# On the Stability of Coalitions: A Rough Set Approach

Jiri Mazurek, Elena Mielcova

***Abstract*****—** The aim of the article is to evaluate coalition stability in multi-party systems with the use of the rough set theory. In this approach an information system including all members of a legislature with their votes is used, and coalitions are considered rough sets defined by their lower and upper approximations. Based on these approximations, three categories of coalitions according to their stability are defined: *stable*, *conditionally stable* and *unstable*; and a stability index is introduced to express a degree of coalition stability, which can provide useful information about coalitions' potential persistence in the long run. The method was applied to the evaluation of government coalition stability in the Czech Parliament during 2006-2010 electoral term.

***Keywords*****—**coalition, coalition stability, legislature, rough sets, voting, Lower House of the Czech Parliament.

## I. Introduction

IN many settings ranging from national parliaments to boards and councils of various kinds, coalitions of subjects naturally arise. A coalition is a (temporary) alliance of subjects such as people, parties, companies or nations for enforcing common interests shared by subjects participating in a coalition. The power and strategy of individual voters, parties or coalitions has been largely studied in the game theory context. In these cooperative *n*-person games a power index expresses a decisional power of each player in a game; and coalitions of players are assigned functions expressing their worth, see e.g. [1]–[3]. In the game theory, a coalition is considered stable, when no player in a coalition can gain from leaving a coalition.

However, there is not much literature on the application of the game theory into the evaluation of coalition stability in a real parliament setting (with rare exception of [4]). In politics a coalition is considered stable, if it poses a simple majority, which means that a coalition disposes of more than $n/2$ members of a legislature. However, in reality many coalitions with formal majority become unstable during electoral terms and this development leads to a rearrangement of votes or new elections, and subsequently to a rise of new coalitions. Therefore, description of a coalition as a precise (and unchanging) set of members doesn't conform to reality.

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The aim of this article is to evaluate stability of coalitions with the use of the rough set approach. The rough set theory was proposed by a Polish computer scientist Zdislaw I. Pawlak in the early 1980s [5]. This theory is based on the idea that objects of the universe are associated with some information, and this information might be uncertain, vague or imprecise. Today, the rough set theory is widely used in many areas such as data mining, knowledge discovery in databases, business failure prediction or medicine; see e.g. [6] –[9].

Decision makers are humans, who must decide upon various subjects with limited amount of information and knowledge, who are not always rational and whose judgment can be biased in many ways (see e.g. [10]). Hence, the rough set approach is more appropriate to a description of real-world voting procedures. When compared to the game theory, the rough set approach has following merits:

- it doesn't require any additional assumptions, e.g. about rationality of decision makers,
- it uses only information contained in an information system (see e.g. Table 1), and it doesn't require any additional information or ad-hoc parameter from 'outside', such as functions describing players' powers or worth of coalitions,
- it can handle uncertainty and imprecision associated with human decision making.

The rough set approach will be explained in a legislature context, but it applies to all situations, where decision making in the form of a voting takes place.

The paper is organized as follows: in Section 2, a brief review of the rough set theory related to the topic is provided, in Section 3 stability of coalitions is defined and Section 4 provides an illustrative example of coalition stability evaluation. In Section V the rough set approach is applied to the evaluation of stability of government coalition in the Lower House of the Czech Parliament from 2006 to 2010. Conclusions close the article.

## II. The Rough Sets: Indiscernibility Relation and Set Approximation

In the rough set theory objects and their evaluation by a set of attributes is represented by a data set called an *information system* – a pair $(U, A)$, where:

- $U$ is a non-empty, finite *universal set* of objects $x$.

- $A$ is a non-empty, finite set of (condition and decision) *attributes* $C = \{C_1, C_2, ..., C_n\}$, and each attribute $C_i \in C$ is a function $C_i : U \rightarrow V(C_i)$, where $V(C_i)$ is a value set (*domain*) of $C_i$.

The *indiscernibility relation* $R_B$ on $U$ associated with a set $B \subseteq A$ is defined as:

$$xR_B y \Leftrightarrow a(x) = a(y); \forall a \in B; x, y \in U$$

A set of objects indiscernible with an object $x$ by a relation $R_B$ is called an *equivalent class* $[x]_B$. Every indiscernibility relation $R$ provides a *partition* $U/R$ of $U$ into equivalent classes.

In the rough set theory a set $X$ cannot be generally expressed exactly by a list of its elements, as some elements from $U$ might be indiscernible by a set of attributes $B$. Let $X \subseteq U$ and $B \subseteq A$, then $\underline{B}(X)$ and $\overline{B}(X)$ denote *lower* and *upper approximation* of a set $X$ with respect to a relation $R_B$, and are defined as:

$$\underline{B}(X) = \{x \in U; [x]_B \subseteq X\}$$

$$\overline{B}(X) = \{x \in U; [x]_B \cap X = \varnothing\}$$

According to definitions above, the lower approximation (a *positive region*) of a set $X$ is a union of all classes which are subsets of $X$ (are contained in $X$), thus objects in $\underline{B}(X)$ *positively* (surely) belong to a set $X$. The upper approximation of the set $X$ is a union of all classes, which have non-empty intersection with $X$, thus objects in $\overline{B}(X)$ can *possibly* be classified as members of $X$.

A tuple $(\underline{B}X, \overline{B}X)$ is called a *rough set*, which means that a rough set is represented by two crisp sets – its *lower* and *upper approximations*, see Figure 1.

A set $BN_B(X) = \overline{B}X - \underline{B}X$ is called a *boundary region* of a set $X$, and it contains objects that cannot be ruled in or out as members of $X$. When $BN_B(X) = \varnothing$, $X$ is a crisp set, otherwise it is a rough set with respect to $B$.

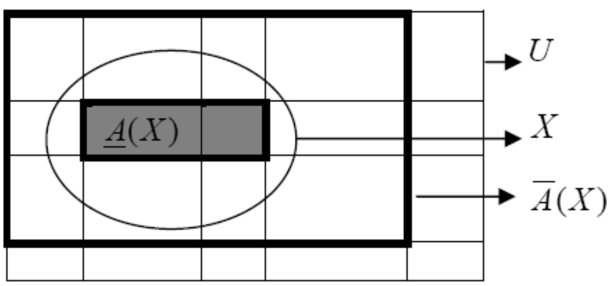


Fig. 1. A rough set $X$ with its lower and upper approximations. Source: [12].

The accuracy of rough set approximation of a set $X$ induced by an indiscernibility relation $R_B$ is given as:

$$\alpha_B(X) = \frac{card(\underline{B}X)}{card(\overline{B}X)}$$

Clearly, $\alpha_B(X) \in [0,1]$. When the lower approximation is equal to the upper approximation, then $\alpha_B(X) = 1$, the approximation is perfect and a set $X$ is a crisp set. Otherwise, a set $X$ is a rough set. When the lower approximation is an empty set, the accuracy is zero. A number $\rho_B(X) = 1 - \alpha_B(X)$ is a *roughness* of a set $X$.

For other interesting features of the rough set theory, such as reducts, cores, dependency of attributes and rule extraction see [6, 7, 11].

To illustrate some of aforementioned concepts we provide Example 1.

***Example 1***. Consider the information system from Table 1. In the table 'MPs' are objects of the universal set $U$, 'Party' is a condition attribute and voting procedures '$v_1$', '$v_2$' and '$v_3$' are decision attributes. Indiscernibility relation $R_B$ induced by the set of decision attributes $B = v_1 \cup v_2 \cup v_3$ provides a partition of $U$ into following seven equivalent classes $[x]_B$:

$\{x_3, x_7, x_8, x_9\}$, $\{x_1, x_2, x_5, x_6\}$, $\{x_{13}, x_{14}, x_{16}, x_{17}\}$, $\{x_{10}, x_{12}\}$, $\{x_4, x_{11}\}$, $\{x_{15}, x_{18}\}$ and $\{x_{19}, x_{20}\}$

From $[x]_B$ we obtain following lower and upper approximations of parties $P_1$, $P_2$ and $P_5$:

$\underline{P_1} = \{x_1, x_2, x_5, x_6\}$,

$\overline{P_1} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{11}\}$

$\underline{P_2} = \{\ \}$, $\overline{P_2} = \{x_3, x_4, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$

$\underline{P_5} = \{x_{19}, x_{20}\}$, $\overline{P_5} = \{x_{19}, x_{20}\}$.

As can be seen, sets (parties) $P_1$ and $P_2$ are rough sets (their lower and upper approximations are not equal), while the set $P_5$ is the crisp set. Lower approximation $\underline{P_1} = \{x_1, x_2, x_5, x_6\}$ of the set $P_1$ includes those elements $x_i$ from $U$, which can be certainly classified as members of $P_1$, while the upper approximation $\overline{P_1} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{11}\}$ lists those elements of $U$, which can be possibly classified as members of $P_1$. The boundary set of the set $P_1$: $BN_B(P_1) = \{x_3, x_4, x_7, x_8, x_9, x_{11}\}$ consist of elements, which cannot be ruled out of the set $P_1$ nor ruled in this set with certainty. The accuracy of rough approximation of the set $P_1$: $\alpha_B(P_1) = \frac{4}{10} = 0.4$ and $\rho_B(P_1) = 1 - \alpha_B(P_1) = 0.6$.

## III. STABILITY OF A COALITION: THE ROUGH SET APPROACH

A coalition of two or more parties is considered stable, if it has more than $n/2$ members (a simple majority), where $n$ is a number of all members of a legislature; and unstable otherwise.

However, there are situations when some members of a coalition vote against a coalition, while some individuals from opposition (or independent ones) might support it. Such 'rebellious' MPs occur because of the existence of various fractions within parties, drop-out and turn-coat individuals, and independent or just irrational or emotional MPs, who change their attitudes more frequently than others. Hence, the strength of a coalition cannot be expressed simply as a sum of all MPs formally belonging to a coalition (which would be a crisp set), as this number might change from one voting to another. Instead, a coalition should be considered a rough set (a set without 'sharp boundary') represented by its lower and upper approximations, which in turn allow for coalition stability evaluation.

**Definition 1**. Let $A_i$ be sets of individuals (MPs) elected for a party $i$ to a legislature, where $i \in \{1,2,...k\}$, $A_i \cap A_j = \varnothing$ for all $i \neq j$.

**Definition 2**. Let $A$ be a set of all members of a legislature, $\bigcup_{i=1}^{k} A_i = A$, $cardA = n$.

**Definition 3**. A coalition is an arbitrary set $C$ satisfying: $C \subseteq A$, $C = \bigcup_{j=1}^{m} A_j$, $2 \le m \le k$, where $A_j$ are parties participating on a coalition $C$.

As mentioned before, a coalition $C$ is considered a rough set, as some members of a coalition may vote against it, while some non-members might support it. Stability of a coalition $C$ depends on cardinalities of its lower and upper approximations. Lower approximation represents a 'pessimistic' view of coalition potential, while upper approximation represents an 'optimistic' view. Definition 4 postulates three categories of coalition stability:

**Definition 4**. Let $C$ be a coalition and let $cardA = n$. Let $\underline{C}$ and $\overline{C}$ be lower and upper approximations of a coalition $C$ respectively, and let $\underline{\alpha}(C) = card\underline{C}$ be *lower coalition potential* and $\overline{\alpha}(C) = card\overline{C}$ be *upper coalition potential*. Then a coalition $C$ is:

- *stable*, if $\underline{\alpha}(C) > n/2$
- *conditionally stable*, if $\underline{\alpha}(C) \le n/2 \wedge \overline{\alpha}(C) > n/2$
- *unstable*, if $\overline{\alpha}(C) \le n/2$

According to Definition 4, a coalition is *stable*, if its lower ('pessimistic') coalition potential gives a majority. A coalition is *conditionally stable*, if the upper ('optimistic') coalition potential gives a majority, but lower coalition potential doesn't. This coalition might be stable to some degree. Finally, a coalition is *unstable*, if even upper coalition potential doesn't exceed $n/2$.

The stability of a conditionally stable coalition can be evaluated by the *stability index SI*.

**Definition 5**. Let $C$ be a coalition of parties with lower coalition potential $\underline{\alpha}(C)$ and upper coalition potential $\overline{\alpha}(C)$, and let $cardA = n$. Then, the *stability index* of a coalition $C$ is defined as follows:

- for a stable coalition $C$: $SI(C) = 1$,
- for a conditionally stable coalition $C$:
$$SI(C) = \frac{\overline{\alpha}(C) + \underline{\alpha}(C) + 1}{n+1} - 0.5, \qquad (1)$$
- for an unstable coalition $C$: $SI(C) = 0$.

The higher is the value of $SI(C)$, the higher is coalition stability.

**Proposition 1.** Stability index $SI$ defined by relation (1) for conditionally stable coalitions satisfies: $SI \in (0,1)$.

*Proof*: To prove Proposition 1, it must be shown that $SI > 0$ and $SI < 1$ for all $n > 1, n \in N$.

i) $SI < 1$: According to Definition 4, a maximum value of $SI$ is achieved, when:

a) $\underline{\alpha}(C) = n/2 \wedge \overline{\alpha}(C) = n$ for $n$ even integer; and b) $\underline{\alpha}(C) = \frac{n-1}{2} \wedge \overline{\alpha}(C) = n$ for $n$ odd integer. From (1) we get:

a) $$SI(C) = \frac{\frac{n}{2} + n + 1}{n+1} - 0.5 = \frac{3n+2}{2(n+1)} - 0.5 < 1, \quad \forall n \in N;$$

and $$\lim_{n \to \infty} \frac{3n+2}{2(n+1)} - 0.5 = 1,$$

b) $$SI(C) = \frac{\frac{n-1}{2} + n + 1}{n+1} - 0.5 = \frac{3n+1}{2(n+1)} - 0.5 < 1, \quad \forall n \in N;$$

and $$\lim_{n \to \infty} \frac{3n-1}{2(n+1)} - 0.5 = 1.$$

ii) $SI > 0$: According to Definition 4, a minimum value of $SI$ is achieved, when:

a) $\underline{\alpha}(C) = 0 \wedge \overline{\alpha}(C) = \frac{n}{2} + 1$ for $n$ even integer; and b) $\underline{\alpha}(C) = 0 \wedge \overline{\alpha}(C) = \frac{n+1}{2}$ for $n$ odd integer. From (1) we get:

a) $SI(C)=\dfrac{\frac{n}{2}+1+1}{n+1}-0.5=\dfrac{n+4}{2(n+1)}-0.5>0$, $\forall n\in N$; and

$\lim_{n\to\infty}\dfrac{n+4}{2(n+1)}-0.5=0$,

b) $SI(C)=\dfrac{\frac{n+1}{2}+1}{n+1}-0.5=\dfrac{n+3}{2(n+1)}-0.5>0$, $\forall n\in N$; and

$\lim_{n\to\infty}\dfrac{n+3}{2(n+1)}-0.5=0$. *Q.E.D.*

In the following section a simple illustrative example on coalition stability evaluation is provided.

## IV. Illustrative Example

Let's consider an information system in Table 1 again. It includes a list of 20 members of a fictional municipal council ($x_1$ to $x_{20}$), 5 parties ($P_1$ to $P_5$) and three voting procedures ($v_1$, $v_2$ and $v_3$) with three possible values: *yes*, *no* and *abstention* (– ). As can be seen, not all party members voted unanimously, and some MPs from different parties voted in accord on all three occasions.

Now, let $C$ be a coalition of parties $P_1$ and $P_2$. This coalition $C$ 'nominally' disposes of 11 MPs: $C=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10},x_{11}\}$, which is a majority; hence this coalition would be traditionally regarded as stable.

However, with the use of indiscernibility relation $R_B$ induced by a set of decision attributes $B=v_1\cup v_2\cup v_3$ and equivalent classes $[x]_B$ listed in Example 1, we obtain following lower and upper coalition approximations:

$\underline{C}=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{11}\}$, $\underline{\alpha}(C)=10$,

$\overline{C}=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10},x_{11},x_{12}\}$,

$\overline{\alpha}(C)=12$.

These results can be interpreted so that the coalition $C$ has from 10 members (a pessimistic view) up to 12 members (an optimistic view). Because $\underline{\alpha}(C)=10\le\frac{n}{2}$ and $\overline{\alpha}(C)=12\ge\frac{n}{2}$, according to Definition 4 the coalition $C$ is classified as *conditionally stable*. The stability index $SI$ obtained from (1): $SI=0.595$.

As can be easily verified, the coalition of parties $P_1$, $P_2$ and $P_5$ would be stable, as $\underline{\alpha}(C)=12$, while the coalition of parties $P_2$ and $P_5$ alone would be unstable, for $\overline{\alpha}(C)=10$.

Table 1. Votes in a fictional municipal council.

| MP | Party | $v_1$ | $v_2$ | $v_3$ |
|---|---|---|---|---|
| $x_1$ | $P_1$ | yes | yes | yes |
| $x_2$ | $P_1$ | yes | yes | yes |
| $x_3$ | $P_1$ | yes | yes | no |
| $x_4$ | $P_1$ | yes | – | yes |
| $x_5$ | $P_1$ | yes | yes | yes |
| $x_6$ | $P_1$ | yes | yes | yes |
| $x_7$ | $P_2$ | yes | yes | no |
| $x_8$ | $P_2$ | yes | yes | no |
| $x_9$ | $P_2$ | yes | yes | no |
| $x_{10}$ | $P_2$ | yes | no | no |
| $x_{11}$ | $P_2$ | yes | – | yes |
| $x_{12}$ | $P_3$ | yes | no | no |
| $x_{13}$ | $P_3$ | no | no | no |
| $x_{14}$ | $P_3$ | no | no | no |
| $x_{15}$ | $P_3$ | – | no | no |
| $x_{16}$ | $P_4$ | no | no | no |
| $x_{17}$ | $P_4$ | no | no | no |
| $x_{18}$ | $P_4$ | – | no | no |
| $x_{19}$ | $P_5$ | no | no | yes |
| $x_{20}$ | $P_5$ | no | no | yes |

## V. Case Study – State Budget Voting in the Czech Parliament 2006-2009

In the paper the rough set theory is applied to the evaluation of coalition stability in multi-party systems, such as, in this case, the state budget voting in the Lower House of the Czech Parliament through 2006-2009.

### A. Data Description

In general, the Czech Parliament is divided into two chambers – the Chamber of Deputies (the Lower House of the Czech Parliament, 200 members), and the Senate (the Upper House of the Czech Parliament, 81 senators). Elections to the Upper House of the Czech Parliament are based on voters' preferences of candidates, while elections to the Lower House are based on voters' political parties' preferences. Hence, it is particularly intriguing point to evaluate accuracy and coalitional stability in the Chamber of Deputies of the Czech Parliament. Therefore, the following analysis is based on data collected from Lower House roll call voting.

The main idea was to compare votes on the same issue through one electoral period. Thus, we have chosen the last complete electoral period, 2006-2010, and agenda that repeats every year – the state budget voting. Parliamentary discussions of state budget for the following year has to be finished before the end of the preceding year; these discussions usually cover tens of amendment votes to a state budget proposal, many of them related to one specific issue. Hence, the number of votes related to state budget issues is different throughout years. However, the accuracy and stability indices are related to number of votes. We wanted to compare the evaluation of these indices in four years; therefore we have reduced the number of votes down to the 10 last votes on the state budget. Higher amount of used votes is causing the distribution of parliamentary members into high amount of groups (each member has her/his own group).

Data are collected with respect to votes of all members. The outcome of every vote for every member can be "no", "yes", "present, abstain", "absent". Every bill to be passed needs at least as many "yes" votes as quota. Quota is based on the sum of all present legislators; hence outcome "present, abstain" serves as "no" outcome; in this analysis this outcome is reclassified as "no" outcome. Basic information on the Czech Parliamentary system as well as the set of all historical votes can be found at the official web site of the Lower House of the Czech Parliament [13].

During the studied period, there were five political parties active in the Lower House of the Czech Parliament (political parties' abbreviations are given in parentheses):

- Civic Democratic Party (ODS)
- Czech Social Democratic Party (CSSD)
- Christian and Democratic Union – Czechoslovak People's Party (KDU-CSL)
- Czech and Moravian Communist Party (KSCM)
- Green Party together with Independent Candidates (later only Green Party, SZ)

Three political parties – ODS, KDU-CSL, and SZ – created governmental coalition; the government consisted of these political parties' members after their mutual decisions.

*B. Results*

The 2006-2010 electoral period started with deadlock in political parties' discussions about a new government setting, as left-wing political parties (CSSD, and KSCM) had gained 100 seats – the same number of seats as right-wing political parties (ODS, KDU-CSL, and SZ). After six months of discussions, (and decrease in seats of CSSD), in January 2007, the coalition of ODS, KDU-CSL, and SZ set the government. In March 2009, the Lower House of the Czech Parliament approved no-confidence of the government; thus the new caretaker government was set and functioned till elections in 2010.

During the 2006-2010 parliamentary period, there were twelve changes in party affiliation, all of them influencing the power of political parties. Power of political parties is usually measured in terms of so-called power indices. To evaluate the power of political parties, we used the Shapley-Shubik Power index of the form:

$$\pi_i^{SS}(q;\vec{w}) = \sum_{S\in W(i)} \frac{(|S|-1)!(n-|S|)!}{n!} \qquad (2)$$

where $n$ is the number of legislators, all of them grouped into political parties with weights denoted by vector $\vec{w}$; the quota of voting is set to be $q$. In (2) the summation is taken over the set of vulnerable coalitions for which a player $i$ is essential, and $|S|$ denotes the cardinality of $S$ (for more information see [2]–[3]).

For illustration of the situation in the Lower House of the Czech Parliament, the political parties' seats distributions, as well as Shapley-Shubik power indices (2) for 2007-2010 state budget voting are given in Table 2. The power distribution over the whole 2006-2010 period is given in Figure 2.

For all political parties, as well as all four periods of state budget voting we calculated the lower and upper approximation of their members, as well as accuracy index. In Table 3, there are given cardinalities of lower and upper approximations, and values of accuracy indices for last ten votes of state budget voting. Figures 3-7 illustrate the time allocation of accuracy indices in the Czech Lower House.

Results show that lower an upper approximations of parties were significantly different, which indicates little coherence and large disunity of parties' members. From the rough set theory point of view this result implies that there are indeed no sharp boundaries among parties, as some individuals vote with different parties or against their own party. The accuracy index of parties varied from 0 to 0.84 and was not stable during the parliamentary period. The interesting point is that the accuracy index of the governmental coalition was the highest during the first budget voting in December 2006, and the smallest in December 2008, before the no-confidence approval procedure that took place in March 2009.

Table 2. Number of seats and Shapley-Shubik power index for political parties present in the Lower House of the Czech Parliament

| Party | ODS | CSSD | KSCM | KDU-CSL | SZ | Independent |
|---|---|---|---|---|---|---|
| 2007 State Budget Voting (December 2006) | | | | | | |
| Seats | 81 | 72 | 26 | 13 | 6 | 2 |
| Power Index | 0.391 | 0.241 | 0.241 | 0,057 | 0.057 | 2x0.0071 |
| 2008 State Budget Voting (December 2007) | | | | | | |
| Seats | 81 | 72 | 26 | 13 | 6 | 2 |
| Power Index | 0.391 | 0.241 | 0.241 | 0,057 | 0.057 | 2x0.0071 |
| 2009 State Budget Voting (December 2008) | | | | | | |
| Seats | 79 | 71 | 26 | 13 | 4 | 7 |
| Power Index | 0.360 | 0.267 | 0.267 | 0.027 | 0.027 | 7x0.0074 |
| 2010 State Budget Voting (December 2009) | | | | | | |
| Seats | 78 | 71 | 26 | 9 | 4 | 12 |
| Power Index | 0.344 | 0.293 | 0.293 | 0.011 | 0.011 | 12x0.004 |

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Table 3. Lower and upper approximations as well as accuracy index for political parties present in the Lower House of the Czech Parliament. The “Coalition” is composed of political parties creating governmental coalition – ODS, KDU-CSL, and SZ

| Party | ODS | CSSD | KSCM | KDU-CSL | SZ | Coalition of ODS, KDU-CSL, and SZ |
|---|---|---|---|---|---|---|
| 2007 State Budget Voting (December 2006) | | | | | | |
| Present legislators | 53 | 72 | 2 | 3 | 1 | 57 |
| Lower approximation | 46 | 23 | 0 | 0 | 0 | 47 |
| Upper approximation | 68 | 86 | 14 | 7 | 14 | 68 |
| Accuracy | 0.676 | 0.267 | 0 | 0 | 0 | 0.691 |
| 2008 State Budget Voting (December 2007) | | | | | | |
| Present legislators | 81 | 72 | 26 | 13 | 6 | 100 |
| Lower approximation | 7 | 11 | 4 | 1 | 1 | 11 |
| Upper approximation | 101 | 96 | 86 | 92 | 93 | 103 |
| Accuracy | 0.069 | 0.115 | 0.047 | 0.011 | 0.011 | 0.107 |
| 2009 State Budget Voting (December 2008) | | | | | | |
| Present legislators | 79 | 69 | 26 | 11 | 3 | 93 |
| Lower approximation | 8 | 23 | 4 | 2 | 0 | 10 |
| Upper approximation | 100 | 94 | 75 | 88 | 86 | 102 |
| Accuracy | 0.08 | 0.245 | 0.053 | 0.023 | 0 | 0.098 |
| 2010 State Budget Voting (December 2009) | | | | | | |
| Present legislators | 78 | 71 | 26 | 9 | 4 | 91 |
| Lower approximation | 14 | 65 | 21 | 6 | 2 | 25 |
| Upper approximation | 89 | 77 | 27 | 24 | 5 | 109 |
| Accuracy | 0.157 | 0.844 | 0.778 | 0.25 | 0.4 | 0.229 |

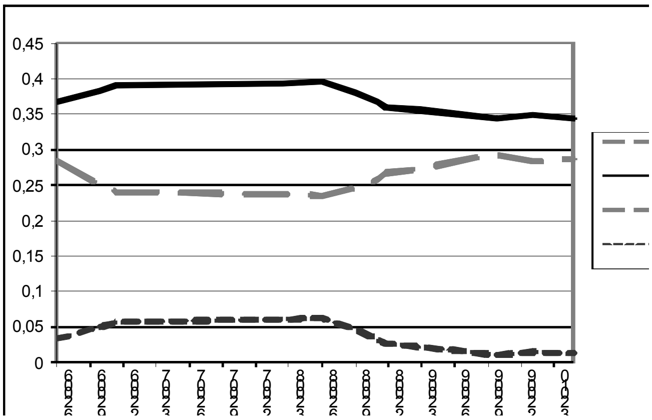


Fig. 2. Shapley-Shubik power index of political parties in 2006-2010 Lower House. Source: own calculations.

The governmental coalition of ODS, KDU-CSL and SZ was only conditionally stable over the period; the stability index is given in Table 4, and Fig. 8. Similarly to previous results, the stability of governmental coalition reached the highest value at the beginning of the electoral period. Even though, the stability index was quite low, it did not exceed the value of 0.4.

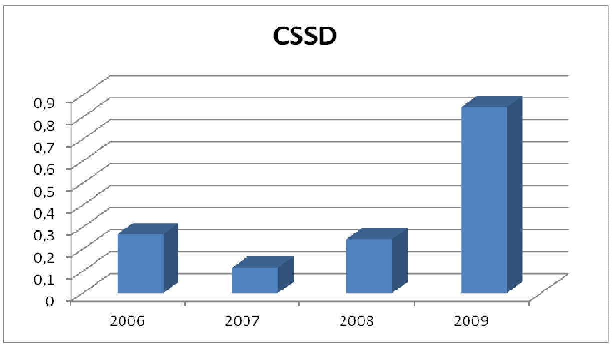


Fig. 3. Accuracy index of CSSD in 2006-2009 Lower House. Source: own calculations.

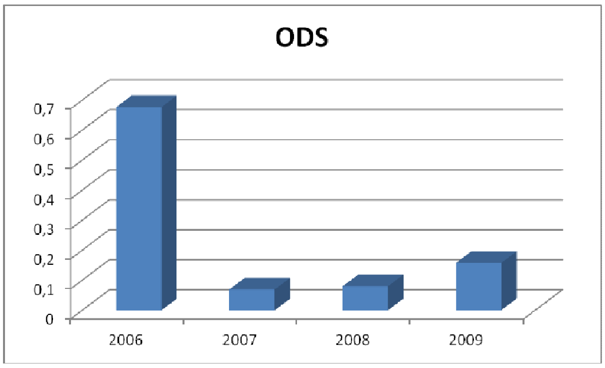


Fig. 4. Accuracy index of ODS in 2006-2009 Lower House. Source: own calculations.

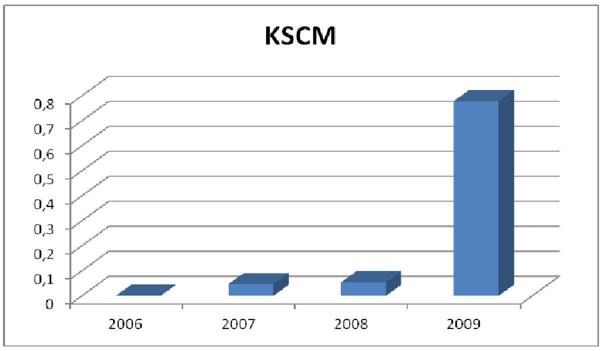


Fig. 5. Accuracy index of KSCM in 2006-2009 Lower House. Source: own calculations.

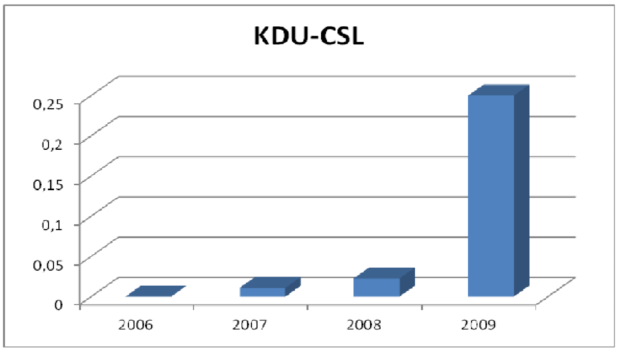


Fig. 6. Accuracy index of KDU-CSL in 2006-2009 Lower House. Source: own calculations.

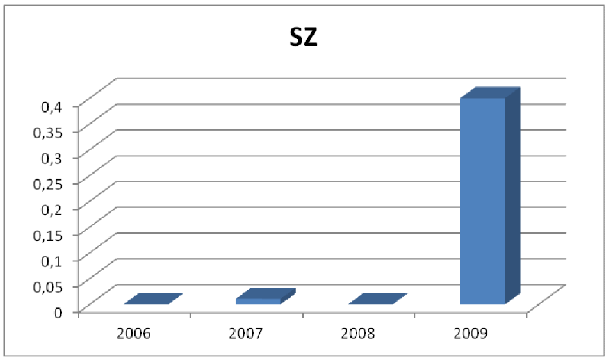


Fig. 7. Accuracy index of Green Party in 2006-2009 Lower House. Source: own calculations.

Table 4. Stability index of governmental coalition in the Lower House of the Czech Parliament.

| Coalition of ODS, KDU-CSL, and SZ | Stability index |
|---|---|
| 2007 State Budget Voting (December 2006) | 0.366 |
| 2008 State Budget Voting (December 2007) | 0.072 |
| 2009 State Budget Voting (December 2008) | 0.074 |
| 2010 State Budget Voting (December 2009) | 0.172 |

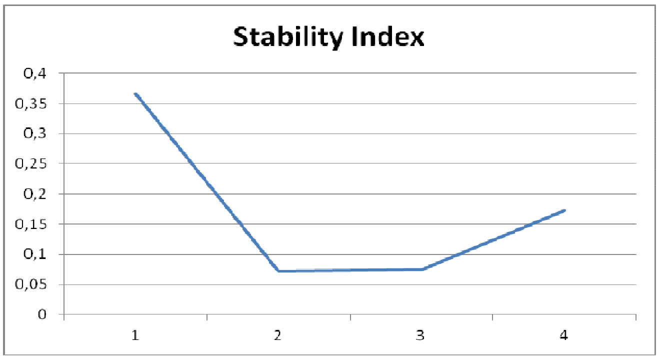


Fig. 8. Stability index of governmental coalition (ODS, KDU-CSL, SZ) in the Lower House of the Czech Parliament 2006-2009. Source: own calculations.

## I. CONCLUSIONS

In the paper the rough set theory is applied to the evaluation of coalition stability in multi-party systems, such as legislatures of various kinds, councils, boards, etc. The rough set approach can be useful for the evaluation of a coalition perspective and its potential persistence during an electoral term mainly because it uses only information of MPs' party memberships and their previous votes (that is actual data); and unlike the game theory framework, it doesn't require any additional assumptions or functions describing MPs' behaviour, for they are often unrealistic in a real legislature setting. Furthermore, the rough set approach enables to divide coalitions into three categories: *stable*, *conditionally stable* and *unstable*; and for the second ones, a stability index providing a degree of coalition stability can be computed.

The method was tested on small set of data, and applied to the state budget voting in the Lower House of the Czech Parliament 2006-2010. During the studied period, the accuracy indices of all political parties varied from 0 to 0.9, while stability index of the governmental coalition was quite low, it did not exceed 0.4 and thus the coalition was only conditionally stable. The stability index reached the highest value at the beginning of the studied period (0.37 in 2006), then decreased down to 0.07 in 2007 and 2008, and slightly increased up to 0.17 at the end of the period in 2009. Obtained results revealed that the rough set approach is appropriate to study of coalition stability and coalition patterns. Furthermore, the moving stability index (over e.g. 10 voting sessions) can be used to model a coalition stability dynamics through a given period.

The rough set approach has some limitations too. It is based (only) on the information of MPs' previous voting, but for example in parliament setting there is a vast number of votes (typically tens or even hundreds) in each parliament sitting, so sets of MPs who vote in the same way might be 'atomized' into sets containing just one MP (our computer simulations revealed that such situation occurs when the number of voting sessions is roughly 50). Another problem stems from the fact that rough sets approach treats all votes with the same weight (importance), while many real votes are just procedural (they concern the programme of a sitting, time of lunch breaks, etc.). But this weakness is shared also by other approaches to the evaluation of voting, such as Shapley's and Shubik's, where power indices also do not distinguish between important and not so important polls. Hence, the proposed rough set approach can be used as an alternative to the classic approaches to the evaluation of voting of various kinds.

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