Comparison of two methods for determination of instantaneous state of dynamical system with LCLC circuit

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Abstract—The paper deals with chosen analytical and numerical methods which make possible to estimate instantaneous state of dynamical system in any time instant. Analytical model of the LCTLC filter uses Laplace-Carson transformation with complex operator \( p \). The method described in the paper using transient component separation makes it possible to use steady state- and transient components to generate of total time waveforms of chosen output state variables or other quantities. The steady state component is created using response of AC input voltage during the first one half-period. Worked-out simulation experiment results are compared to common numerical solution done in Matlab/Simulink environment using discrete type of dynamical model of the filter system which is modelled and analyzed by second method for determination of instantaneous state of discrete dynamical system Theoretical analysis, computer simulation, and experimental verification are given in the paper.

Keywords—Circuit analysis, modelling and simulation, Laplace-Carson transform, state-space equation, non-harmonic supply, linear discrete control

I. INTRODUCTION

Conceptions of resonant converters greatly expanded into the various spheres of industry and consumer applications. Generally known switched mode power supplies as well as for power converters, to target the highest switching frequency together with the highest efficiency that is possible. If will be increased both phenomes together, simultaneously the power density increases. In order to reach the satisfactory electrical parameters and behaviour of converter, it is necessary to utilize new concepts of its main circuit [1]. In every industrial and consumer application became energy efficiency, power density and harmonic current emissions main qualitative indicators of power semiconductor converters. LCTLC resonant converter belongs between generally known topologies used in various applications. Basically, the multi-element topologies are based on serial and parallel connection of accumulation components. Their combinations along with high frequency transformer creates varies topologies with individual specific properties [2]. Analysing of resonant systems may help to improve the design and final properties of devices.

II. THEORETICAL ASPECTS OF USED METHODS

Generally, we can use analytical and/or numerical solution for the analysing and investigating of dynamical system. Consequently, we have to create either continuous or discrete dynamical model of the system. The first one is well known in the state-space form [1],[5]

\[
\frac{dx(t)}{dt} = A \cdot x(t) + B \cdot u(t)
\]

where \( x(t) \) is vector of state variables, \( u(t) \) exciting vector, \( A \) and \( B \) are system matrices. Vector of output quantities is expressed as

\[
y(t) = C \cdot x(t) + D \cdot u(t)
\]

By numerical integration of (1) we obtain discrete form of dynamical model

\[
x_{k+1} = F_x \cdot x_k + G_u \cdot u_k
\]

More detailed description of discrete model is given in B. subchapter later.

A. Analytical method using steady-state and transient components under non-harmonic supply

Using method of operator calculus Laplace or Laplace-Carson (L-C), respectively, transform [6],[7]

\[
U_1(p) = \frac{U_1^{1/2}(p)}{1 + e^{-\pi T/2}}
\]

where operator voltage during one half-period is in Laplace-Carson

\[
U_1^{1/2}(p) = U(1 - e^{-\pi T/2}) \quad \text{or} \quad u_1^{1/2}(s) = U \frac{(1 - e^{-\pi T/2})}{s}
\]

using Laplace transform.

Generally, the current response on such a voltage consists of steady state and transient components [8]

\[
F(p) = U \frac{(1 - e^{-\pi T/2}) A(p)}{1 + e^{-\pi T/2} B(p)}
\]

The response of any state variable during one half-period of transient phenomenon can be obtained and evaluated just by roots of polynomial of denominator \( B(p) \); and \( T_s = T/2 \).
Determination of matrices discretized by integration step exciting vector, using of:

\[ C = 1 : N \]

Now, after inverse L-C transformation the steady state [3],[5],[9]

Runge-Kutta-, Taylor expansion methods of (1) are giving

As mentioned above the numerical integration using, Euler-,

components similarly as in the DC linear circuit [5],[8].

Such methodology makes it possible to separate both

 transient component during one half-period

\[ f_{trans}^{T/2}(p) = U \left( 1 - e^{-pT/2} \right) A(p) B(p) \]

Steady state component during one half-period can be simply obtained by their subtracting

\[ f_{steady}^{T/2}(p) = f_{trans}^{T/2}(p) - f_{trans}^{T/2}(p) \]

Now, after inverse L-C transformation the steady state component for one half-period

\[ f_{steady}(t) = f_{trans}^{T/2}(t) - f_{trans}^{T/2}(t) \]

And

\[ f(t) = f_{trans}(t) \pm \left[ f_{trans}^{T/2}(t) - f_{trans}^{T/2}(t) \right] \]

Such methodology makes it possible to separate both components similarly as in the DC linear circuit [5],[8].

B. Method for determination of instantaneous state of discrete dynamical system

As mentioned above the numerical integration using, Euler-, Runge-Kutta-, Taylor expansion methods of (1) are giving [3],[5],[9]

\[ x_{k+1} = F_{\Delta} x_k + G_{\Delta} u_k \]

where \( x(k) \) is vector of state variables in discrete form, \( u(k) \) exciting vector, \( k \) is order of calculation, \( F \) and \( G \) are system matrices discretized by integration step \( \Delta \).

Determination of \( F_{\Delta}, G_{\Delta} \) matrices is possible to provide by using of:

- analytical method (suitable for low order system);
- numerical methods depending on their type:
  - Euler direct explicit method (\( h \equiv \Delta \)):
    \[ x(h) = (1 + hA)x_0 + (hB)u_0 \]
    \[ \rightarrow F_{\Delta} = 1 + hA, \quad G_{\Delta} = hB. \] (11a)
  - Euler indirect implicit method:
    \[ x(h) = \text{inv}(1 - hA)x_0 + \text{inv}(1 - hA)hB.u_0 \]
    \[ \rightarrow F_{\Delta} = \text{inv}(1 - hA), \quad G_{\Delta} = \text{inv}(1 - hA)hB. \]
  - Taylor expansion:
    \[ x(h) = e^{Ah} + \int_0^h e^{A(h-t)}B.u_0dt \ldots \]
    \[ = \sum_{i=0}^{n} \frac{(Ah)^i}{i!}x_0 + h \sum_{i=0}^{n} \frac{(Ah)^i}{(i+1)!}B.u_0 \]
    \[ \rightarrow F_{\Delta} = \sum_{i=0}^{n} \frac{(Ah)^i}{i!}A = h \sum_{i=0}^{n} \frac{(Ah)^i}{(i+1)!}B. \]

- Z-transformation method;
- method of experiment when state variables \( x(h) \) and \( F_{\Delta}, G_{\Delta} \) can be obtain at discrete time \( \Delta \equiv h \).

Then, by gradual sovereign and generalization /mathematical induction/ we get [xx],[xx]:

\[ x_1 = F_{\Delta}^1 x_0 + F_{\Delta}^1 G_{\Delta} u_0 = G_{\Delta} u_0 \] (12a)

\[ x_2 = F_{\Delta}^2 x_0 + F_{\Delta}^1 G_{\Delta} u_0 + F_{\Delta}^1 G_{\Delta} u_1 \]

\[ x_3 = F_{\Delta}^3 x_0 + F_{\Delta}^2 G_{\Delta} u_0 + F_{\Delta}^1 G_{\Delta} u_1 + F_{\Delta}^0 G_{\Delta} u_2 \]

\[ x_k = F_{\Delta}^k x_0 + F_{\Delta}^{k-1} G_{\Delta} u_0 + F_{\Delta}^{k-2} G_{\Delta} u_1 + F_{\Delta}^{k-1} G_{\Delta} u_{k-1} + F_{\Delta}^k G_{\Delta} u_k \] (12b)

Thus

\[ x_k = (F_{\Delta})^k x_0 + G_{\Delta} \sum_{i=k-1}^{0} F_{\Delta}^1 \{ u_{k-(i+1)} \} \] (13)

III. MODELING AND SIMULATION OF LCLC FILTER CIRCUIT

Using A method of steady-state and transient components

Considering, the basic scheme of LCTLC inverter Fig. 1 and equivalent scheme of LCLC filter circuit, Fig. 2 [12].

\[ \text{Fig. 1 Basic schematic of LCTLC inverter} \]

\[ \text{Fig. 2 Equivalent scheme of LCTLC inverter with HF transformer} \]
\[ l_{11}(p) - U_{c2}(p) \left[ \left( \frac{1}{R_2} + \frac{1}{pL_2} + pC_2 \right) \right] \]  

(16b)

Since \( U_{c2}(p) \equiv U_2(p) \) and \( l_{11} + pL_1 + \frac{1}{pC_1} = Z_1(p) \), 
\[
\left[ \frac{1}{R_2} + \frac{1}{pL_2} + pC_2 \right] = Y_2(p) \]

it can be written

\[
U_2(p) = U_1(p) \left( \frac{1}{1 + Z_1(p)Y_2(p)} = U_1(p)F(p) \right) \]

(17)

where \( \frac{1}{Z_1(p)Y_2(p)} \) is the operator transfer function of the system.

Introducing

\[ \frac{1}{Z_1(p)Y_2(p)} \]

For \( r = 0.05; \quad \omega = 1 \) control vector \( c = \left[ 1; 1.1; 3.0525; 1.1; 1 \right] \) (ideal elements, nominal load) the roots are

\[
p_{1,2} = \left(-a \pm \omega \cdot i \right) \quad \text{and} \quad p_{3,4} = \left(-x \pm \omega \cdot i \right) \]

(20)

For \( r = 0.05; \quad z = 1 \) control vector \( c = \left[ 1; 1.1; 3.0525; 1.1; 1 \right] \) (ideal elements, zero load) the roots are

\[
p_{1,2} = \left(-0.3887 \pm 1.5027i \right) \quad \text{and} \quad p_{3,4} = \left(-0.1613 \pm 0.6238i \right) \]

(20a)

\[ \gg \quad \text{control vector} \quad c = \left[ 1; 0.0; 3.00; 0.0; 1 \right] \quad \text{(ideal elements, zero load)} \]

the roots are

\[
p_{1,2} = \left(-0.0000 \pm 1.6180i \right) \quad \text{and} \quad p_{3,4} = \left(-0.0000 \pm 0.6180i \right) \]

(20b)

Based on the transfer function obtained from operator form eq.(19) the bode diagram in Matlab environment has been created, Fig. 3.
It is important that \( \omega_{1,2} \) and \( \omega_{3,4} \) are frequencies when input impedance features zero values, and voltage transfer by infinite values.

The inverse Laplace transform can be worked out [1],[8]

\[
H(s) = \frac{A(p_k)}{p_k B'(p_k)} e^{p_k t}
\]

since \( A(0)/B(0) = 0 \).

\[
\sum_{k=1}^{N} U_k \left( 1 - e^{-p_k t/2} \right) / \left( 1 + e^{-p_k t/2} \right) = \sum_{k=1}^{N} U_k \left( A(p_k) / p_k B'(p_k) \right) e^{p_k t}
\]

Then

\[
u_{\text{steady}}(t) = \sum_{k=1}^{N} U_k \left( A(p_k) / p_k B'(p_k) \right) e^{p_k t}
\]

Regarding the complex conjugated roots the members of \( A(p_k) / p_k B'(p_k) \) will give \( |R_k| e^{i\rho_k} \)

and, similarly

\[
1 - e^{-p_k t/2} \quad \text{will give} \quad |R_k| e^{j\rho_k}
\]

so

\[
\sum_{k=1}^{N} U_k \left( 1 - e^{-p_k t/2} \right) / \left( 1 + e^{-p_k t/2} \right) = \sum_{k=1}^{N} U_k \left( A(p_k) / p_k B'(p_k) \right) e^{p_k t}
\]

It is possible to show

\[
|R_1| e^{i\rho_1 e^{(-a+j)b} \alpha_{\omega}} + |R_2| e^{i\rho_2 e^{(-a-j)b} \alpha_{\omega}} = 2|R_1| e^{-a \alpha_{\omega} \cos(\rho_1 + b \omega_t + \alpha_{\omega})} \quad \text{etc. (26)}
\]

and

\[
|R_3| e^{i\rho_3 e^{(x+j)y} \alpha_{\omega}} + |R_4| e^{i\rho_4 e^{(x-j)y} \alpha_{\omega}} = 2|R_3| e^{x \alpha_{\omega} \cos(\rho_3 + y \omega_t)} \quad \text{etc. (27)}
\]

\[
1 - e^{-p_T/2} = |T_1| e^{j\tau_1},
\]

\[
1 + e^{-p_T/2} = |T_2| e^{j\tau_2}, \quad \text{etc. (29)}
\]

And also

\[
|T_1| e^{j\tau_1 e^{(-a+j)b} \alpha_{\omega}} + |T_2| e^{j\tau_2 e^{(-a-j)b} \alpha_{\omega}} = 2|T_1| e^{-a \alpha_{\omega} \cos(\tau_1 + b \omega_t)} \quad \text{(30)}
\]

\[
|T_3| e^{j\tau_3 e^{(x+j)y} \alpha_{\omega}} + |T_4| e^{j\tau_4 e^{(x-j)y} \alpha_{\omega}} = 2|T_3| e^{x \alpha_{\omega} \cos(\tau_3 + y \omega_t)} \quad \text{(31)}
\]

Then, for output voltage

\[
u_2(t) = 2U[(R_1) e^{-a \alpha_{\omega} \cos(\rho_1 + b \omega_t)} + (R_3) e^{-x \alpha_{\omega} \cos(\rho_3 + y \omega_t)}] \quad \text{(32)}
\]

Steady state component for one half-period

\[
u_{\text{steady}}(t) = u_{2\text{steady}}(t) - u_{\text{trans}(t)} \quad \text{(34)}
\]

And

\[
u_2(t) = u_{\text{trans}(t)} \pm [u_{2\text{steady}}(t) - u_{\text{trans}(t)}] \quad \text{(35)}
\]

Results of simulation are shown in Fig. 4a,b.

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**Fig. 4 Time waveforms of the model for real parameters a) \( Z_{\text{load}}=100 \% \) (nominal value) and b) ideal parasite-less with \( Z_{\text{load}}=0 \% \)**

Important, when ideal elements, zero load i.e. \( r = g = 0 \); \( z = 0 \) >> \( c = [1; 0.0; 3.0; 0.0; 1] \) then \( e^{-a \alpha_{\omega} t} = 0; e^{-x \alpha_{\omega} t} = 0 \) thus \( u_{\text{trans}(t)} \) will never be zero.
Using B method for determination of instantaneous state of
discrete dynamical system

For calculation \( x_k = (F_\Delta)^k x_0 + G_\Delta \sum_{\ell=0}^{k-1} F_\Delta^{-\ell} u_{k-(\ell+1)} \) at
any time instant \( t = k \Delta \) we need, at first, to know \( F_\Delta, G_\Delta \) and
\( x_0, u_k \).

Then continuous dynamic state space model yields the for
following system equations [19]

\[
\begin{align*}
\frac{d i_{l1}(t)}{dt} &= -\frac{R_1}{L_1} i_{l1}(t) - \frac{1}{L_1} u_{c1}(t) - \frac{1}{L_1} u_{c2}(t) + \frac{1}{L_1} u(t) \quad (36a) \\
\frac{d i_{l2}(t)}{dt} &= \frac{1}{L_2} u_{c2}(t) \quad (36b) \\
\frac{d u_{c1}(t)}{dt} &= \frac{1}{C_1} i_{l1}(t) \quad (36c) \\
\frac{d u_{c2}(t)}{dt} &= \frac{1}{C_1} i_{l2}(t) - \left( \frac{1}{R_2} + \frac{1}{R_1} \right) u_{c2}(t) \quad (36d)
\end{align*}
\]

Thus for parameters designed by [13]

\( L_1 = L_2 = L = 0.1, C_1 = C_2 = C = 5 \times 10^{-3}, R_1 = 1, r \equiv R_1 = 0.01, g \equiv 1 \)
\( R_2 = 0.01 \)

and elements of \( A \) and \( B \) matrices:

\[
\begin{align*}
& a_{11} = \frac{r}{L} = -0.1; \quad a_{12} = 0; \quad a_{13} = -\frac{1}{L} = -10; \quad a_{14} = -\frac{1}{L} = -10; \\
& a_{21} = a_{22} = a_{23} = 0; \quad a_{24} = \frac{1}{L} = 10; \\
& a_{31} = \frac{1}{C} = 200; \quad a_{32} = a_{33} = a_{34} = 0; \\
& a_{41} = \frac{1}{C_1}; \quad a_{42} = -\frac{1}{C_2}; \quad a_{43} = 0; \quad a_{44} = -\left( g + \frac{1}{R_1} \right) \frac{1}{C} = -\frac{1}{RC} = -202; \\
& b_{11} = \frac{1}{L} = 10; \quad b_{12} + b_{44} = 0.
\end{align*}
\]

The \( F_\Delta \) and \( G_\Delta \) will be

\[
F_\Delta = [E - \Delta \, A]; \quad G_\Delta = \Delta \, B \quad (37)
\]

and \( u_k = [u_k; 0; 0; 0]^T \) where \( u_k = \sqrt{2} U_{DC} \sin \left[ \text{fix} \left( \frac{\Delta}{T} k \right) \frac{\pi}{2} \right] \) and \( \pi \) is unit matrix and \( \Delta \equiv 10^{-4} \).

Then

\[
F_\Delta = 
\begin{bmatrix}
1.0000 & 0 & 0.0010 & 0.0010 \\
0 & 1.0000 & 0 & -0.0010 \\
-0.0200 & 0 & 1.0000 & 0 \\
-0.0200 & 0.0200 & 0 & 1.0201
\end{bmatrix}
\]

\[
G_\Delta = 
\begin{bmatrix}
0.0010 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

If \( k \) goes from 0 to 2, 160 (6 \( T \)) the result will be as in Fig. 5.

By comparison of voltage waveforms in Fig. 5 and Fig. 4a we can conclude that are in very good agreement.

Finally, there is shown the experimental verification at steady state.

By comparison of voltage waveform to those in Fig. 5 and Fig. 4a at steady state we can conclude again that is in good agreement.

IV. CONCLUSION

The method of steady-state and transient components (A), and method of determination of instantaneous state of discrete dynamical system (B) have been introduced. Both methods make possible to calculate the system response to input non-harmonic voltage signal at any continuous or discrete time instant (\( t \) or \( k \), \( \Delta \) respectively) regardless if input voltage is continuous or discrete impulse one. Comparison of simulation results of both methods is possible from Fig. 4a and Fig. 5 – the results are practically identical. Main difference between methods can be seen in used approaches. Since the method A deals with evaluation of the roots of denominator of transfer voltage function, the method (B) works directly - without the evaluation. If we need to know the poles due to behaviour of the discrete impulse system, so the inverse Z-transform using residua lemma should be used [10], [14]. The mentioned in the paper methods make it possible to solve any dynamical state of the system such as step changes of the load (switch on/off), step changes of the switching frequency (+/-), short circuiting of the filter circuit, etc. Analyzing of resonant filter system may help to improve the design and final properties of devices.
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