sqrt{x^{*}\nu_e/U_e}$ and the time scale $t=\delta/U_e$, where x^* is the distance from the leading edge of a plate, ν_e and U_e are the kinematic viscosity and velocity at the boundary layer edge, respectively. We will consider here the boundary layer on a flat impermeable plate, and the viscosity, velocity and temperature are related to the corresponding values at the external boundary of the boundary layer U_e, T_e, ν_e A linear approximation is used for the description of the non-stationary flow parameters which is valid at small amplitudes

Manuscript received December 14, 2011: This work was supported by Russian Foundation for Basic Research (project 11-01-00195-a).
S. A. Gaponov is with the S.A. Khristianovich Institute of Theoretical and Applied Mechanics (ITAM) SB RAS, 630090, Novosibirsk, Russia (phone:+7(383)3301228; fax:+7(383)3307268;
e-mail: gaponov@itam.nsc.ru).
B. V. Smorodsky is with the S.A. Khristianovich Institute of Theoretical and Applied Mechanics (ITAM) SB RAS, 630090, Novosibirsk, Russia (phone:+7(383)3301228; fax:+7(383)3307268;
e-mail: smorodsk@itam.nsc.ru).

of the sound wave. The flow parameters of the incident sound wave are described by the vector function:

$$\mathbf{Q}^{*}\left(x_1, y_1, z_1, t\right) = \varepsilon \mathbf{q_0} \exp\left[i\left(\alpha_1 x_1 + \beta_1 z_1 + y_1\lambda_1 - \omega t\right)\right]$$

Where ε is the wave intensity at the adopted normalization, x_1 is directed along the main stream, y_1 − normal to the plate surface coordinate, z_1− coordinate in the lateral direction (normal to x_1 and y_1). The disturbances inside the boundary layer are described by the dependence:

$$\mathbf{Q}\left(x_1, y_1, z_1, t\right) = \varepsilon \mathbf{q_1}(y_1)\ \exp\left[i\left(\alpha_1 x_1 + \beta_1 z_1 - \omega_1 t_1\right)\right]$$

By using a conventional procedure of the linearization of the Navier-Stokes equations, the energy and continuity one can show that the components of the vector **q**(y) will satisfy a system of the eighth-order ordinary differential equations [15] depending on the wave numbers α_1, β_1 the frequency ω_1 and the main stream parameters: $U(y_1)$ and $T(y_1)$. If we introduce the angle $\chi=arctg(\beta_1/\alpha_1)$ and go over from x_1 and z_1 to the new variables $x=x_1\cos(\chi)-z_1\sin(\chi)$ and $z= x_1\sin(\chi) +z_1\cos(\chi)$, then the disturbances will not depend on z. The system of the eighth-order differential equations can be reduced under insignificant additional assumptions to a sixth-order system [2], which was obtained earlier Dunn and Lin [16]:

$$\frac{dq_i}{dy_1} = \sum_{j=1}^{6} a_{ij} q_j \qquad (1)$$

We have used the following notations: q_1 and αq_3 are the disturbance amplitudes of the velocities in the x- and y-directions, $\gamma M_1^2(\cos^2\chi)q_2$, q_5 are the disturbances of the pressures and the temperature; q_2 and q_6 are derivatives with respect to y_1 of q_1 and $q_5$respectively; γ is ratio of specific heats, M_1 is the Mach number. The coefficients a_{ij} depend on y_1, the wave number $\alpha = \sqrt{\alpha_1^2 + \beta_1^2}$ and $\omega_1=\alpha_1 c$; the Reynolds numbers $R = R_1 \cos\chi$ and the Mach number $M = M_1 \cos\chi$ where $R_1=U_e\delta/\nu_e$, $M_1 =U_e/a$, a is the sound velocity.

Let the velocity and temperature disturbance vanish on the surface, i.e.:

$$q_1(0)= q_3(0)= q_5(0)=0 \qquad (2)$$

and outside the boundary layer where the main stream parameters do not depend on y_1,

$$\mathbf{q} = \mathbf{q}^0\, e^{i\lambda y_1} + \mathbf{q}^1 I_1\, e^{-i\lambda y_1} + \mathbf{q}^3 I_3\, e^{-\lambda_3 y_1} + \mathbf{q}^4 I_4\, e^{-\lambda_4 y_1}; \qquad (3)$$

where $\mathbf{q}^{0,1}=[1,\pm i\lambda, \pm i\lambda/\alpha^2, c-1, M_1^2(c-1)(\gamma-1)]$;
approximate values of $\mathbf{q}^{3,4}$ look like:$\mathbf{q}^3=[1 ,\lambda^3 , 0, 0, 0, 0]$, $\mathbf{q}^4=[0, 0, 0, 0, 1, \lambda^4]$;

$$\lambda = \alpha\sqrt{M^2(1-c)^2 - 1};\ \lambda_3 \approx \sqrt{i\alpha(1-c)Re};$$

$\lambda_4 \approx \sqrt{i\alpha \Pr(1-c)R}$, $c = \omega_1/\alpha_1$, Pr is the Prandtl number. The third and the fourth terms of the above dependence describe the thermal and vortical waves whose intensity rapidly decreases with increasing y_1, if $c\neq 1$. The second term corresponds to the reflected sound wave whose intensity is proportional to the incident wave intensity and to the reflection factor. Thus outside the boundary layer only the first two terms remain significant which taken into account (the incident and eflected waves). The problem (l)-(3) enable one to compute both the reflection factor and the amplitude of the disturbances inside the boundary layer. In the present paper computations were carried out for the boundary layer on a flat plate for different Mach and Reynolds numbers and different orientations of the sound wave. The dependence of the viscosity on temperature was used as the Sutherland's law, the Prandtl number $Pr=0.72$.

III. THE REFLECTION FACTORS AND OSCILLATIONS IN A BOUNDARY LAYER

For a problem of a laminar-turbulent transition disturbances in boundary layer play important role. In Fig. 1 we present a comparison of distributions amplitudes of the mass flow rate, A, of the "acoustic" (AC) ($M_1=2$, $R_1=545$, $c=0.25$, $F=\alpha_1 c/R_1 =10^{-5}$, $\chi=0°$) and the “Tollmin-Schlihtihg” (TS) wave ($M_1 =2$, $R_1=720$, $c=0.51$, $F=0.36\cdot 10^{-4}$, $\chi =600$). Inside the boundary layer the amplitude of the "acoustic" wave corresponds to the incident sound wave with the unit amplitude of the longitudinal velocity oscillations. Since the eigenoscillations (TS) are determined theoretically with the accuracy up to an arbitrary multiplier, the maximum value of the mass flow rate oscillations was assumed to be the same as for the "acoustic" wave. It can be seen from the presented data that under the influence of the sound the oscillations arise in the boundary

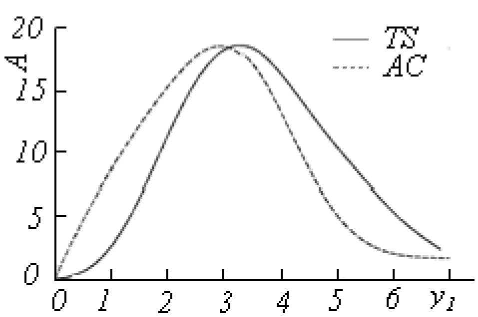

Fig. 1. Profiles of mass flow amplitude (A) for a “Tollmin-Shlichting” wave (curve TS) at $M_1 = 2$, $R_1 = 720$,$F = 36\cdot 10^{-6}$, $\chi = 60°$ and for an acoustic wave (curve AC): $M_1 = 2$, $R_1 = 545$, $F = 10\cdot 10^{-6}$, $\chi = 0$.

layer, which exceed in their intensity considerably the corresponding quantities in the incident sound wave. Despite the fact that the parameters of the sound wave and the eigenfrequencies differ strongly, the behavior of their amplitudes inside the layer are similar. The magnitude of the maximum of the mass flow rate amplitude and the reflection factor depend on the boundary layer properties and on the acoustic wave parameters. boundary layer.

It was shown in the work [12] that its value can vary within wide limits, and it can either exceed unity in modulus or

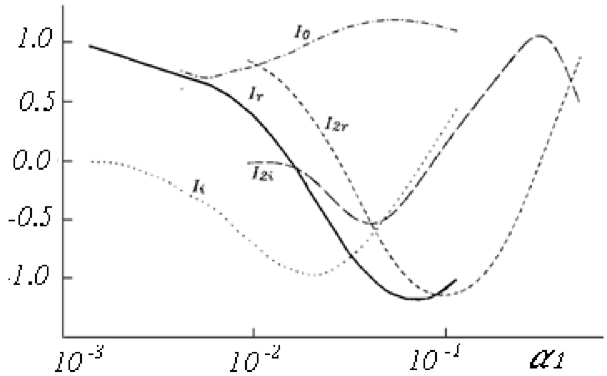

Fig.2. Dependence of the reflection factor on a wave number

vanish. A possibility of the reflection with a factor being less then unity is in contradiction with the data [17].

It should be noted that the results of [17] were obtained without regard for the viscosity effect which in the stability theory is related to the quantity αR. Therefore the theory of [17] can describe satisfactorily the phenomena only at large values of αR. Our data taking the viscosity into account ($M_1=2$) are presented in Fig.2. The real and imaginary values of reflection factors are shown versus the wave number α_1 for different frequencies, $F=2\pi f\, \nu_e/U_e^2$: $I_r\, I_i$ ($R_1 =545$, $c=0.38$) and Reynolds numbers: I_{2r}, I_{2i} ($F=10^{-4}$, $c=0{,}4$). We also present here the modulus of the reflection factor $|I_r +iI_i| = I_0$. The range of the variation of F was $0.8{\cdot}10^{-4} \div 2{\cdot}10^{-4}$ and the range of the variation of the Reynolds number R_I was ($50\div3000$). It can be seen that the approach of $I=I_r+iI_i$ and $I_2=I_{2r}+iI_{2i}$ is observed at $\alpha_1 \approx 0.05$. An analysis of these data shows that a significant influence of the viscosity manifests itself at $\alpha_1 R_1 \leq 10$. A small difference in I and I_2 in the region of the large values of α_1 is related to the difference in the phase velocities. With the diminution of α_1 and as a consequence of $\alpha_1 R_1$ the modulus of the reflection factor has a minimum which is less than unity. Such a possibility was already noted in [12] and it does not agree with the data of [17].

A question on the receptivity of the supersonic boundary layer of the acoustic disturbances was already discussed in [12]. It was noted therein that there exists a region of an efficient receptivity of the sound waves in a boundary layer between the lower branch of the neutral stability curve and the leading edge. There is the continuous degeneration of an incident sound wave into a "Tollmien-Schlichting" wave in this region. The corresponding sound wave is absorbed completely by the boundary layer (the reflection factor is equal to zero). On the other hand, experimentally it was found [11] that more intense oscillations are observed in this region. They are induced by the sound field. Therefore it is interest to compare the magnitude of the reflection factor with the magnitude of the maximum of the mass flow rate oscillations. From the general considerations it appears to be logical to assume that the intensity of the oscillations inside the boundary layer itself must increase with increasing absorption of the sound wave energy. The results of a such comparison are shown in Fig. 3. For the two values of the phase velocity c = = 0.08 (solid line) and *0.28* (dashed line) at $R_1 = 545$ and $M_1 = 2$ we present the

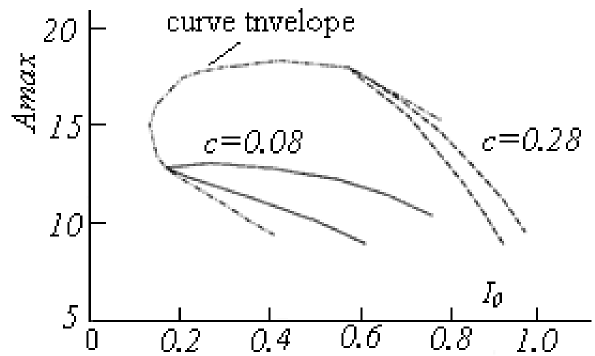

Fig. 3. Dependence of the maximum of mass flow oscillations inside the layer on the reflection factor.

values of the maximum of the mass flow rate oscillations versus the reflection factor modulus, whose changing was achieved by varying the quantities F. The boundary values of the frequency parameter are indicated at the curves ends. The obtained data confirm on the whole the hypothesis that at a fixed value of c the largest values of A_{max} correspond to the minimum values of the reflection factor I_0 . The envelope of the curves for different c is shown by a dash-dot line. In Fig. 4 we present the values of A_{max} and I_0 on the envelope. It can be seen that the maximum values of the amplitude A_{max} do not correspond completely to the minimum values of the reflection factor. But at the same time the maximum is achieved in the region of small values $I_0 < 0.5$.

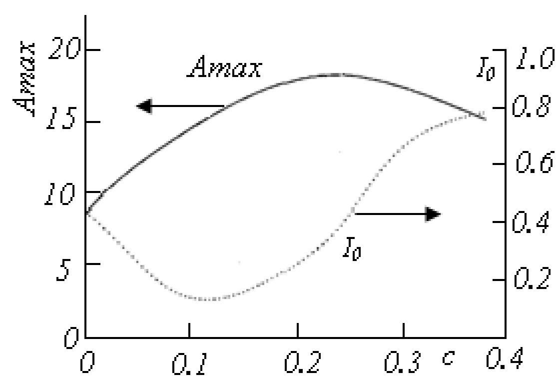

Fig. 4. The maximum values of amplitude of pulsations of the mass expense inside boundary layer (dot line) and the absolute value of reflection factor (a continuous line) in depending on phase speed of an acoustic wave

Taking into account the fact that the absorption energy is determined by the quantity $(1 - I_0^2)$, one can draw a conclusion that with its increase the oscillations in the boundary layer also increase. Therefore, it is not incidental that the efficiency of the sound wave effect in this region is high from the viewpoint of excitation of unstable waves [11].

Fig. 5 shows the dependence of the greatest intensity of fluctuations on M_1. With increase of the Mach number fluctuations in a layer are increased. However in the field of high values of M_1 this effect weakens. From the resulted data it is important to notice that fluctuations intensity in a layer can exceed level of external disturbances in a few dozen times.

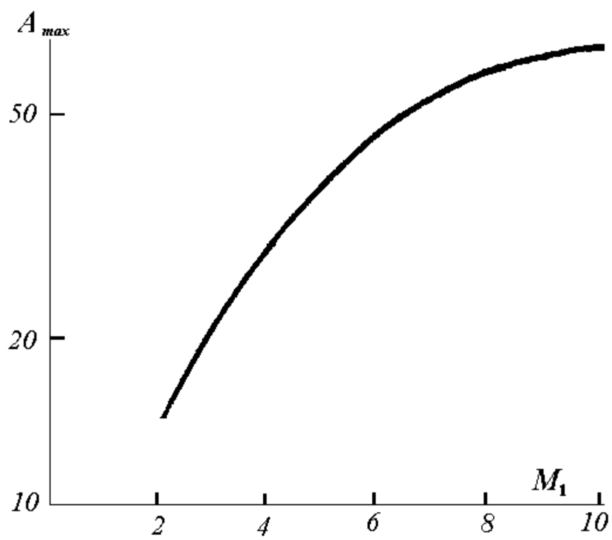

Fig. 5 Dependence of the greatest intensity of fluctuations on M_1

IV. INTERACTION OF LONGITUDINAL SOUND WAVE WITH A BOUNDARY LAYER

In the case of a longitudinal sound wave ($c = 1 \pm 1/M$) the notions of the incident and reflected waves lose their meaning and hence the uncertainty in the formulation of problem (l)-(3) is revealed. In this case the disturbances for $y \gg \delta$ normalized with respect to the amplitude of the longitudinal velocity oscillations in each of the incident and reflected waves have the form [2]:

$$\mathbf{q}^{0,1}=[1, \pm i\lambda, \pm i\lambda/\alpha^2, c-1, M_1^2(c-1)(\gamma-1)], \qquad (4)$$

where the plus sign corresponds to the incident wave and the minus sign corresponds to the reflected wave. At $c \rightarrow 1-1/M_1$ $\lambda = \alpha\Lambda = \alpha\sqrt{M_1^2(1-c)^2-1} \rightarrow 0$ and the vectors q^0 and q^1 coincide. If the reflection factor, I, also tends to zero, then the solution describes well the external motion of the wave whose normal velocity component a αq_3 is equal to zero. Since equations (1) depend on the Reynolds number and the wave number, the case of the vanishing reflection factor is a special case. For the arbitrary values of α and R_1 only the trivial solution **q=0** will be obtained. On the other hand there is another approach to the construction of the solution. Let us specify the solution at the external boundary in the form $\mathbf{q}^{01} = (\mathbf{q}^0 + I\mathbf{q}^1)/(1+I)$. Then the components of the vector $\mathbf{q}^{01}$ have the form:

$$\mathbf{q}^{01} = [1, \frac{1-I}{1+I}, \frac{\lambda}{\alpha^2}\frac{1-I}{1+I}, c-1, M_1^2(c-1)(\gamma-1), i\lambda M_1^2(c-1)(\gamma-1)\frac{1-I}{1+I}]$$

If we introduce the quantity $\lambda(1-I)/(1+I)=b$ determined from the solution of system (1) with the boundary conditions (2), (3), then the total value of the normal velocity amplitude can be written in the form $V_0=b/\alpha=\Lambda(1-I)/(1+I)$. After V_0 is determined if we compute the reflection factor: $I = (\Lambda - V_0)/(\Lambda + V_0)$.The vector $\mathbf{q}^{01}$ has a good agreement with the main parameters of the longitudinal sound wave: the longitudinal velocity, the pressure and the temperature. However in the result of the interaction of the sound wave with the boundary layer the normal velocity components $V_0 \sim \alpha$ and the gradients of the longitudinal velocity and temperature proportional to α^2 arise on the external boundary of the boundary layer. It is well known from the stability theory [2] that the gradient of the pressure amplitude in the normal direction is proportional to α^2 and for the long wave approximations it can be taken to be equal to zero ($dq_4/dy=0$ in (1)). In this case the interaction of a longitudinal sound wave with a boundary layer is described by the same equations as in the problem on a complete stabilization of a supersonic boundary layer by cooling [19].

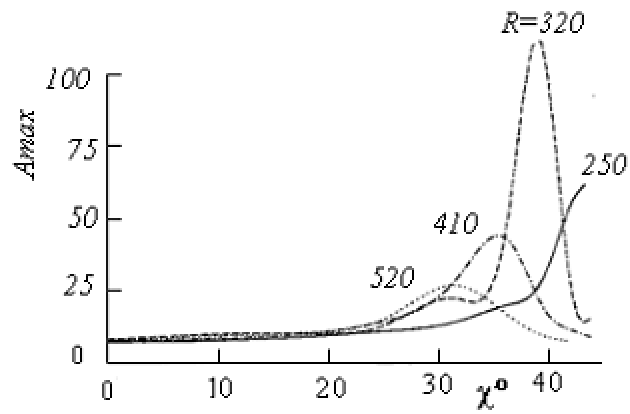

Fig. 6. Maximal oscillations of the mass flow in the layer in dependence of the angle between the wave vector of the sound wave and the main stream direction.

Taking into account that $c=1-1/M_1$ we have computed the amplitude of the mass flow rate oscillations caused by the longitudinal sound wave at different values of the angle χ. The dependence of the maxima of the mass flow rate amplitudes on the angle χ for different R_1 is shown in Fig. 6 ($M_1=2.0$, $F=0.9 \cdot 10^{-4}$). It is seen that the sound wave with the amplitude of the longitudinal velocity equal to unity excites an intense oscillation of the mass flow rate inside the boundary layer. This can be observed especially clearly for $R_1=320$. It is interesting to note that the most intense oscillations are achieved at $\chi \neq 0$. With increasing of Reynolds number χ_{cr} decreases, and at $R_1=250$ $\chi_{cr} > 45°$. The exact value of χ_{cr} at $R_1=250$ was indeterminated, since the computations were not carried out at $\chi > 45°$. In connection with the data obtained one should pay attention to the experiments of [18] in which the disturbances with $\chi=45°$ were revealed in the spectrum. However the results were obtained therein at $R_1=565$, $F=0.5 \cdot 10^{-4}$. Taking into account the fact that the theoretical results depend mainly only on $\alpha_1 R_1$, M_1, and $F=\alpha_1 c/R_l$, we find that $R_1^T = R_1^E\sqrt{F^E/F^T}$ at equal values of $\alpha_1 R_1$. (The superscripts "E" and "T" denote here the experimental and theoretical values of the parameters, respectively). Thus the theoretical results with $F=0.9l\cdot 10^{-4}$ and $R_1=418$ are the most close to the data presented in Fig. 6 at $R_1=410$, correspond to the above experiments. In accordance with the obtained data the waves with $\chi \approx 35°$ should be amplified the most intensively, which is an agreement with [19] at least qualitatively.

Another important fact is that in the case of a longitudinal sound wave the oscillations inside the boundary layer exceed by several times the external oscillations (see Fig. 6). It appears that this is the main reason for the excitation of the oscillations in the boundary layer on the upper surface of a plate, when the sound wave source was located below the plate level [20, 21]. Only the longitudinal waves with different χ could reach the boundary layer zone. The basic energy of the oscillation before the plate was generated at a finite angle to it passing outside the boundary layer. However, the disturbances inside the boundary layer exceeded multiply the oscillations in the external sound wave. It was the main reason why they were observed in the experiment.

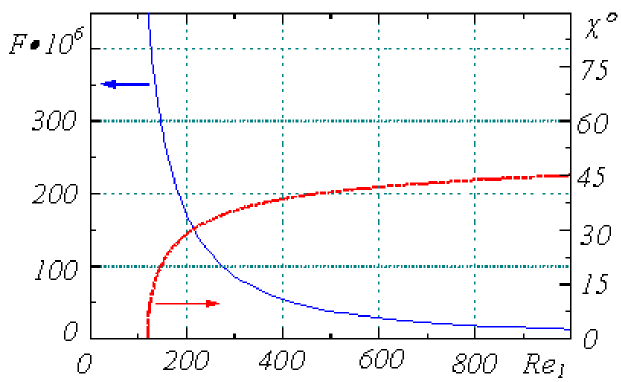

Fig. 7. Frequency parameter and the orientation angle of the resonant longitudinal acoustic wave in depending on the Reynolds number, $M = 2$.

Fig. 6 shows that there are critical values of R_{cr}, at which interaction is the strongest (maximum intensity of disturbance within the boundary layer). Analysis of the data showed that there is a combination of χ and R_1 for given frequency F when the mass flow inside the boundary layer can greatly exceed the relevant values on the outside edge of the boundary layer. Explanation of the big intensity oscillations exciting in a boundary layer by an acoustic wave can be given on the base of the resonance theory. TS waves are described by eigen solutions of the stability equation (1) with $q_1= q_3= q_5=0$ at $y=0,\infty$. It is known, that the phase velocity of the Tollmin-Shlichting wave goes to $1-1/M$ at the lower branche of the neutral stability curves and parameters of two types of waves also converge [18]. Then it becomes clear that the acoustic wave with the same parameters will affect as a resonant force for boundary layer and the fluctuations amplitude inside a layer in this case will be infinite. Thus high values A_{max} on Fig. 6 can be explained by a closeness of the parameters combination of a sound wave (χ and R_1) to a resonant combination of these parameters.

Dependence of frequency parameter and an orientation angle of a resonant sound wave on R_1 are shown on Fig 7. Calculations showed that frequency the acoustic resonance decreases with increasing of the Reynolds number and the orientation angle goes to $45°$ when $R_1 > 800$. The existence of acoustic resonance is limited by area of $R_1 > 120$.

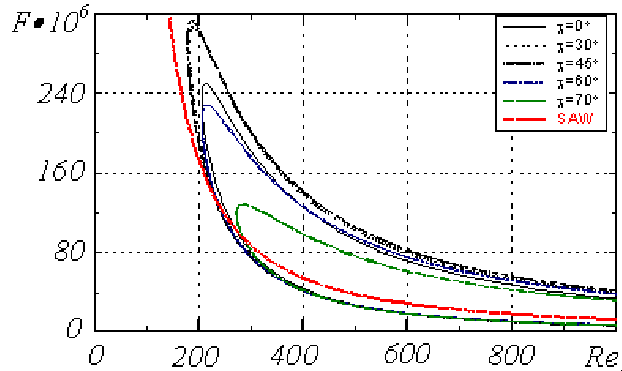

Fig. 8. Neutral stability curves and frequency of an acoustic resonance, F^* (dark circles) in depending on the Reynolds number, $M = 2$.

In Fig. 8 the position of an acoustic resonance frequency (line with mark) is compared with the position of the neutral stability curve of “Tollmin-Shlichting” waves for different orientation angles in the plane (F, R_1). As it was found the curve of the acoustic resonance are in close of the lower branches of the neutral stability curves. In addition it was confirmed by calculations that the phase velocity of the “Tollmin-Shlichting” wave goes to to $1-1/M$ with the growth of R_1, and disturbances profiles of two types of waves also converge. It leads to result that eigen waves and longitudinal sound waves become similar each other. Therefore one should expect that receptivity of the supersonic boundary layer to longitudinal sound field can be a maximum in the neighborhood of the lower branches of the neutral stability curves.

V. INTERACTION OF STATIONARY MACH WAVES WITH A BOUNDARY LAYER

Within the framework of a problem about interaction of acoustic waves with a boundary layer we will consider influence of stationary Mach waves on flow parameters in a supersonic boundary layer. Let's notice that the problem about interaction of Mach waves with a boundary layer arises at treatment of the experimental data received in wind tunnels. The flows in them aren't homogeneous in space because of a roughness of walls of a wind tunnel in particular. Such disturbances of an external flow promote earlier laminar-turbulent transition of a boundary layer at test models. The external Mach waves are described by $\mathbf{q}_{\alpha\beta\lambda} = \mathbf{q_0}\, exp(i(\alpha x + \beta z \mp \lambda y))$ and inside of a boundary layer by vector-functions $\mathbf{q}_{\alpha\beta} = \mathbf{q}\,(y)\, exp\,(i(\alpha x + \beta\, z))$. The vector $\mathbf{q}$ satisfies to the equations (1) with boundary conditions (2, 3) in which $c=0$. Thus calculations of an interaction of Mach waves with a boundary layer are similar to calculations which we did in problem of a sonic waves influence on a boundary layer. The calculations results of $A_{max} = A_{max}\,(\chi)$ are presented in Fig. 9. They are similar to obtained data in the case of interaction between longitudinal sound wave with boundary layer (see Fig. 6), although the maximum values below ($A_{max} < 10^2$ *at* $\chi \approx 40°$).

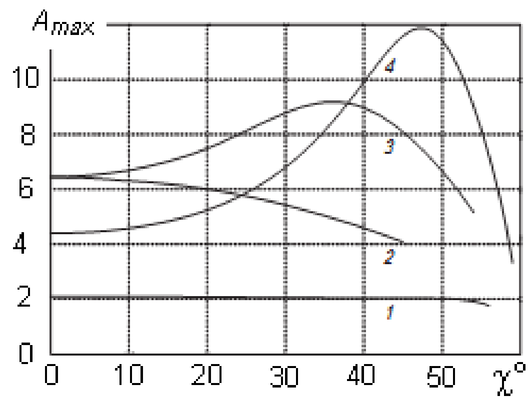

Fig. 9 Stationary amplitudes maximum of mass flow in depending on the orientation angle χ for $M_1=2$, $R_1=500$, $\alpha_1=0.2(1)$, $0.05(2)$, $0.03(3)$, $0.02(4)$

It is possible to see that for the external short-wave nonuniformity ($\alpha_1 \geq 0.05$) interaction is stronger for two demensional (2D) ($\chi = 0°$) waves, while for $\alpha_1 < 0.03$ three-dimensional external perturbations lead to greater nonuniformity inside boundary layer heterogeneity (A_{max} is peak at $\chi \neq 0$).

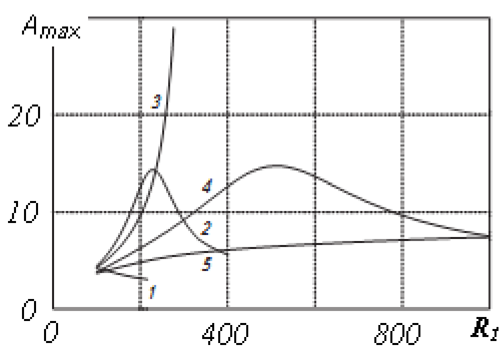

Fig. 10. Stationary disturbances: an amplitude maximum of mass flow A_{max} in depending on the Reynolds number for $M_1=3.5$, $\chi = 0°$, $\alpha_1 = 0.1(1)$, $0.05(2)$, $0.04(3)$. $0.03(4)$, $0.02(5)$.

Dependences $A_{max}=A_{max}(R_1)$ for $\chi = 0°$ and $M_1=3.5$ are shown in Fig. 10. Again there is a critical R_{1cr} at which A_{max} has a peak and hence the nonuniformity intensity within the boundary layer should be the maximum.

VI. NONLINEAR EFFECT OF EXTERNAL LOW-FREQUENCY ACOUSTICS ON EIGEN-OSCILLATIONS IN A SUPERSONIC BOUNDARY LAYER

The problem of nonlinear interaction of acoustic waves and eigen-oscillations in a supersonic boundary layer is directly related to the problem of receptivity of steady flows to external actions. In the linear formulation, the latter problem involves determination the vibrations amplitude in a boundary layer excited external acoustic waves. It should be emphasized that, if the main flow is parallel external monochromatic waves do not excite eigen-oscillations [2]. The problem of excitation of eigen-oscillations by a monochromatic acoustic wave owing to nonparallelism of the main flow was considered in the linear formulation for the first time by Gaponov [12].

In the nonlinear formulation of the problem the external sound wave can be considered as a pumping wave. Eigen-oscillations develop in its field. An example of such a process is the development of disturbances in the boundary layer on a model located in the test section of a usual supersonic wind tunnel. The external acoustic field is generated by a turbulent boundary layer on the wind-tunnel walls. This leads us to the question of the principal possibility of conducting experiments on linear stability theory, since there are no estimates of the admissible level of external disturbances at present. At the same time, a large number of experiments on stability of a supersonic boundary layer was conducted in the T-325 wind tunnel of the Khristianovich Institute of Theoretical and Applied Mechanics of Siberian Division of the Russian Academy of Sciences (ITAM SB RAS) [21]. With respect to linear instability, these results are in agreement with the theory, although the possible influence of acoustics on the development of instability waves is feared. Therefore, apart from the general theoretical importance, the question of nonlinear interaction of external acoustics and eigen-oscillations in the boundary layer is relevant from the viewpoint of applications, apart from the general theoretical importance, which is related to the possibility of modeling unsteady phenomena. In the present paper, we consider the interaction of hydrodynamic waves exponentially decaying at infinity and an external acoustic wave within the framework of weakly nonlinear theory. Acoustic disturbances with the greatest amplitude are located in the low-frequency range, whereas the frequency of disturbances responsible for the transition is greater by an order of magnitude. The objective of the present work is to determine the degree of influence of weakly nonlinear interaction of the waves at these different frequencies.

Amplitude equation [22]. to the triplet of interacting waves can be written as:

$$\frac{dA_1}{dx_1} = -\alpha_{1i}^1 A_1 + k_1(\mathbf{q^2}, \mathbf{q^3}) A_2 A_3 \exp(i\Delta\varphi)$$

$$\frac{dA_2}{dx_1} = -\alpha_{1i}^2 A_2 + k_2(\mathbf{q}^1, \mathbf{q}^{3*}) A_1 A_3^* \exp(i\Delta\varphi)$$

$$\frac{dA_3}{dx_1} = -\alpha_{1i}^3 A_3 + k_3(\mathbf{q}^1, \mathbf{q}^{2*}) A_1 A_2^* \exp(i\Delta\varphi)$$

Here Δ –phase detuning, complex variables are labeled by asterisk. The factors of nonlinear communication k_m characterize a force field generated by interacting waves, $\mathbf{q^m}$ **–** eigen-solutions of the linear problem. Detailed output of amplitude equations can be found in [22].

The calculations were conducted for $M_1 = 2$, $R_1 = 220 \div 640$, fundamental wave frequency $F_1 = (0.250 \div 0.90) \cdot 10^{-4}$, acoustic wave frequency $F_3 = (0.447 \div 0.950) \cdot 10^{-5}$, and angles of the fundamental wave relative to the flow $\chi = 30 \div 60°$. Spanwise wave numbers satisfied condition: $\beta_1 = \beta_2$ and $\beta_3 = 0$.

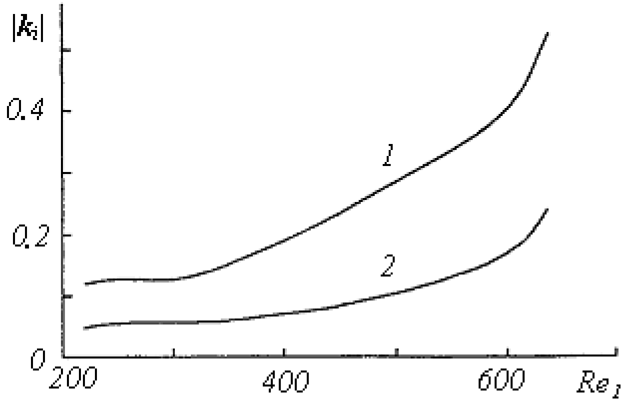

Fig. 11 Dependences of absolute coefficients of the nonlinear connection k_j $(j = 1.2)$ on the Reynolds number: $F_1 = 35 \cdot 10\text{-}6$, $F_3 = 4.7 \cdot 10\text{-}6$, $\chi_1 = 50°$.

The phase velocity of the acoustic wave was synchronized with the phase velocity of the second Tollmien-Schlichting wave, and the detunings Δφ remained small everywhere. As a result of the calculations, we determined the fields of interaction coefficients, wave numbers, velocities, and reflection factors for the acoustic wave as functions of the Reynolds numbers for different frequencies and waves angles χ at $M_1 = 2$ on a flat plate.

It follows from the calculations that, for the frequencies of the fundamental wave $F_1 = 0.35 \cdot l0^{-4}$ and acoustic wave $F_3 = 0.047 \cdot 10^{-4}$, there exists a range of triplet angles (about $50°$) where the growth rate of the interaction coefficients is maximum. Fig. 11 shows absolute values of k_i (i = 1, 2) as functions of Re_1 for the triplet angle $\chi_1 =$ 50 and the above-mentioned wave frequencies. For other values of F_1 and F_3 the maximum growth rates of the interaction coefficients were obtained for the same triplet angle. It is seen from Fig. 11 that the interaction coefficient of the fundamental (first) hydrodynamic wave is approximately twice the interaction coefficient of the second wave. This ratio is also observed for different parameters of the triplet of a given configuration. The calculations for different F_1 at a fixed frequency of the acoustic wave $F_3 = 0.047 \cdot 10^{-4}$ and triplet angle $\chi_1 =$ 50° showed that the maximum values of $k_{1,2}$ are obtained for the frequency $F_1 = 0.5 \cdot 10^{-4}$. The calculation results for these

parameters are plotted in Fig. 12 (the notation is the same as in Fig. 11). It follows from the calculation that the phase velocities of the Tollmien-Schlichting waves increase with increasing Re_1; therefore any triplet synchronized in phase velocities is destroyed at rather high Re_1. This instant is seen in the figure as a drastic decrease in the interaction coefficients.

Thus hydrodynamic waves with $\chi_1 = 50$ and the dimensionless frequency parameter close to $F_1 = 0.5 \cdot l0^{-4}$ receive the maximum effect of acoustics. As already noted all the calculations were conducted for $\beta_3 = 0$. However the maximum linear effect of acoustics is observed at β_3 other than zero (see Fig. 6, 9). Hence further calculations should be performed for $\beta_3 \neq 0$. Finally we note that the interaction coefficients in our case are small. Since the weakly nonlinear action is proportional to the amplitude of the external acoustic

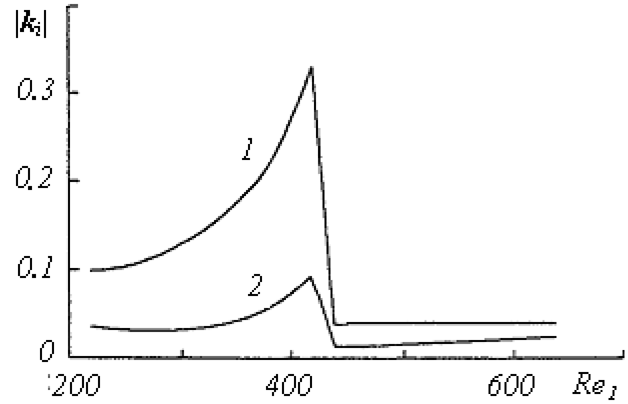

Fig. 12 Dependences of absolute coefficients of the nonlinear connection k_j (j = 1.2), on the Reynolds number: $F_1 = 50 \cdot 10^{-6}$, $F_3 = 4{,}7 \cdot 10^{-6}$, $\chi_1 = 50°$.

wave it is negligibly small for low-turbulence wind tunnels (for example, T-325 at the ITAM SB RAS).

As the Mach number increases the hydrodynamic and acoustic frequencies should become closer. In addition, the levels of acoustic and induced oscillations inside the boundary layer increase with increasing Mach number (see Fig. 5). Therefore the conclusion about the weak effect of acoustics on the degree of amplification of the Tollmien-Schlichting waves cannot be automatically extended to the case of high Math numbers.

REFERENCES

[1] Morkovin M.V. “On transition experiments at moderate supersonic speeds,” J. Aeronaut. Sci., vol. 24, pp. 480-486, 1957.

[2] Gaponov S. A. and Maslov A. A. “Development of Disturbances in Compressible Flows” (in Russian). Nauka (Siberian Division), Novosibirsk, 1980.

[3] Goldshtein M.E. “Aeroacoustics”. McGraw-Hill, 1976.

[4] Ho Chih-Ming H.P. “Perturbed free shear layers.” Ann. Rev. Fluid Mech., vol. 16, pp. 365-424, 1984.

[5] Tarn C.K.W., Burton D.E. “Sound generated by instability waves of supersonic flows. 1. Two-dimensional mixing layers.” J. Fluid. Mech. vol. 138, pp. 249-273, 1984.

[6] Gaponov S.A. and Zheltukhin N. A. “Instability and acoustics of supersonic jets and boundary layers.” In Models of the Mechanics of Inhomogeneous Systems, ed. G.V.Gadiyak (in Russian). Novosibirsk, Inst. Theor. and Appl. Mech. SB RAS 1979, pp. 203-222, 1979.

[7] Mack L.M. “Linear stability theory and the problem of supersonic boundary layer transition. “AlAA Journal, vol. 13, pp. 278-289, 1975.

[8] Gaponov S.A. “Interaction between a supersonic boundary layer and acoustic disturbances.” Fluid Dynamics vol.12, no.6, pp. 858-862, 1977.

[9] Gaponov S.A. “Excitation by sound of Tollmien-Schlichting waves in a supersonic boundary layer.” Fluid Dynamics, vol. 18, no. 3, pp. 384-390, 1983.

[10] Gaponov S.A., Lebiga V.A., Maslov A.A., Pridanov V.G. “Development of disturbances caused by external sound field in a supersonic boundary layer”. IXth All-Union Acoustic Conf., Acoustic Institute, Moscow (in Russian), pp. 49-52,1977.

[11] Maslov A. A. and Semionov N.V. “Excitation of natural oscillations in a boundary layer by an external acoustic field.” Fluid Dynamics.” vol 21, no. 3, pp. 400-404, 1986.

[12] Gaponov S.A. “Excitation of instability waves in the supersonic boundary layer by sound.” In: Nonlinear Instability of Nonparallel Flows, Eds. S.P. Lin, W.R.C. Phillips and D.T. Valentine, Springer-Verlag, pp. 207-212, 1994.

[13] Maslov A.A. and Semionov N.V.” Generation of acoustic oscillations by a supersonic boundary layer.” Izv. SO AN SSSR. Ser. Tech. Nauk (in Russian), no. 7, pp. 58-63, 1987.

[14] **Gaponov S.A.** Interaction of External Vortical and Thermal Disturbances with Boundary Layer //International journal of mechanics. 2008. Vol. 1. No 2. P. 15-19.

[15] Gaponov S.A., V. Ya. LevchenkoV. Ya., Smorodsky B. V Linear Stability of Three-Dimensional Subsonic and Supersonic Swept-Wing Boundary Layers// International Journal of Mechanics, 2008, vol. 2, No 2. P. 25-34.

[16] Dunn D.W. and Lin C.C. “On the stability of the laminar boundary layer in a compressible fluid.” J. Aeronaut. Sci., vol. 1, pp. 455-477, 1955.

[17] Duck P.W. “The response of a laminar boundary layer in supersonic flow to small amplitude progressive waves.” J. Fluid Mech., vol. 219, pp. 423-448, 1990.

[18] Maslov A A. and Semionov N.V. “Structure of artificial disturbances induced by an external acoustic field in a supersonic boundary layer,” Fluid Dynamics, vol. 24 , no. 3, pp. 394-398, 1989.

[19] Gaponov S.A. and Maslov A.A.. “Numerical solution to the problem of the complete stabilization of a supersonic boundary layer.” J. of App. mech. and tech. physics, vol. 13, no. 2, pp, 164-167, 1972.

[20] Kosinov A.D., Maslov A.A. and Semionov N.V. “Experimental investigation of supersonic boundary layer receptivity.” Transitional Boundary Layers in Aeronautics (eds. R.A.W.M.Henkes, J.L. van Ingen). - Amsterdam, pp. 413-420, 1996.

[21] Semionov N. V., Kosinov **A. D.** “An Experimental Study of Receptivity of Supersonic Boundary Layer on a Blunted Plate.” //International Journal of Mechanics. 2008. Vol. 2, No 3. P. 87-95

[22] Gaponov S.A. and Terekhova N. M. Three-Wave Interactions between Disturbancesin the Hypersonic Boundary Layer on Impermeable and Porous Surfaces. //Fluid Dynamics, 2009, Vol. 44, No. 3, pp. 362–371.

[23] Gaponov S. A., Maslennikova I. I., Tyushin V. Yu. Nonlinear effect of external low frequency acoustics on eigen oscillations in a supersonic boundary layer. //Journal of Applied Mechanics and Technical Physics, 1999, Vol. 40, No. 5, pp. 865-870