A Colored Petri Net for the France-Paris Metro

A. Spiteri Staines

***Abstract*—** This work tries to explain how colored Petri nets (CPNs) can be useful for modeling a real life complex networked transport system. Petri nets are expressive graphical formalisms that are useful for modeling discrete system behavior. Petri nets are well documented and have an extensive amount of well researched areas and interesting applications. Colored Petri nets (CPNs) are very useful extensions to traditional Petri nets, increasing their expressivity and modeling power. An underground metro system is a good case of complexity. The real French Paris Metro system that forms part of the RER is considered and analyzed with part of it being modeled as a colored Petri net or (CPN). The CPN used is as close as possible to the real life system and is fully executable. Different scenarios and results are obtainable from this apart from its usefulness for validation and verification. This paper thus assesses the suitability of colored Petri nets for modeling real world high complexity transport systems. The findings are briefly discussed.

***Keywords*—** Directed Graphs, Colored Petri nets (CPNs), Modeling, Metro System, Transport Systems

I. Introduction

PLACE transition nets defined over 3 decades ago have given rise to numerous extensions for applying them to different systems of all types. One very important extension to Petri nets is definitely the introduction of token coloring and arc and transition inscriptions based on high level specification languages such as ML. These Petri nets are known as Colored Petri nets (CPNs). Consequently because of the special arc properties and firing rules, the CPN is capable of modeling many different types of scenarios ranging from simple to complex instruction sets and programmed dynamic behavior.

Obviously Petri nets have already found their use for modeling train systems [1]-[3],[6]-[7]. This particular area is highly applicable to Petri net modeling, as Petri nets are particularly oriented towards discrete event modeling and synchronous control [4]-[5]. Their use for train modeling is one of their proposed uses.

Traditionally, in existing papers, the focus is more on solving theoretically a problem, so that actual solutions given for train system simulation do not really consider practical issues like the system complexity and other points that need to be managed.

On many occasions two/three views are possible for modeling these types of systems, such as a microscopic view of a limited part of the system is considered and analyzed in great detail [11]. Another view is the macroscopic view which tries to analyze the model from the top level functionality, ignoring lower level details. Both approaches are not always suitable, as some details are always lost. A better possibility could be to employ a meoscopic view. This should fall somewhere in between. The latter approach is adopted here although the CPNs support all three views simultaneously, because they allow different levels of detail and also decomposition.

The possibility of simulation and reachability analysis that can be carried out by many software tools is very important for system modeling. This feature implies the possibility of carrying out controlled experiments and is readily understood even by system stakeholders that do not have a proper technical background. Because of the development in technology, software and modeling tools, today it is possible to model complex systems using colored Petri nets. The models can be conveniently constructed with some patience on a tool like the CPNtool and full simulation and analysis is conductible. CPNTool seems to be very useful for modeling actor scenarios [8]-[10],[12]-[13]. CPNs can be considered to be a formal method for expressing concurrency, non-determinism, choice and other discrete system properties at various levels. The complex net system can also be visualized either in part or in full.

An underground metro organization system is a collection of trains and rails that exist at different levels. This is a highly organized network with many temporal constraints. The system needs to satisfy certain conditions like timings, stopovers, sequential ordering of journeys, min – max waiting time. Normally mathematical models and other graph based methods can be used to model a metro system. Most of these methods are possibly non visual. Petri nets have extensive use for discrete event modeling apart from other uses in communications and software expression. Petri nets offer the advantage of being executable.

II. Motivation

The motivation in this work is to show the usefulness of CPNs in modeling transport systems such as a complex underground metro system from a practical perspective. The model needs to be executable. System functional requirements define how a system is expected to behave. Traditionally different notations and formalisms are available for modeling. However these are more representational rather than executable. Using CPNs for modeling a real metro system is quite challenging, at the same time from one central model it

Manuscript received Feb 23, 2012. Anthony (Tony) Spiteri Staines, is with the Department of Information Systems, Faculty of ICT, University of Malta, (corresponding phone: 00356-21373402,e-mail: toni_staines@yahoo.com)

is possible to create many other models that would focus on very specific issues. Computer simulation of a real system provides for an environment for learning more about the real system. The model that will be obtained can be used for testing and analysis. With the use of proper simulation tools many practical applications are possible.

Esthetically driven perception of a metro network, both from its geographical layout or schematic plan or diagram, can be used to create or design a Petri net model layout. Obviously the schematic diagram presents a complex topological layout of the entire metro system together with the stops. This is not necessarily drawn to scale. This layout serves as the initial inspirational factor for creating a Petri net model.

Despite some parts of the model being quite confusing it is suitable for redesigning or re-conceptualizing it as a Petri net.

The flow from one station to another is understood easily through the underlying structure. The configuration or the layout of this network offers the possibility for abstracting the real distributed system execution whilst exploring many different routes and combinations. These still occur and are valid in the Petri net model which will allow for in-depth exploration. For modeling these routes the use of tokens, places and transitions including time dimensionality are indispensable. So creating the Petri net is a way of conceptualizing or abstracting the metro system.

III. Problem Definition

A. *Brief Description of the Paris-Metro System*

The Paris Metro System or Metro'de Paris was selected for this work for various reasons such as its large size, complexity and distribution.

The Paris Metro System can be considered a very important symbol of the Paris city. It has an elaborate architectural system. There are about 14 main lines that use different levels underground. There are over 300 stops which can also be considered to be stations. A large number of stops overlap other lines to facilitate transfer.

This metro system is considered to be one of the densest and most complex metro systems in the world. Lines are numbered 1..14. These are identified on the appropriate travel maps by their various colors and names. Lines are bi-directional. i.e. travel takes place in two directions for each line.

Most places in Paris are covered by a metro station and some are within close walking distances of 400m. The distribution of the lines create overlap, intersection and meeting points at very important locations. These stop, over points or metro points can have many or a few metro lines. This is clearly seen in the metro maps and other synoptic maps showing the areas of validity for travel. These are divided into a number of zones.

The underground station layouts allow travelers to shift quickly from one line to another. It can be estimated that the trips between stations take from 60 secs to 2 mins max. So normally travelers wait for a max time of 2 mins to board the metro train.

Two possibilities are given. I.e. i) class reduction and ii) structural reduction. As previously stated, the best Petri nets used to obtain graphs have to be structurally reduced and limited.

B. *Problem Statement*

The aim of this work is to develop a comprehensive working model that represents / parallels characteristics of the system in the real world, as closely as possible. The model must present the actual topological features of the metro system. It should be suitable for execution and analysis. Simulation of the model needs to be reactive in the sense that any change in state 'outside' can also be performed inside the model. The idea is to create a CPN model that is fully functional and concise, but simultaneously represents and models the major characteristics of the Paris Metro System. For likeness to the real system to be achievable, the trains must be able to travel concurrently over different lines even though no temporal constraints or relationships between the trains of the different lines exist.

Some important assumptions are required in order to construct the CPN model. These assumptions are made both about i) the real metro system and ii) the actual CPN that is constructed to model it..

C. *Assumptions about the Real Metro Systems*

For the real metro system the following assumptions are considered: i) Trains stop at all stations, ii) Lines do not share tracks of other lines even at interchange (transfer stations), iii) Trains belong to a particular line only (i.e. they have unique identities), iv) Trains travel at a speed of about 20 km/h to a maximum speed of 70 km/h. iv) Trains that reach the end of the line (i.e. the last station) change direction to travel down the line (in the opposite direction). v) The spread of the stations is properly distributed. vi) Trains must traveling in opposite directions to service the same line.

D. *Assumptions about the Colored Petri Net Model*

For the Petri net model, the following assumptions are made: i) Every metro stop from one station to another can be associated with exactly one transition, ii) Every station, even ones with overlapping lines can be associated with exactly one place, iii) Transitions, arcs, places and tokens can all be made to include delays, iv) Transitions are used to transfer a token from one place to another. I.e. to transfer a metro train from one station to another destination occurs via token firing, v) The net marking would indicate the state of the line/s, vi) Arc inscriptions can be used to control what is allowed to happen by checking or testing the token types, vii) The tokens based on colored sets can contain useful information. I.e. the train identity, number, direction, line, etc., viii) Tokens represent metro trains. Tokens must have a unique identity, ix) Lines are represented as arcs and have identity through specific inscriptions and labeling, x) At termination point of a line the train returns to the other side i.e. its direction is reversed or a sort of switch over, xi) Places represent particular physical locations or stops in the real metro system.

IV. Proposed Solution

The proposed solution is to construct a CPN model that closely reflects the real functioning of the Paris Metro System. This can be achieved by examining the actual map of the metro system and constructing a CPN from this graphical layout.

The actual map of the Paris-Metro/ RER system is a symbolic graph or symbolical graphical representation of the entire metro networks and its connectivity. This map is composed of a linked set of nodes and edges, but the actual direction is not actually indicated. Each line has its own set of nodes and edges that are represented in the form shown below. This is an actual undirected simple finite path graph of the general form G= (E,V) where E= (e1, e2, e3,....., en) represent the connections from one station to another in a given metro line and V=(v1, v2, v3,....., vn). Normally there are no repeated edges in this graph and sequence of vertices such that from the starting node or vertex, following the sequence of vertices in succession to the termination node or vertex, there is always exactly only one edge, joining any two vertices. i.e. this is the path between the vertices. The length of a path is the number of edges in that path.

In the map the directionality is not properly represented so this needs to be transformed or changed before the map can actually be used to implement a Colored Petri net model. In practical terms each metro line can be represented as a digraph. The incoming and outgoing edges to the vertices represent the incoming and outgoing lines from a particular station which is given as a vertex.

The map simple path graph is transformed into something like a Markov Chain graph. This is because from each station, in the metro, it is possible to move out to other stations in a chain like fashion. This is analogous to the Markov chain where from one state it is possible to transit from one state to another in a chain like manner and the next state only depends on the current state only and not on past states.

This is a closer representation of what is actually happening in the metro. It shows that an actual location or station, that services 1 line only, must have at least one entry point and one exit point. Each line normally has at least two metro trains traveling in opposite directions. One from the station and the other to the station, this could be happening simultaneously.

Normal place transition nets or timed Petri nets can be used for modeling, however they are not as expressive as CPNs. CPNs can be used to model different timing issues and even their execution.

Transition firing is an atomic action which has to satisfy certain conditions like token availability. Each token has a unique identity and direction in which it is allowed to travel.

Token firing must occur in a predefined temporal order to actually model what is happening in real life.

By using CPNs it is possible to describe, in the minutest detail, the different locations of a particular metro route and even its overlapping stations with other metro routes. The metro train is given a unique identity that prevents it from going onto other routes when it is not supposed to do this. We have used the unique modeling features and capabilities of CPN's for the metro scenario.

Special places and their physical drawing size can be used to reflect the actual importance of the station. i.e. a large hub can be drawn as a larger place i.e. more area, when compared to smaller stations. These facts are indicated below: i) Place type is small- single metro line, ii) Medium- Mixed line capacity (i.e. can overlap more than one metro line). This would symbolize 2 or 3 different metro lines converging at the same location. iii) Large- Mix central point or hub, i.e. where more than 3 metro lines meet, cross or converge. Places represent locations or Metro station stops.

The steps for the solution are outlined in fig. 1.

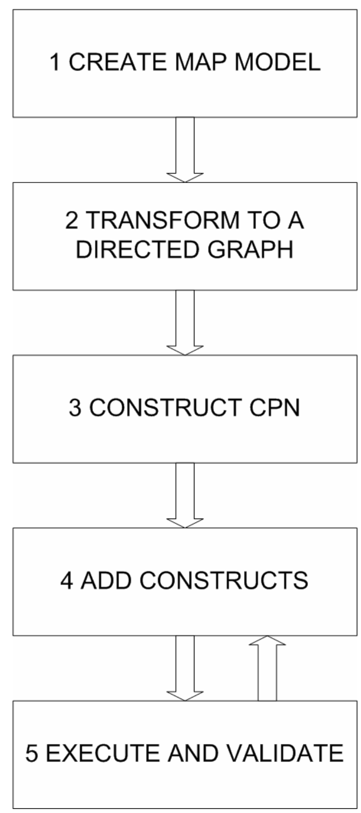

Fig. 1 Main steps for the proposed solution

V. Implementation

For implementing and illustrating the solution, three main lines from the Paris- Metro System were selected. The lines are 1,3,13 and these have been considered in detail. These lines overlap each other at certain stations. The graphical layout of these lines along with the stations/stops is given in fig. 2. The more stations or stops included the more complex the final CPN model becomes to draw.

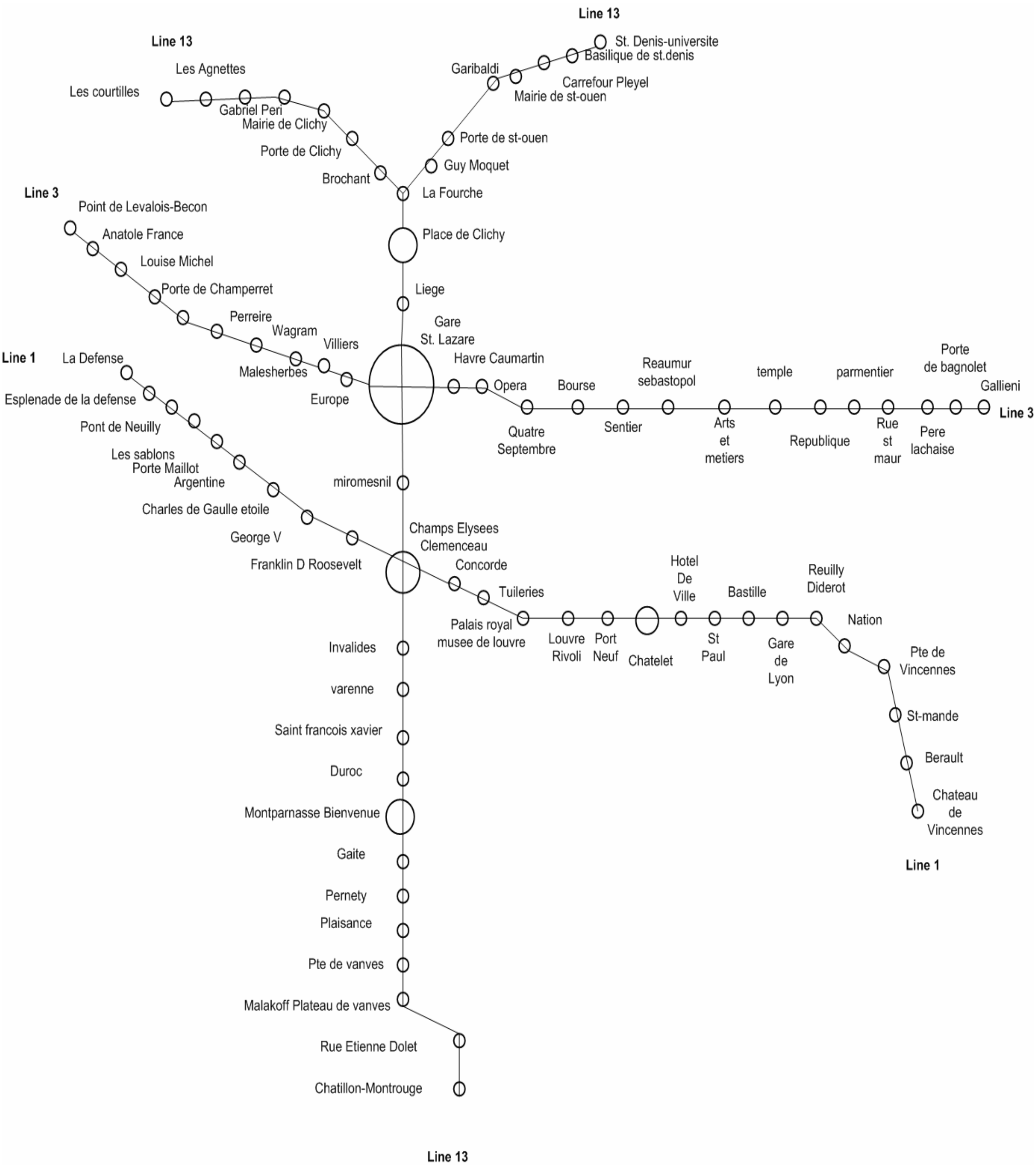

Fig. 2 Basic map showing Paris-metro lines 1,3,13

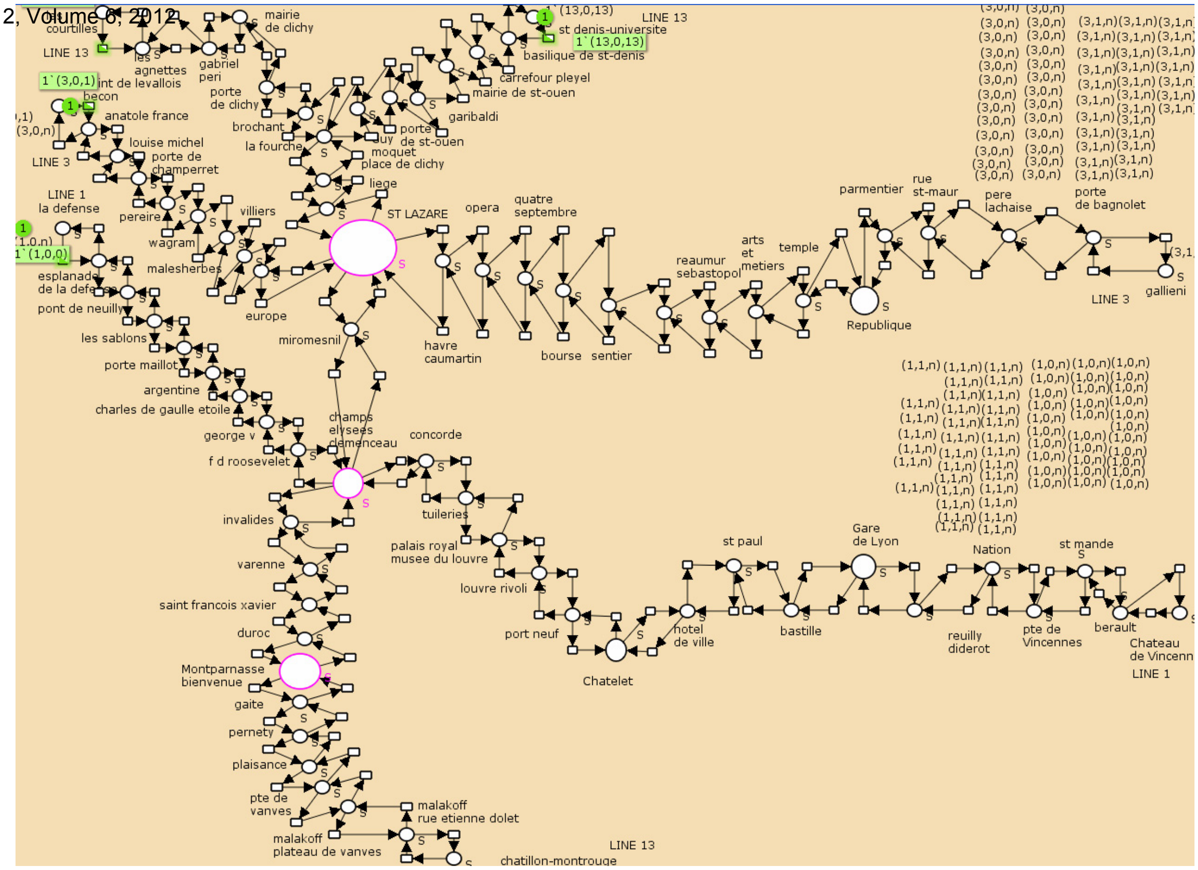

Fig. 3 CPN model for Paris-metro Lines 1,3,13

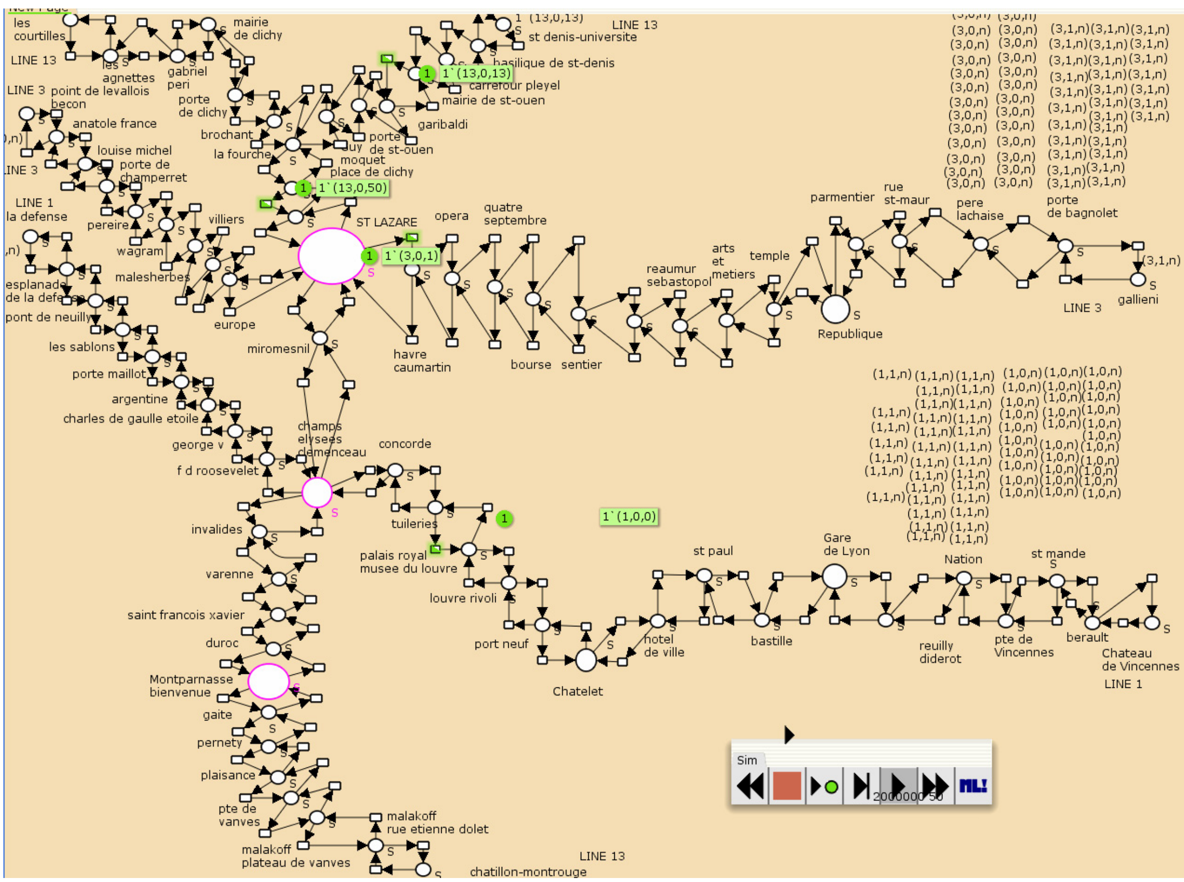

Fig. 4 CPN model execution for Paris-metro Lines 1,3,13

One important step is to ensure that the map model is accurate. The undirected graphical topology of the metro system is the starting point for constructing the CPN. A directed graph model can be constructed for better comprehension and visualization of the CPN model to be built. The graph model is a simple modification of fig. 2. The edges shown in fig. 2 are not indicative of direction. The graph model indicates direction.

After the CPN is built, more details are necessarily added to make the model executable. E.g. token types have to be added. Here the token types used is that of a 3-tuple integer color set of the form (n1,n2,n3) where n1 represents the line number, n2 represents the direction of the train and n3 is for the train id.

The lines are separated from each other by using arc inscriptions that restrict or prevent it from accessing other lines. E.g. a token having values (1,0,2) and representing an actual train can only travel on line 1 in a particular direction because the arc inscriptions would prevent it from travel on other lines.

Other things like functions, variables, etc, need to be defined and used. This is an incremental process and the Petri net is improved as we go along.

VI. Model Execution and Results

The model was executed and tested in detail and the following data was used for the initial model execution. The results shown in this section are just a brief summary of the actual results that show the basic execution results obtainable.

Line No	Station	Token	Direction
1	La Defense	(1,0,0)	Down
3	Point de Levalois-Becon	(3,0,1)	Down
13	Les Courtilles	(13,0,50)	Down
13	St. Denis Universite	(13,0,13)	Down

Table 1 Initial configuration for all lines

A. Initial Configuration

Table 1 has the initial information or token information that has been used to start to operate the three lines. This data is the token data that is found in the CPN model for the Paris Metro system. This data can be used to model different conditions and scenarios.

B. Route Test Data

Tables 2 and 3 show part of the test results obtained for Line 3 using the CPN model. Only some stations are shown otherwise the tables would be too large.

Table 3 indicates that from Gallieni the token representing the train on line 3 travels successfully to Porte de bagnolet. For this the token values are (3,1,1). When the train arrives at the end of the line its direction is reversed by changing the direction value from 0 to 1 or vice-versa. I.e. (3,0,1) is for going down the line and (3,1,1) is for going up the line. When the train for line 3 arrives at Gallieni end of line station from a going down direction the 0 value is set to 1 reversing the direction of the train. This is achieved via arc inscriptions.

From	To	Check	Direction
Point de levalois-becon	Anatole France	OK	Down
Anatole France	Louise Michel	OK	Down
Louise Michel	Porte de champerret	OK	Down
Porte de champerret	Perreire	OK	Down

Table 2 Fragment of route test data – Line 3 Down

From	To	Check	Direction
Gallieni	Porte de bagnolet	OK	Up
Porte de bagnolet	Pere lachaise	OK	Up
Pere lachaise	Rue st maur	OK	Up

Table 3 Fragment of route test data – Line 3 Up

Line No	Repeated Cyclical Behavior	Return to Initial State	Traverse All Stations
1	Yes	Yes	Yes
3	Yes	Yes	Yes
13	Yes	Yes	Yes
13	Yes	Yes	Yes

Table 4 Basic Tests

C. Other Results

In all cases the trains for the metro system complete a full circuit which means that there is continuity and continuous travelling up and down a line as happens in the real world. The trains also keep to their respective lines as was originally suggested. Results were obtained for all the three lines functioning together. Some of the tests are shown in Table 4.

Basically the model exhibits the following key properties: i) Model is fully functional and operates properly, ii) Model is deadlock free, this implies that there are no dead states or unused places, iii) the model can be used for incrementally modeling behavior or train & stops, v) reversing of train direction properly takes place when final station is reached or arrived at.

The CPN model of the metro system can be subject to further work, tests and improvements. It can be used to model diverse scenarios and analysis that have not even yet been considered. E.g. just by extending this color set it would be possible to add even further details like specific time calculations, other identities, etc.

The CPN can be interpreted and analyzed using Petri net mathematics e.g. liveness, reversibility, deadlock, etc. and state space exploration. It can be used for visualization and reduction. It can be transformed or reduced into other Petri net classes for other forms of analysis like those previously mentioned. The behavioral properties of the model can be explored.

VII. Limitations and Other Conclusions

A. Desirable Properties

Even though the model is functional, when using a single token per line it is desirable to have multiple tokens per line that would represent the actual conditions of real life. However this would mean that in the model safety issues are required and restrictions of a single place containing only one token for direction is normally desirable. It means that more than two tokens traveling in the same direction can possibly be placed in the same place at the same time. In the real world this is prevented as it would indicate a collision unless some other form of buffering the trains is used. Depending on what is being modeled this restriction can be ignored.

Theoretically it is possible to restrict place capacities to a maximum number of tokens. It is also possible to put a restriction on the types of tokens that can be located in the same place in a particular time, i.e. a form of temporal ordering of tokens. However these restrictions depend on the modeling tool being used. Normally a simple place restriction in ordinary Petri nets would suffice but in colored nets the places are unbounded in the CPN tool, so a locking mechanism to prevent places from having more than 1 token in the same direction on the same line might have to constructed. One limitation of the CPN model developed is that the places have unbounded capacities. This is because of the CPN tools. Some possible solutions are suggested below.

B. Limiting Place Capacities

If there are multiple tokens per line and the transitions are fired manually using consistent and proper temporal ordering for the firing sequences then there is no problem because the firing will reflect what happens in the real world.

If automatic transition firing is used a problem will occur, i.e. the transition firing might not properly represent token ordering and a particular place might contain more than one token (i.e. symbolizing train) traveling in the same direction. Whilst this could be possible in real life, here we are trying to prevent this from actually happening. A solution is to use a place locking mechanism using anti places which would actually serve as switches ensuring that a place can have only one token from a line specific time in one direction at a time. Obviously tokens from other lines are still possible using the same criteria and they do not interfere with each other. The number of tokens in a place would then actually reflect the number of trains in a station at a given point in time. To insert anti places it implies that the model will become more complex and require further extensions to the network. Consequently having many places that reflect the actual stops, will make matters more complex. I.e. by inserting anti places it is possible to have double the amount of places, complicating the model unnecessarily.

C. Including Time

Estimating the temporal performance analysis of discrete event systems requires the inclusion of the time dimension. In CPNs including time is not really an issue. Just by modifying the token definition it is possible to have a numeric value for time. This is because the time value would be placed in another variable. Alternatively a timed Petri net or TPN can be constructed. A TPN can focus on various timing issues whilst other details are simplified. If timings are the most important property that needs to be analyzed, then it is possible to focus only on this aspect. The class of timed Petri nets that closely fits the metro's system actual behavior would have to be used, because there are various categories of TPNs. Time analysis would be useful for things like cycle time analysis, max and min waiting time, arrival estimation time. Time analysis can also be used to find performance bottlenecks and improve the overall system performance.

D. Simplicity and Readability

These properties are achievable by taking a macroscopic view of the system and by reducing the total number of places and transitions in the system. Unfortunately if the network is simplified the overall result is that certain details and information is inevitably lost.

Another possibility that can be explored is that of using a top level net topology that decomposes onto other subnets for each line. This can be done using the CPN tool that supports hierarchical decomposition.

VIII. Conclusion

Petri net formalisms have been successfully applied to different areas like manufacturing, transport, robotics, automated control systems, communications networking, software engineering, etc. However their real strength lies in the description of discrete event systems. A metro system fits such behavior perfectly.

This work can definitely be developed further. It has been shown how CPNs can be useful for modeling a discrete complex system such as a complex underground metro system or a train metro system. Results are obtainable and a fully functional model is constructed. This shows us that CPNs are relevant for modeling Metro or train traffic networks. More work can be done in this respect. The results presented here are by no means exhaustive. Many other tests, execution and verification techniques can be devised and used. To improve the model it might be necessary to limit place capacities and this requires additional places and more complexity.

Ideally the model should be constructed to contain the entire Paris Metro system. This is definitely achievable theoretically. However in real life, the larger the real system, the more complex will the resultant CPN will be. This implies that it is tedious and time consuming to construct and validate.

Decomposition techniques would have to be used to simplify it and reduce detail thus less information will be visible.

It is possible to use other classes of Petri nets for specific issues. I.e. Timed Petri nets would be useful for modeling the timings in detail.

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