Invariant properties of cascaded six-pole networks

A. A. Penin

Abstract— The invariant properties of input and output of two-port circuits, established previously, are generalized for a multiport network on the example of six-pole network.

The six-pole network is interpreted as two interconnected two-port circuits because of final resistance of the general wire of these circuits.

For the preset load conductivities, the projective coordinates of a running regime point are introduced concerning of the characteristic regime points, which set the projective coordinate systems on the input and output of six-pole networks. The invariance or preservation of the projective coordinate in these coordinate systems is shown.

The direct and reverse formulas of recalculation of currents and non-uniform coordinates are obtained in the form of fractionally - linear expressions of identical type.

The results allow separating or restoring two sensing signals via input currents of the six-pole circuit or the three-wire line inputs without determination of their transmission parameters.

Keywords— interference of loads, invariant properties, loading characteristic, multi-port network, projective coordinates.

I. Introduction

In the electric circuits theory two-port networks are usually considered, including their cascade connection with fixed value of load conductivity [1]. But the special research of influence of load changes reveals invariant properties of the input and output regime parameters of such networks [2].

It is obvious, when there is the quantity expressed via conductivities or currents, which keeps the value in all the sub circuits or cross-section of circuit in the form of cascaded two-port networks, then it is interesting for the theory and useful in practice.

It is natural to discuss the question about detection of invariant properties of multiport networks on an example of the six-pole network which contains two output loads and two input voltage sources. In this case, the interference of load conductivities is observed. This six-pole network can be interpreted as two interconnected two-port networks. In practice, it can be the manifestation of final resistance of the general wire of two circuits. The three-wire communication line with use, for example, of physical "earth" as the third wire can also be the example of such circuit.

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II. Projective coordinates of a six-pole network output

Let us consider the six-pole network in Fig. 1. This circuit represents, in fact, two two-port networks which are connected among them by the conductivity y_{KL}. Therefore, the interference of load conductivities Y_{L1}, Y_{L2} is observed.

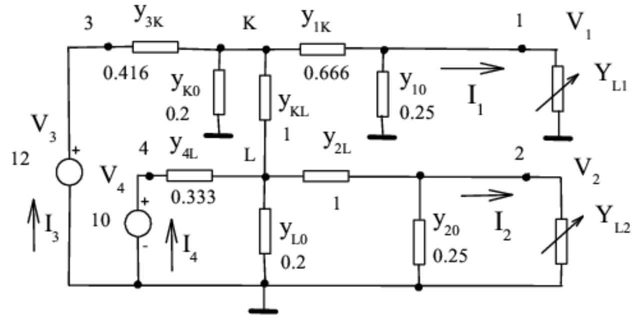

Fig. 1 Six-pole network represents two two-port networks connected via conductivity y_{KL}

This circuit concerning loads represents an active two-port network. As it was shown [3], the family of load characteristics $(I_1, I_2, Y_{L1}) = 0$, $(I_1, I_2, Y_{L2}) = 0$ at change of conductivities Y_{L1}, Y_{L2} is represented by two bunches of straight lines in system of coordinates $(I_1 0 I_2)$ in Fig. 2. For simplification, we consider DC circuit. The bunch center, the point G_1, corresponds to the bunch of the straight lines with the parameter Y_{L2}. Physically, the bunch center corresponds to such regime of the second load Y_{L2} which does not depend on its values. It is carried out for its current $I_2 = 0$ on account of the first load parameters, $Y_{L1} = Y_{L1}^{G1} < 0$, $I_1 = I_1^{G1}$.

The parameters of the center G_2 of the bunch Y_{L1} are expressed similarly, $I_1 = 0$, $Y_{L2} = Y_{l2}^{G2} < 0$, $I_2 = I_2^{G2}$.

The specified parameters of such characteristic regimes are determined either by the matrix of Y - parameters of a six-pole network or by the direct calculation of the circuit, taking into account one of the conditions $I_2 = 0$ and $I_1 = 0$.

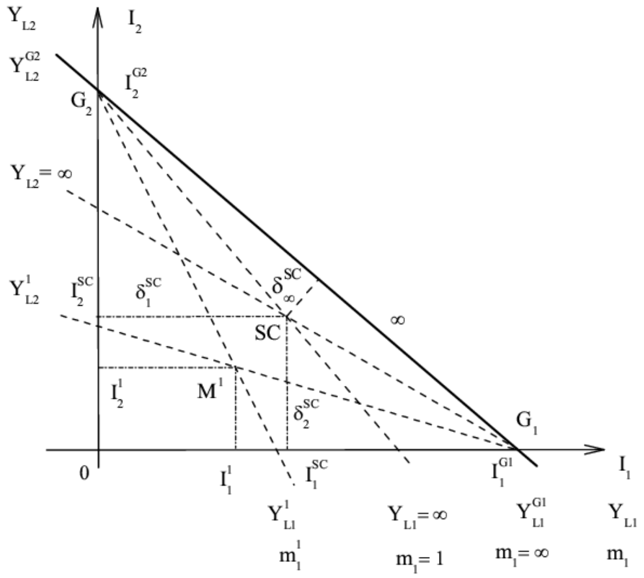

Fig. 2 Two bunches of load characteristics with the parameters Y_{L1}, Y_{L2}.

Next, we use the idea of projective coordinates of the point of a running regime [3]. Let the initial or running regime corresponds to the point M^1 which is set by the values of conductivities Y_{L1}^1, Y_{L2}^1 and currents I_1^1, I_2^1. In addition, this point is defined by projective non-uniform coordinates m_1^1, m_2^1 and homogeneous coordinates $\xi_1^1, \xi_2^1, \xi_\infty^1$ which are set by the coordinate triangle $G_1\ 0\ G_2$ and the unit point SC [4]. The unit point corresponds to the short circuit regime, the point 0 is the beginning of coordinates as open circuit regime, and the straight line $G_1\ G_2$ is the infinitely remote straight line ∞.

The non-uniform projective coordinate m_1^1 is set by a cross ratio of four points

$$m_1^1 = (0\ Y_{L1}^1\ \infty\ Y_{L1}^{G1}) = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} \div \frac{\infty - 0}{\infty - Y_{L1}^{G1}} = = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} . \quad (1)$$

The points $Y_{L1}=0,\ Y_{L1}^1 = Y_{L1}^{G1}$ correspond to extreme or base values. The point $Y_{L1}=\infty$ is the unit point. The values of m_1 are shown in Fig.2. For the point $Y_{L1}^1 = Y_{L1}^{G1}$, coordinate $m_1 = \infty$ defines the sense of line of infinity $G_1\ G_2$. The cross ratio for the projective coordinate m_2^1 is expressed similarly.

In addition, the homogeneous projective coordinates ξ_1, ξ_2, ξ_∞ set the non-uniform coordinates as follows

$$m_1 = \frac{\rho\xi_1}{\rho\xi_\infty},\ m_2 = \frac{\rho\xi_2}{\rho\xi_\infty}, \quad (2)$$

where ρ is the coefficient of proportionality.

The homogeneous coordinates are defined as the ratio of distances of the points M^1, SC to the sides of the coordinate triangle. Taking into account the side equations, we obtain

$$\rho\,\xi_1^1 = \frac{\delta_1^1}{\delta_1^{SC}} = \frac{I_1^1}{I_1^{SC}},\ \rho\,\xi_2^1 = \frac{\delta_2^1}{\delta_2^{SC}} = \frac{I_2^1}{I_2^{SC}},$$

$$\rho\,\xi_\infty^1 = \frac{\delta_\infty^1}{\delta_\infty^{SC}} \quad (3)$$

$$\mu_\infty \delta_\infty^{SC} = \left(\frac{I_1^{SC}}{I_1^{G1}} + \frac{I_2^{SC}}{I_2^{G2}} - 1 \right),$$

$$\mu_\infty \delta_\infty^1 = \left(\frac{I_1^1}{I_1^{G1}} + \frac{I_2^1}{I_2^{G2}} - 1 \right), \quad (4)$$

where $\mu_\infty = \sqrt{\frac{1}{\left(I_1^{G1}\right)^2} + \frac{1}{\left(I_2^{G2}\right)^2}}$ is the normalizing factor.

The homogeneous projective coordinates (3) have a matrix form

$$\begin{bmatrix} \rho\,\xi_1 \\ \rho\,\xi_2 \\ \rho\,\xi_\infty \end{bmatrix} = [\mathbf{C}] \cdot \begin{bmatrix} I_1 \\ I_2 \\ 1 \end{bmatrix}, \quad (5)$$

$$[\mathbf{C}] = \begin{bmatrix} \frac{1}{I_1^{SC}} & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 \\ \frac{1}{I_1^{G1}\mu_\infty\delta_\infty^{SC}} & \frac{1}{I_2^{G2}\mu_\infty\delta_\infty^{SC}} & -\frac{1}{\mu_\infty\delta_\infty^{SC}} \end{bmatrix} .$$

From here, the expressions (2) of non-uniform coordinates assume a convenient form

$$m_1 = \frac{\frac{1}{I_1^{SC}} I_1}{\frac{I_1}{I_1^{G1}\mu_\infty\delta_\infty^{SC}} + \frac{I_2}{I_2^{G2}\mu_\infty\delta_\infty^{SC}} - \frac{1}{\mu_\infty\delta_\infty^{SC}}},$$

$$m_2 = \frac{\frac{1}{I_2^{SC}} I_2}{\frac{I_1}{I_1^{G1}\mu_\infty\delta_\infty^{SC}} + \frac{I_2}{I_2^{G2}\mu_\infty\delta_\infty^{SC}} - \frac{1}{\mu_\infty\delta_\infty^{SC}}}. \quad (6)$$

The inverse transformation

$$\begin{bmatrix} \rho I_1 \\ \rho I_2 \\ \rho 1 \end{bmatrix} = [\mathbf{C}]^{-1} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_\infty \end{bmatrix}, \quad (7)$$

$$[\mathbf{C}]^{-1} = \begin{bmatrix} I_1^{SC} & 0 & 0 \\ 0 & I_2^{SC} & 0 \\ \frac{I_1^{SC}}{I_1^{G1}} & \frac{I_2^{SC}}{I_2^{G2}} & -\mu_\infty\delta_\infty^{SC} \end{bmatrix},$$

where the components of current vector define homogeneous coordinates of a current.
From here, we find the current

$$I_1 = \frac{\rho I_1}{\rho 1} = \frac{I_1^{SC} \cdot m_1}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2 - \mu_\infty\delta_\infty^{SC}},$$

$$I_2 = \frac{I_2^{SC} \cdot m_2}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2 - \mu_\infty\delta_\infty^{SC}}. \quad (8)$$

III. Projective Coordinates of a Six-Pole Network Input

Let us superpose the system of coordinates $(I_3\, 0\, I_4)$ of input currents with the system of coordinates $(I_1\, 0\, I_2)$ in Fig. 3. Then any point with coordinates (I_1, I_2) corresponds to a point with coordinates (I_3, I_4).

In the terms of geometry, the projective transformation takes place which transfers points of plane (I_1, I_2) into the points of plane (I_3, I_4). Therefore, the coordinate triangle $G_1\, 0\, G_2$, unit point SC and running regime point M^1 correspond to the triangle $\overline{G_1}\ \overline{0}\ \overline{G_2}$, point $\overline{SC}$, and point $\overline{M}^1$, as it is shown by arrows in Fig. 3.

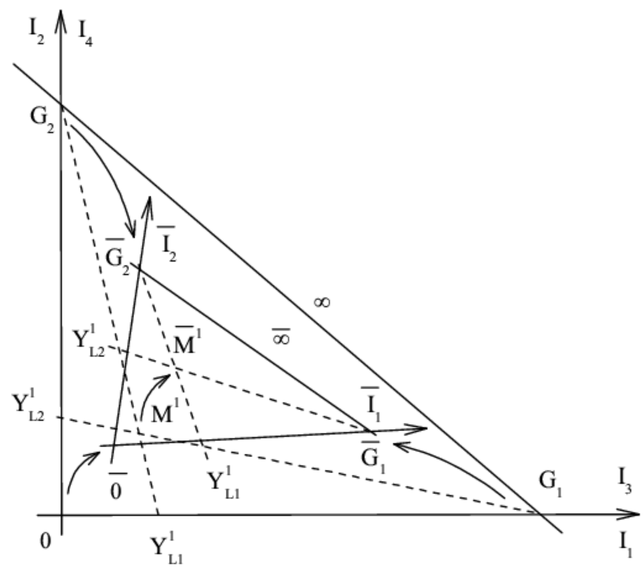

Fig. 3 Projective transformation transfers points of the plane (I_1, I_2) into points of the plane (I_3, I_4).

Then, the axes of currents I_1, I_2 correspond to the axes $\bar{I}_1, \bar{I}_2$. In addition, two bunches of the characteristics $(I_1, I_2, Y_{L1}) = 0$, $(I_1, I_2, Y_{L2}) = 0$ correspond to two bunches of the characteristics $(I_3, I_4, Y_{L1}) = 0$, $(I_3, I_4, Y_{L2}) = 0$ with the centers in the points $\overline{G}_2$, $\overline{G}_1$. In the electric circuit theory, the linear property of currents in different branches of a circuit at change of resistance in any other branch is known. It just also corresponds to projective nature of such property. Thus, the point $\overline{M}^1$ is set by other values of currents, the currents I_3^1, I_4^1. Besides, this point is defined by projective non-uniform and homogeneous coordinates which are set by the coordinate triangle $\overline{G_1}\ \overline{0}\ \overline{G_2}$ and unit point $\overline{SC}$.

The property of projective transformations shows that these coordinates of point $\overline{M}^1$ are equal to the coordinates of point M^1, as the points $M^1, \overline{M}^1$ are set by the same loads Y_{L1}^1, Y_{L2}^1. Therefore, this property gives required invariant relations between the input and output currents.

For finding of the point $\overline{M}^1$ projective coordinates, it is necessary to obtain the equations of sides of a coordinate triangle. According to Fig. 4, the normalized equation of the side $\overline{0}\overline{G}_1$ or the axis $\bar{I}_1$ looks like

$$\frac{I_4}{I_4^{OC}-\bar{k}_1 I_3^{OC}}-\frac{\bar{k}_1 I_3}{I_4^{OC}-\bar{k}_1 I_3^{OC}}-1=0,$$

$$\bar{k}_1 = tg\alpha_1 = \frac{I_4^{G1}-I_4^{OC}}{I_3^{G1}-I_3^{OC}},$$

where $\bar{k}_1$ is the angular coefficient or slope ratio.

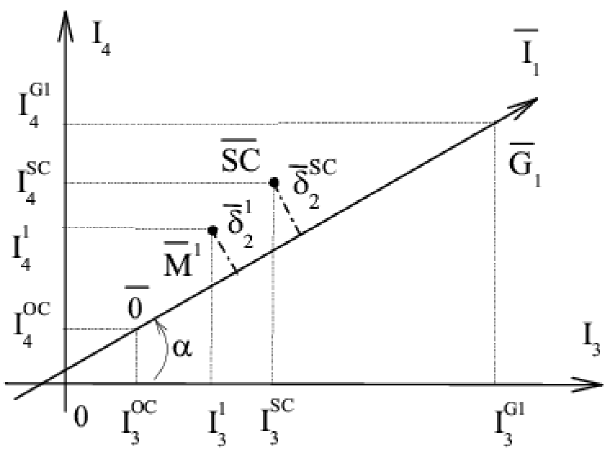

Fig. 4 Point's $\overline{M}^1$, $\overline{SC}$ distances to the axis $\bar{I}_1$

Then, the point's $\overline{M}^1$ distance $\bar{\delta}_2^1$ to the axis $\bar{I}_1$ is defined by expression

$$\bar{\mu}_2\bar{\delta}_2^1 = \frac{I_4^1}{I_4^{OC}-\bar{k}_1 I_3^{OC}}-\frac{\bar{k}_1 I_3^1}{I_4^{OC}-\bar{k}_1 I_3^{OC}}-1,$$

$$\bar{\mu}_2 = \sqrt{\left(\frac{1}{I_4^{OC}-\bar{k}_1 I_3^{OC}}\right)^2+\left(\frac{\bar{k}_1}{I_4^{OC}-\bar{k}_1 I_3^{OC}}\right)^2},$$

where $\bar{\mu}_2$ is the normalizing factor.

The point's $\overline{SC}$ distance $\bar{\delta}_2^{SC}$ to the axis $\bar{I}_1$ is

$$\bar{\mu}_2\bar{\delta}_2^{SC} = \frac{I_4^{SC}}{I_4^{OC}-\bar{k}_1 I_3^{OC}}-\frac{\bar{k}_1 I_3^{SC}}{I_4^{OC}-\bar{k}_1 I_3^{OC}}-1.$$

Similarly, the axis $\bar{I}_2$ equation is

$$\frac{I_4}{\bar{k}_2 I_3^{OC}-I_4^{OC}}-\frac{\bar{k}_2 I_3}{\bar{k}_2 I_3^{OC}-I_4^{OC}}+1=0,$$

$$\bar{k}_2 = tg\alpha_2 = \frac{I_4^{G2}-I_4^{OC}}{I_3^{G2}-I_3^{OC}}.$$

Then, the point's $\overline{M}^1$ distance $\bar{\delta}_1^1$ to the axis $\bar{I}_2$ is

$$\bar{\mu}_1\bar{\delta}_1^1 = \frac{I_4^1}{\bar{k}_2 I_3^{OC}-I_4^{OC}}-\frac{\bar{k}_2 I_3^1}{\bar{k}_2 I_3^{OC}-I_4^{OC}}+1,$$

$$\bar{\mu}_1 = \sqrt{\left(\frac{1}{\bar{k}_2 I_3^{OC}-I_4^{OC}}\right)^2+\left(\frac{\bar{k}_2}{\bar{k}_2 I_3^{OC}-I_4^{OC}}\right)^2}.$$

The point's $\overline{SC}$ distance $\bar{\delta}_1^{SC}$ to the axis $\bar{I}_2$ is

$$\bar{\mu}_1\bar{\delta}_1^{SC} = \frac{I_4^{SC}}{\bar{k}_2 I_3^{OC}-I_4^{OC}}-\frac{\bar{k}_2 I_3^{SC}}{\bar{k}_2 I_3^{OC}-I_4^{OC}}+1.$$

Similarly, the infinitely remote straight line $\overline{\infty}$ equation is

$$\frac{I_4}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}+\frac{\bar{k}_\infty I_3}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}-1=0,\quad \bar{k}_\infty = \frac{I_4^{G2}-I_4^{G1}}{I_3^{G1}-I_3^{G2}}.$$

The point's $\overline{M}^1$ distance $\bar{\delta}_\infty^1$ to the line $\overline{\infty}$ is

$$\bar{\mu}_\infty\bar{\delta}_\infty^1 = \frac{I_4^1}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}+\frac{\bar{k}_\infty I_3^1}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}-1,$$

$$\bar{\mu}_\infty = \sqrt{\left(\frac{1}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}\right)^2+\left(\frac{\bar{k}_\infty}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}\right)^2}.$$

The point's $\overline{SC}$ distance $\bar{\delta}_\infty^{SC}$ to the line $\overline{\infty}$ is

$$\bar{\mu}_\infty\bar{\delta}_\infty^{SC} = \frac{I_4^{SC}}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}+\frac{\bar{k}_\infty I_3^{SC}}{I_4^{G1}+\bar{k}_\infty I_3^{G1}}-1.$$

The homogeneous projective coordinates are

$$\rho\xi_1^1 = \frac{\bar{\delta}_1^1}{\bar{\delta}_1^{SC}} = -\frac{\bar{k}_2}{(\bar{k}_2 I_3^{OC}-I_4^{OC})\bar{\mu}_1\bar{\delta}_1^{SC}} I_3^1 + \frac{1}{(\bar{k}_2 I_3^{OC}-I_4^{OC})\bar{\mu}_1\bar{\delta}_1^{SC}} I_4^1 + \frac{1}{\bar{\mu}_1\bar{\delta}_1^{SC}},$$

$$\rho\xi_2^1 = \frac{\bar{\delta}_2^1}{\bar{\delta}_2^{SC}} = -\frac{\bar{k}_1}{(I_4^{OC}-\bar{k}_1 I_3^{OC})\bar{\mu}_2\bar{\delta}_2^{SC}} I_3^1 + \frac{1}{(I_4^{OC}-\bar{k}_1 I_3^{OC})\bar{\mu}_2\bar{\delta}_2^{SC}} I_4^1 - \frac{1}{\bar{\mu}_2\bar{\delta}_2^{SC}},$$

$$\rho\xi_{\infty}^{1} = \frac{\bar{\delta}_{\infty}^{1}}{\bar{\delta}_{\infty}^{SC}} = \frac{\bar{k}_{\infty}}{(I_4^{G1} + \bar{k}_{\infty} I_3^{G1})\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}} I_3^1 + $$
$$+ \frac{1}{(I_4^{G1} + \bar{k}_{\infty} I_3^{G1})\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}} I_4^1 - \frac{1}{\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}} .$$

and have a matrix form

$$\begin{bmatrix} \rho\,\xi_1 \\ \rho\,\xi_2 \\ \rho\,\xi_{\infty} \end{bmatrix} = [\overline{\mathbf{C}}] \cdot \begin{bmatrix} I_3 \\ I_4 \\ 1 \end{bmatrix},$$

$$[\overline{\mathbf{C}}] = \begin{bmatrix} -\bar{C}_{11} & \dfrac{\bar{C}_{11}}{\bar{k}_2} & \dfrac{1}{\bar{\mu}_1\,\bar{\delta}_1^{SC}} \\ -\bar{C}_{21} & \dfrac{\bar{C}_{21}}{\bar{k}_1} & -\dfrac{1}{\bar{\mu}_2\,\bar{\delta}_2^{SC}} \\ \bar{C}_{31} & \dfrac{\bar{C}_{31}}{\bar{k}_{\infty}} & -\dfrac{1}{\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}} \end{bmatrix}, \qquad (9)$$

where the constituents of matrix are

$$\bar{C}_{11} = \frac{\bar{k}_2}{(\bar{k}_2 I_3^{OC} - I_4^{OC})\bar{\mu}_1\,\bar{\delta}_1^{SC}},$$
$$\bar{C}_{21} = \frac{\bar{k}_1}{(I_4^{OC} - \bar{k}_1 I_3^{OC})\bar{\mu}_2\,\bar{\delta}_2^{SC}},$$
$$\bar{C}_{31} = \frac{\bar{k}_{\infty}}{(I_4^{G1} + \bar{k}_{\infty} I_3^{G1})\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}}$$

From here, the non-uniform coordinates have the form similar to (6)

$$m_1 = \frac{-\bar{C}_{11} I_3 + \dfrac{\bar{C}_{11}}{\bar{k}_2} I_4 + \dfrac{1}{\bar{\mu}_1\,\bar{\delta}_1^{SC}}}{\bar{C}_{31} I_3 + \dfrac{\bar{C}_{31}}{\bar{k}_{\infty}} I_4 - \dfrac{1}{\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}}},$$

$$m_2 = \frac{-\bar{C}_{21} I_3 + \dfrac{\bar{C}_{21}}{\bar{k}_1} I_4 - \dfrac{1}{\bar{\mu}_2\,\bar{\delta}_2^{SC}}}{\bar{C}_{31} I_3 + \dfrac{\bar{C}_{31}}{\bar{k}_{\infty}} I_4 - \dfrac{1}{\bar{\mu}_{\infty}\bar{\delta}_{\infty}^{SC}}} \qquad (10)$$

The obtained expressions have a general appearance in comparison to (6) because of nonorthogonal coordinates $\bar{I}_1\,\bar{0}\,\bar{I}_2$. Thus, in practice, the characteristic values of input and output currents (vertexes of coordinate triangles) and the characteristic load values are precomputed or preprogrammed by the calculation or testing the six-pole network. Further, using the running values of input currents, we find or, more precisely, restore the values of non-uniform coordinates (10) and given load conductivities according to the expressions $Y_{H1}(m_1), Y_{H2}(m_2)$ which are reverse to expression (1).

The formulated algorithm represents practical interest for transfer of two sensing signals via an unstable six-pole network or a three-wire line; it is by analogy to the signal transmission via a two-port network [2].

Two cascaded six-pole networks. Let us consider the cascaded six-pole networks in the Fig. 5.

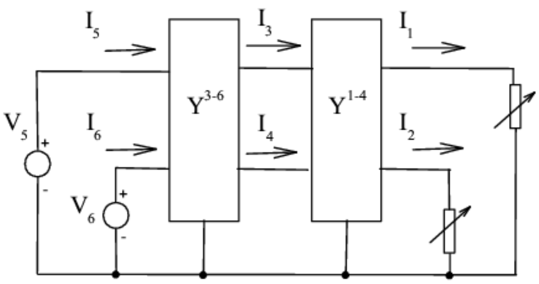

Fig. 5 Cascaded six-pole networks.

Similarly, we superpose the system of coordinates $(I_5\,0\,I_6)$ of input currents of the first six-pole network with the matrix Y^{3-6} of Y parameters with the systems of coordinates $(I_3\,0\,I_4)$, $(I_1\,0\,I_2)$. Then, the projective transformation, which transfers the plane (I_1, I_2) points to the plane (I_5, I_6) points, takes place. Therefore, the coordinate triangle $G_1\,0\,G_2$ corresponds to the triangle $\tilde{G}_1\,\tilde{0}\,\tilde{G}_2$ in Fig.6.

Also, the unit point SC, the running regime point M^1 will correspond to the points $\widetilde{SC}$, $\tilde{M}^1$. Moreover, two bunches of characteristics $(I_1, I_2, Y_{L1}) = 0$, $(I_1, I_2, Y_{L2}) = 0$ correspond to two bunches of characteristics $(I_5, I_6, Y_{L1}) = 0$, $(I_5, I_6, Y_{L2}) = 0$ with the point centers $\tilde{G}_2, \tilde{G}_1$.

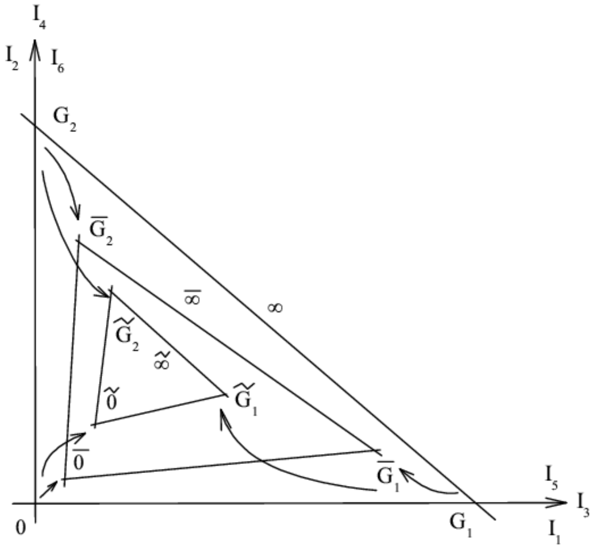

Fig. 6 Mapping of the coordinate triangles.

Too, the point $\widetilde{M}^1$ is defined by the projective non-uniform and homogeneous coordinates which are set by the coordinate triangle $\widetilde{G}_1\, \widetilde{0}\, \widetilde{G}_2$ and the unit point $\widetilde{S}\widetilde{C}$. These projective coordinates of the point $\widetilde{M}^1$ are equal to the projective coordinates of the points M^1, $\overline{M}^1$ according to the property of projective transformations. Thus, the invariant relationships between the input and output currents of cascaded six-pole networks take place.

The projective coordinates of the point $\widetilde{M}^1$ are obtained similarly to projective coordinates of the point $\overline{M}^1$. For this purpose, it is necessary to form the sides equations of the triangle $\widetilde{G}_1\, \widetilde{0}\, \widetilde{G}_2$.

IV. Invariance of Regime Changes

Besides the invariance of projective coordinates, for example, in the form of non-uniform coordinates (1), the invariance of changes of these non-uniform coordinates on account of changes of load conductivities takes place. Let the subsequent regime corresponds to the point M^2 with the parameters of loads Y_{L1}^2, Y_{L2}^2, currents I_1^2, I_2^2, and non-uniform coordinates m_1^2, m_2^2.

The subsequent currents according to (7), (8)

$$I_1^2 = \frac{I_1^{SC} \cdot m_1^2}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1^2 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2^2 - \mu_\infty \delta_\infty^{SC}},$$

$$I_2^2 = \frac{I_2^{SC} \cdot m_2^2}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1^2 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2^2 - \mu_\infty \delta_\infty^{SC}}. \tag{11}$$

Further, we are using the results [3]. Let us present the subsequent values of non-uniform coordinates via the initial values m_1^1, m_2^1 and their changes m_1^{21}, m_2^{21}

$$m_1^2 = m_1^{21} \cdot m_1^1 = m_1^{21} \cdot \frac{\xi_1^1}{\xi_\infty^1},$$

$$m_2^2 = m_2^{21} \cdot m_2^1 = m_2^{21} \cdot \frac{\xi_2^1}{\xi_\infty^1}. \tag{12}$$

Then, the expressions (11) are

$$I_1^2 = \frac{I_1^{SC} \cdot m_1^{21} \cdot \xi_1^1}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1^{21} \cdot \xi_1^1 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2^{21} \cdot \xi_2^1 - \mu_\infty \delta_\infty^{SC} \xi_\infty^1},$$

$$I_2^2 = \frac{I_2^{SC} \cdot m_2^{21} \cdot \xi_2^1}{\frac{I_1^{SC}}{I_1^{G1}} \cdot m_1^{21} \cdot \xi_1^1 + \frac{I_2^{SC}}{I_2^{G2}} \cdot m_2^{21} \cdot \xi_2^1 - \mu_\infty \delta_\infty^{SC} \xi_\infty^1},$$

or takes the matrix form

$$\begin{bmatrix} \rho I_1^2 \\ \rho I_2^2 \\ \rho 1 \end{bmatrix} = [C]^{-1} \cdot \begin{bmatrix} m_1^{21} & 0 & 0 \\ 0 & m_2^{21} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \xi_1^1 \\ \xi_2^1 \\ \xi_\infty^1 \end{bmatrix} = [C]^{-1} \cdot [m^{21}] \cdot \begin{bmatrix} \xi_1^1 \\ \xi_2^1 \\ \xi_\infty^1 \end{bmatrix}.$$

Taking into account (5), we obtain

$$\begin{bmatrix} \rho I_1^2 \\ \rho I_2^2 \\ \rho 1 \end{bmatrix} = [C]^{-1} \cdot [m^{21}] \cdot [C] \cdot \begin{bmatrix} I_1^1 \\ I_2^1 \\ 1 \end{bmatrix} = [M^{21}] \cdot \begin{bmatrix} I_1^1 \\ I_2^1 \\ 1 \end{bmatrix}, \tag{13}$$

where the matrix of current changes is

$$[M^{21}]=\begin{bmatrix} M_{11}^{21} & 0 & 0 \\ 0 & M_{22}^{21} & 0 \\ M_{31}^{21} & M_{32}^{21} & 1 \end{bmatrix}=\begin{bmatrix} m_1^{21} & 0 & 0 \\ 0 & m_2^{21} & 0 \\ \dfrac{m_1^{21}-1}{I_1^{G1}} & \dfrac{m_2^{21}-1}{I_2^{G2}} & 1 \end{bmatrix}. \quad (14)$$

The obtained relationship (13) allows carrying out the recalculation of the load currents for the preset value of load changes in the form of non-uniform coordinate changes.

These changes of non-uniform coordinates are also true for input currents. Therefore, it is possible to obtain similar relationships for recalculation of the input currents.

The reverse transformation to the (9) is

$$\begin{bmatrix} \rho I_3 \\ \rho I_4 \\ \rho 1 \end{bmatrix}=[\overline{C}]^{-1}\cdot\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_\infty \end{bmatrix}, \quad (15)$$

where

$$[\overline{\mathbf{C}}]^{-1}=\begin{bmatrix} \overline{C}_{11}^{-1} & \overline{C}_{12}^{-1} & \overline{C}_{13}^{-1} \\ \overline{C}_{11}^{-1}\dfrac{I_4^{G1}}{I_3^{G1}} & \overline{C}_{12}^{-1}\dfrac{I_4^{G2}}{I_3^{G2}} & \overline{C}_{13}^{-1}\dfrac{I_4^{OC}}{I_3^{OC}} \\ \overline{C}_{11}^{-1}\dfrac{1}{I_3^{G1}} & \overline{C}_{12}^{-1}\dfrac{1}{I_3^{G2}} & \overline{C}_{13}^{-1}\dfrac{1}{I_3^{OC}} \end{bmatrix},$$

and the constituents of matrix are

$$\overline{C}_{11}^{-1}=\frac{\overline{k}_2 I_3^{OC}-I_4^{OC}}{(\overline{k}_1-\overline{k}_2)(I_3^{OC}-I_3^{G1})}I_3^{G1}\overline{\mu}_1\overline{\delta}_1^{SC},$$

$$\overline{C}_{12}^{-1}=-\frac{\overline{k}_1 I_3^{OC}-I_4^{OC}}{(\overline{k}_1-\overline{k}_2)(I_3^{OC}-I_3^{G2})}I_3^{G2}\overline{\mu}_2\overline{\delta}_2^{SC},$$

$$\overline{C}_{13}^{-1}=\frac{\overline{k}_\infty I_3^{G1}+I_4^{G1}}{(\overline{k}_1+\overline{k}_\infty)(I_3^{OC}-I_3^{G1})}I_3^{OC}\overline{\mu}_\infty\overline{\delta}_\infty^{SC}.$$

From here, similarly to (11), we pass to the subsequent currents

$$I_3^2=\frac{\overline{C}_{11}^{-1}m_1^2+\overline{C}_{12}^{-1}m_2^2+\overline{C}_{13}^{-1}}{\overline{C}_{11}^{-1}\dfrac{1}{I_3^{G1}}m_1^2+\overline{C}_{12}^{-1}\dfrac{1}{I_3^{G2}}m_2^2+\overline{C}_{13}^{-1}\dfrac{1}{I_3^{OC}}},$$

$$I_4^2=\frac{\overline{C}_{11}^{-1}\dfrac{I_4^{G1}}{I_3^{G1}}m_1^2+\overline{C}_{12}^{-1}\dfrac{I_4^{G2}}{I_3^{G2}}m_2^2+\overline{C}_{13}^{-1}\dfrac{I_4^{OC}}{I_3^{OC}}}{\overline{C}_{11}^{-1}\dfrac{1}{I_3^{G1}}m_1^2+\overline{C}_{12}^{-1}\dfrac{1}{I_3^{G2}}m_2^2+\overline{C}_{13}^{-1}\dfrac{1}{I_3^{OC}}}. \quad (16)$$

The obtained expressions have a general appearance in comparison to (11) because of nonorthogonal coordinates. Convenience of reverse to each other the expressions (10), (16) consists in their identical form. It is being reached on account of change of variables; we replace the load conductivities by the non-uniform projective coordinates, and currents are already non-uniform coordinates.

Further, we use the changes of non-uniform coordinates according to (12), and obtain the matrix expression

$$\begin{bmatrix} \rho I_3^2 \\ \rho I_4^2 \\ \rho 1 \end{bmatrix}=[\overline{\mathbf{C}}]^{-1}\cdot\begin{bmatrix} m_1^{21} & 0 & 0 \\ 0 & m_2^{21} & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} \xi_1^1 \\ \xi_2^1 \\ \xi_\infty^1 \end{bmatrix}=$$

$$=[\overline{\mathbf{C}}]^{-1}\cdot[\mathbf{m}^{21}]\cdot\begin{bmatrix} \xi_1^1 \\ \xi_2^1 \\ \xi_\infty^1 \end{bmatrix}.$$

Using the transformation (9), we obtain

$$\begin{bmatrix} \rho I_3^2 \\ \rho I_4^2 \\ \rho 1 \end{bmatrix}=[\overline{\mathbf{C}}]^{-1}\cdot[\mathbf{m}^{21}]\cdot[\overline{\mathbf{C}}]\cdot\begin{bmatrix} I_3^1 \\ I_4^1 \\ 1 \end{bmatrix}=[\overline{\mathbf{M}}^{21}]\cdot\begin{bmatrix} I_3^1 \\ I_4^1 \\ 1 \end{bmatrix}. \quad (17)$$

If to carry out calculations, we receive the matrix $[\overline{\mathbf{M}}^{21}]$. The matrix $[\overline{\mathbf{M}}^{21}]$ of change of currents carries a general view in comparison to (14).

Using the transformations (13), (17), we find the subsequent currents

$$I_1^2=\frac{\rho I_1^2}{\rho 1}=\frac{m_1^{21}\cdot I_1^1}{\dfrac{m_1^{21}-1}{I_1^{G1}}\cdot I_1^1+\dfrac{m_2^{21}-1}{I_2^{G2}}\cdot I_2^1+1},$$

$$I_2^2=\frac{\rho I_2^2}{\rho 1}, \quad (18)$$

$$I_3^2 = \frac{\rho I_3^2}{\rho 1} = \frac{\overline{M}_{11}^{21} \cdot I_3^1 + \overline{M}_{12}^{21} \cdot I_4^1 + \overline{M}_{13}^{21}}{\overline{M}_{31}^{21} \cdot I_3^1 + \overline{M}_{32}^{21} \cdot I_4^1 + \overline{M}_{33}^{21}},$$

$$I_4^2 = \frac{\rho I_4^2}{\rho 1}. \tag{19}$$

The calculation shows the equal values of the denominators of the expressions (18), (19).

Example. The active network in Fig.1 is described by the following system of the equations

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{12} & -Y_{22} & Y_{23} & Y_{24} \\ -Y_{13} & -Y_{23} & Y_{33} & -Y_{34} \\ -Y_{14} & -Y_{24} & -Y_{34} & Y_{44} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = [\mathbf{Y}][\mathbf{U}],$$

$$[\mathbf{Y}] = \begin{bmatrix} -0.6813 & 0.1393 & 0.147 & 0.0464 \\ 0.1393 & -0.7727 & 0.087 & 0.159 \\ -0.147 & -0.087 & 0.3247 & -0.029 \\ -0.0464 & -0.159 & -0.029 & 0.2803 \end{bmatrix}.$$

Hereinafter, the dimensions of values are not specified for simplifying of record.

For the output of multiport we have the next results.

The parameters of the bunch centers G_1, G_2,

$$I_1^{G1} = 15.1172, \; Y_{L1}^{G1} = -0.7991;$$
$$I_2^{G2} = 15, \; Y_{L2}^{G2} = -0.9375.$$

The short circuit currents, $I_1^{SC} = 2.229, \; I_2^{SC} = 2.636$.

The parameters of the initial regime, the point M^1

$$Y_{L1}^1 = 0.5, \; Y_{L2}^1 = 0.333, \; I_1^1 = 1.101, \; I_2^1 = 0.8868.$$

The non-uniform projective coordinates (1)

$$m_1^1 = \frac{Y_{L1}^1}{Y_{L1}^1 - Y_{L1}^{G1}} = \frac{0.5}{0.5 + 0.7991} = 0.3848,$$

$$m_2^1 = \frac{Y_{L2}^1}{Y_{L2}^1 - Y_{L2}^{G2}} = \frac{0.333}{0.333 + 0.9375} = 0.2622.$$

The homogeneous projective coordinates (3), (4)

$$\rho\xi_1^1 = \frac{I_1^1}{I_1^{SC}} = \frac{1.101}{2.229} = 0.4939,$$

$$\rho\xi_2^1 = \frac{I_2^1}{I_2^{SC}} = 0.3364, \quad \rho\xi_\infty^1 = \frac{\delta_\infty^1}{\delta_\infty^{SC}} = 1.2825,$$

where

$$\mu_\infty \delta_\infty^1 = \left(\frac{1.101}{15.1172} + \frac{0.8868}{15} - 1 \right) = -0.868,$$

$$\mu_\infty \delta_\infty^{SC} = \left(\frac{2.229}{15.1172} + \frac{2.636}{15} - 1 \right) = -0.6768.$$

Let us check up the values of the non-uniform projective coordinates (2),

$$m_1^1 = \frac{\rho\xi_1}{\rho\xi_\infty} = \frac{0.4939}{1.2825} = 0.3851,$$

$$m_2^1 = \frac{\rho\xi_2}{\rho\xi_\infty} = \frac{0.3364}{1.2825} = 0.2622.$$

Matrix $[\mathbf{C}]$ according to (5)

$$[\mathbf{C}] = \begin{bmatrix} \frac{1}{2.229} & 0 & 0 \\ 0 & \frac{1}{2.6358} & 0 \\ \frac{-1}{15.1172 \cdot 0.6768} & \frac{-1}{15 \cdot 0.6768} & \frac{1}{0.6768} \end{bmatrix}.$$

Let us check up the value of the non-uniform projective coordinate (6)

$$m_1^1 = \frac{\frac{1}{2.229} 1.101}{-\frac{1.101}{15.1172 \cdot 0.6768} - \frac{0.8868}{15 \cdot 0.6768} + \frac{1}{0.6768}} = \frac{0.494}{1.286}.$$

The inverse transformation matrix (7)

$$[\mathbf{C}]^{-1} = \begin{bmatrix} 2.229 & 0 & 0 \\ 0 & 2.6358 & 0 \\ 0.1474 & 0.1757 & 0.6768 \end{bmatrix}.$$

Then, the current (8)

$$I_1^1 = \frac{2.229 \cdot 0.3858}{0.1474 \cdot 0.3858 + 0.1757 \cdot 0.2623 + 0.6768} = \frac{0.86}{0.7797} = 1.101.$$

For the input of multiport we have the next results.

The parameters of the bunch centers $\overline{G}_1$, $\overline{G}_2$

$$I_3^{G1} = 6.3886, \; I_4^{G1} = 3.333, \; I_3^{G2} = 5, \; I_4^{G2} = 5.$$

The currents corresponding to the short circuit,

$$I_3^{SC} = 3.607, \; I_4^{SC} = 2.455.$$

The currents corresponding to the open circuit,
$I_3^{OC} = 2.639,\ I_4^{OC} = 1.602$.

The parameters of the initial regime, the point $\overline{M}^1$
$I_3^1 = 3.051,\ I_4^1 = 1.929$.

The normalized equation of the axis $\overline{I}_1$

$$\frac{I_4}{I_4^{OC} - \bar{k}_1 I_3^{OC}} - \frac{\bar{k}_1 I_3}{I_4^{OC} - \bar{k}_1 I_3^{OC}} - 1 = ,$$

$$= \frac{I_4}{0.3835} - \frac{I_3}{0.8305} - 1 = 0$$

$$\bar{k}_1 = \frac{I_4^{G1} - I_4^{OC}}{I_3^{G1} - I_3^{OC}} = \frac{3.333 - 1.602}{6.3886 - 2.639} = 0.4617 .$$

The point $\overline{M}^1$ distance to the axis $\overline{I}_1$

$$\bar{\mu}_2 \bar{\delta}_2^1 = \frac{1.929}{0.3835} - \frac{3.051}{0.8305} - 1 = 0.3566 ,$$

$$\bar{\mu}_2 = \sqrt{\left(\frac{1}{0.3835}\right)^2 + \left(\frac{1}{0.8305}\right)^2} = 2.8721 .$$

The point $\overline{SC}$ distance to the axis $\overline{I}_1$

$$\bar{\mu}_2 \bar{\delta}_2^{SC} = \frac{2.455}{0.3835} - \frac{3.606}{0.8305} - 1 = 1.0576 .$$

The normalized equation of the axis $\overline{I}_2$

$$\frac{I_4}{\bar{k}_2 I_3^{OC} - I_4^{OC}} - \frac{\bar{k}_2 I_3}{\bar{k}_2 I_3^{OC} - I_4^{OC}} + 1 = ,$$

$$= \frac{I_4}{2.1961} - \frac{I_3}{1.5258} + 1 = 0$$

$$\bar{k}_2 = \frac{I_4^{G2} - I_4^{OC}}{I_3^{G2} - I_3^{OC}} = \frac{5 - 1.602}{5 - 2.639} = 1.4392 .$$

The point $\overline{M}^1$ distance to the axis $\overline{I}_2$

$$\bar{\mu}_1 \bar{\delta}_1^1 = \frac{1.929}{2.196} - \frac{3.051}{1.5258} + 1 = -0.1216 ,$$

$$\bar{\mu}_1 = \sqrt{\left(\frac{1}{2.1961}\right)^2 + \left(\frac{1}{1.5258}\right)^2} = 0.798 .$$

The point $\overline{SC}$ distance to the axis $\overline{I}_2$

$$\bar{\mu}_1 \bar{\delta}_1^{SC} = \frac{2.455}{2.1961} - \frac{3.606}{1.5258} + 1 = -0.2458 .$$

The normalized equation of the infinitely remote line $\overline{\infty}$

$$\frac{I_4}{I_4^{G1} + \bar{k}_\infty I_3^{G1}} + \frac{\bar{k}_\infty I_3}{I_4^{G1} + \bar{k}_\infty I_3^{G1}} - 1 = ,$$

$$= \frac{I_4}{11} + \frac{I_3}{9.166} - 1 = 0$$

$$\bar{k}_\infty = \frac{I_4^{G2} - I_4^{G1}}{I_3^{G1} - I_3^{G2}} = \frac{5 - 3.333}{6.3886 - 5} = 1.2 .$$

The point $\overline{M}^1$ distance to the line $\overline{\infty}$

$$\bar{\mu}_\infty \bar{\delta}_\infty^1 = \frac{1.929}{11} + \frac{3.051}{9.166} - 1 = -0.4918 ,$$

$$\bar{\mu}_\infty = \sqrt{\left(\frac{1}{11}\right)^2 + \left(\frac{1}{9.166}\right)^2} = 0.142 .$$

The point $\overline{SC}$ distance to the line $\overline{\infty}$

$$\bar{\mu}_\infty \bar{\delta}_\infty^{SC} = \frac{2.455}{11} + \frac{3.606}{9.166} - 1 = -0.3835 .$$

The homogeneous projective coordinates have the same values

$$\rho \xi_1^1 = \frac{\bar{\delta}_1^1}{\bar{\delta}_1^{SC}} = \frac{0.1216}{0.2458} = 0.4947 ,$$

$$\rho \xi_2^1 = \frac{\bar{\delta}_2^1}{\bar{\delta}_2^{SC}} = \frac{0.3566}{1.057} = 0.3364 ,$$

$$\rho \xi_\infty^1 = \frac{\bar{\delta}_\infty^1}{\bar{\delta}_\infty^{SC}} = \frac{0.4918}{0.3835} = 1.2823 .$$

The matrix $[\overline{\mathbf{C}}]$ according to (9)

$$[\overline{\mathbf{C}}] = \begin{bmatrix} \frac{1}{0.2458 \cdot 1.5258} & \frac{1}{-0.2458 \cdot 2.1961} & \frac{1}{-0.2458} \\ \frac{1}{-1.057 \cdot 0.8305} & \frac{1}{1.057 \cdot 0.3834} & \frac{1}{-1.057} \\ -\frac{1}{0.3835 \cdot 9.166} & -\frac{1}{0.3835 \cdot 11} & \frac{1}{0.3835} \end{bmatrix} =$$

$$= \begin{bmatrix} 2.666 & -1.852 & -4.067 \\ -1.1383 & 2.4653 & -0.9454 \\ -0.2844 & -0.237 & 2.607 \end{bmatrix}$$

.

Let us carry out the requalification of the value of the non-uniform projective coordinate according to (10)

$$m_1^1 = \frac{2.666 \cdot 3.051 - 1.8522 \cdot 1.929 - 4.067}{-0.2844 \cdot 3.051 - 0.237 \cdot 1.929 + 2.607} = \frac{0.495}{1.282} .$$

For the invariance of regime changes we have the next results.

The parameters of the subsequent regime, the point M^2,
$Y_{L1}^2 = 1,\ Y_{L2}^2 = 1,\ I_1^2 = 1.459,\ I_2^2 = 1.602,$
$I_3^2 = 3.253,\ I_4^2 = 2.132.$

$$m_1^2 = \frac{1}{1+0.7991} = 0.555,\ m_2^2 = 0.516.$$

The non-uniform projective coordinate changes (12)

$$m_1^{21} = m_1^2 \div m_1^1 = 0.555 \div 0.3848 = 1.442,$$
$$m_2^{21} = m_2^2 \div m_2^1 = 0.516 \div 0.2622 = 1.968.$$

The matrix of the current changes (14)

$$[\mathbf{M}^{21}] = \begin{bmatrix} 1.442 & 0 & 0 \\ 0 & 1.968 & 0 \\ 0.0292 & 0.0645 & 1 \end{bmatrix}.$$

The matrix of the reverse transformation (15)

$$[\overline{\mathbf{C}}]^{-1} = \begin{bmatrix} 0.9422 & 0.8789 & 1.7868 \\ 0.4915 & 0.8791 & 1.0847 \\ 0.1474 & 0.1757 & 0.6768 \end{bmatrix}.$$

The matrix of the change of the input current I_3 (17)

$$[\overline{\mathbf{M}}^{21}] = \begin{bmatrix} 1.1463 & 1.3225 & -2.503 \\ \overline{M}_{21}^{21} & \overline{M}_{22}^{21} & \overline{M}_{23}^{21} \\ -0.01927 & 0.2982 & 0.5728 \end{bmatrix}.$$

Let us check up the recalculation of the output current I_1 (13), and input current I_3 (17)

$$I_1^2 = \frac{\rho I_1^2}{\rho 1} = \frac{1.442 \cdot 1.101}{0.0292 \cdot 1.101 + 0.0645 \cdot 0.8868 + 1},$$
$$== \frac{1.5876}{1.0893} = 1.457$$

$$I_3^2 = \frac{1.1463 \cdot 3.051 + 1.3225 \cdot 1.929 - 2.503}{-0.01927 \cdot 3.051 + 0.2982 \cdot 1.929 + 0.5728} =$$
$$= \frac{3.545}{1.0893} = 3.254.$$

V. Conclusions

1. For the preset load conductivities, the projective coordinates of a running regime point are introduced concerning of the characteristic regime points, which set the projective coordinate systems for input and output of the six-pole networks.
2. The invariance or preservation of the projective coordinates of running regimes in these coordinate systems is shown.
3. The direct and reverse formulas of recalculation of currents and non-uniform coordinates are obtained in the form of fractionally - linear expressions of identical type.
4. The results allow separating or restoring two sensing signals via input currents of the six-pole circuit or the three-wire line inputs without determination of their transmission parameters.
5. The offered approach can be generalized on AC circuits.

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