Legal Values and Legal Entropy: a suggested Mathematical Model

Massimiliano Ferrara and Angelo, Roberto Gaglioti

***Abstract*—** We will describe the fundamentals of a mathematical model for the quantitative analysis of the legal phenomena, intended to provide with a framework of legal general theory, allegedly applicable to every legal situation. In particular, the model can identify any legally material event using a logical hypothetical tool (the Model Situation) and associating to any Situation a certain axiological potential, what makes possible to determine even the amount of axiological potential, at disposal for the discretionary policies of the legal operator. Legal conflicts among axiological potentials may be easily and property rights objectively entitled and adjudicated amongst many challengers. We will try and apply the model to one legal rule of universal applicability (art. 3 of the Universal Declaration of Human Rights, attributing the right to life) and put some seminal considerations regarding the concept of Legal Entropy as related to the welfare level within the legal system.

***Keywords*—**Values, Law, Entropy, Mathematics

I. Introduction

IT seems that up to now no mathematical model of legal general theory is of common use, neither among the legal scholars, nor in the legal current practice. Some steps in this sense emerge from the literature on Legal Informatics, defined as "*the study of the structure and properties of information, as well as the application of technology to the organization, storage, retrieval, and dissemination of information*" [1].

Perhaps one could tend to underestimate the benefits that such a kind of model could provide, both in theory and in practice. The rhetorical traditional framework of legal studies and the need of an undeniable margin of discretionary power for the owner of the legal process (the legislator, the judge, the administrator, the private contracting party, etc.) might prevent one from even trying to develop such a model.

Instead we will try to explain the basics of such a model that one of the Authors (as a former PhD student in Civil Law and barrister) has been developing since 2004 in three monographs [2– 3 – 4]. The general rationale underlying our attempt is that of describing the mathematical proportions governing every legal assessment.

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The search for "objective indicators and objective evaluation process" in public administration organizations is a cutting-edge issue [5].

We will set out a model of general legal theory, hence neither confined to a certain sector of law, nor to a specific legal system: in other terms it would embody a newly conceived interpretation of *Natur der Sache* (a specific version of the doctrine of Natural Law [6]), based on mathematical rules.

It only needs to be able to manage technical tools and elementary skills of introductory mathematical literature [7]. In effects, some simple equations of second degree allow the conversion of legal values into numbers.

II. Framework Of Legal Theory

Our model assumes a certain technical idea of legal rules, a framework about how material Facts are converted into legal Values by the Legal Rule, about how legally Facts produce Effects. It is essential that legal Facts and legal Effects are shaped by the legal Rule under the vests of model / hypothetical Situations (*Tatbestanden* or Species facti). The concept of *Species facti* has been developed in Civil Law Countries [8], although even Common Law describes legal Situations by means of orderly Elements (for instance, while defining the concept of *Crime* [9]), what seems to be enough to make our model applicable thereto. Indeed, it is widely agreed that Law is a science of Values, and that it has to be axiologically founded and understood [10].

What seems to be innovative is the way we intend to derive from it a truly mathematical model.

III. Logical Structure Of The Rule

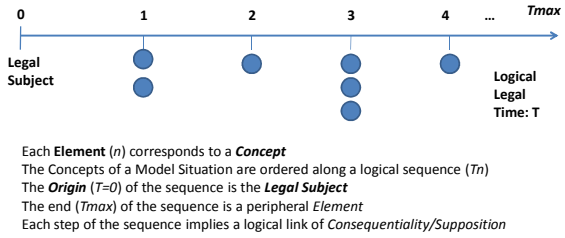
A Legal Rule can be defined as a device associating a Model Fact to a Model Effect, both linked by legal Causation. The fact is relevant when it triggers material human interests, according to the judgment of the legal system. Assuming such judgment as exogenous, the Rule vests the Facts translating them into a Model Fact Situation. This allows the Rule to be applicable in more than one single individual case: the model describes the features of Facts ordering them as a sequence of Elements, which compose the Model Situation. For ex., a material fact such as the birth of a baby "NN", is described under a model fact Situation of the birth of a baby in general: the birth implies the procreation, which implies a male and a female, which imply the humanity of both; finally any human being is a Subject (under the legal sense of the term). Every

Element entails a concept and accounts for one unit of Conceptual Mass (CM). Any step of the logical sequence of Elements can be associated with a Legal Logical Time (T).

In our above example:

Concept *a*: Legal Subjectivity: T=0 (origin)
Concept *b*: Mankind: T=1
Concepts *c* and *d*: Male, Female: T=2
Concept *e*: Procreation: T=3
Concept *f*: Birth: T=4

Orderly Sequence of Elements



Each **Element** (*n*) corresponds to a ***Concept***
The Concepts of a Model Situation are ordered along a logical sequence (*Tn*)
The ***Origin*** (*T=0*) of the sequence is the ***Legal Subject***
The end (*Tmax*) of the sequence is a peripheral *Element*
Each step of the sequence implies a logical link of *Consequentiality/Supposition*

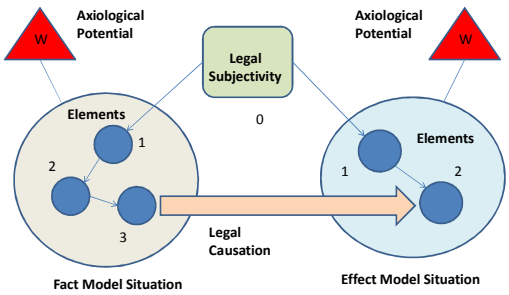
The Rule then links a Model Fact and a Model Effect: in our case the born child has the right to life. Even the legal effect is to be described as a Model (Effect) Situation composed of Elements ordered in a chain of logical sequential steps.

The Model Fact (F) and the Model Effect (E) of any Legal Rule (R) are linked by the Legal Causation (K): whenever F occurs (*Sein*), E has legally to follow (*Sein Sollen*).

Concept *g*: Life: T=5

The Situation converts the material Contingent Fact or the material Contingent Effect, corresponding to the Situation, in a legal axiological potential, in other terms in Legal Values (W).

A Diagram Scheme for the Legal Rules



IV. The Dimensions Of Legal Values

According to a renowned legal doctrine [11], the Legal Values possess two dimensions: Strength (how easy can the operator acknowledge the axiological potential of each certain Situation?); Height (how important is the human interest at stake in each Situation?). Moving from this theory, other scholars have observed, in qualitative terms, that Strength (S) and Height (H) are inversely linked [12]. The more a value own S, the less H is to be estimated. More H is associated with immaterial/spiritual values, whilst more S is associated with material/concrete values.

It seems to be the time to describe mathematically the axiological legal process.

V. The Basic Equations Of The Model

Is it possible to formulate equations for the process associating legal Values to legal Situations?

Yes, it is, in our humble opinions.

A. Conceptual Mass

In order to reduce the axiological process in quantitative terms via mathematical functions, one has to analytically describe the simplified fundamentals of such process.

The more we put CM in the lowest steps of the chain (Tn→ 0), the more each n unit of CM assumes H.

The more we put CM in the peripheral steps of the chain (T→Tmax i.e. its maximal cipher in the given situation) the more each unit of CM gains S.

S and H could be described as *momenta* of the CM, towards the origin of the sequence (the legal subjectivity: T=0) or towards the last chain of the sequence (the ultimate elements at Tmax).

Why?

Because when we put CM nearer to the Subjectivity, it impacts more directly the human interest: by definition, the Legal Subject is the center of attribution of any legally relevant interest [9].

Moreover, when we put CM nearer to the ultimately peripheral elements of the sequence, we make it more tangible and visible to the legal observer. In effects, the eye of the observer (i.e. the legal operator) is placed by the last Element, in the sphere of reality, the everyday world. To her what is nearer to the periphery, rather than to the legal origin, is more easily tangible.

The momentum of CM towards the origin/center of the chain might be named Functional Weight (FW); the momentum of CM towards the periphery might be named Structural Weight (SW).

For any single unit ("n") of CM, FW and SW can be described as f(T) in a string. The slope of the function is characterized by the following distinguishing features:

As regards FWn:

If Tn→ 0, then FWn→ its *maximum* (FWn→ 1 for each n unit of CM);

If Tn→T_max, then FWn↓.

As regards SWn:

If Tn→Tmax, then SWn→ 1 (i.e.its *maximum*: SWn→ 1 for each n unit of CM);

If Tn→0, then SWn ↓, the more the distance between Tmax and Tn.

It seems to be possible to express the above simple and simplifying ideas by means of the following equations:

$$n\epsilon\{N\},\ T\epsilon\{N\}\ .\alpha>0,\ 0 < FW_n \leq 1$$
$$n\epsilon\{N\},\ T\epsilon\{N\}\ .\beta>0,\ 0 < SW_n \leq 1$$

$$FW_{n=}\ f(T_n) = \frac{\alpha}{T_n}$$
$$SW_n = f(T_{max}-T_n)=f(\Delta T_n)=\beta/(\Delta T_n + \beta)$$

Right now we have associated the two *momenta* to each *n* unit of CM. Once we have described F and E of a certain legal rule R, we can calculate FWn and SWn of any n element of F and of E. The sum of FWn and SWn for all the n elements of the situation, gives the total amount of SW and FW associated with the specified Situation.

The legal synthesis of F and E via K adds other questions. One of the Authors has already addressed such issues hem in a previous work [2].

The value of the parameters α and β determines a series of trajectories of SWn and FWn curves. It could be suggested the use of Euler's Scheme to build up such series, as it is used for derivatives' payoffs evaluation in the realm of financial economics [13].

Now we can aggregate the *momenta* of a Situation:

$$FW_{tot} = \sum_{n=1}^{\infty}(FW_n)$$
$$SW_{tot} = \sum_{n=1}^{\infty}(SW_n)$$

(α, β)>0,

$$0 < SW_n \leq 1$$
$$0 < FW_n \leq 1$$

Summing up FWtot and SWtot, we obtain the total axiological potential (Wtot) of the Situation at issue:

$$W_{tot} = (FW_{tot} + SW_{tot})$$

FIG. 1 - Functional Weight FW=*f*(Tn)

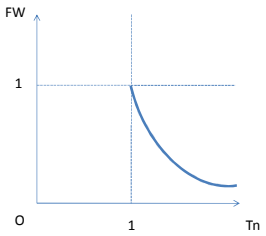


FIG. 2 - Structural Weight SW=f(ΔTn)

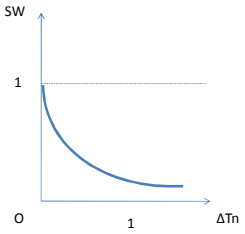


Fig. 1 and Fig. 2 above show FWn and SWn curves as functions respectively of Tn and ΔTn. Instead, the next table sums up the features of the two axiological dimensions, and describes their quantitative features allowing their mathematical representation.

The two Dimensions of Legal Values

Dimensions of Legal Values	Meaning in General Legal Theory	Basic Assumptions of this Article	Variables and Forms of the Functions
Height "**H**" (in this Article: Functional Weight, "**FW**")	How much is important the human interest at stake?	The more the Mass (**n**) is near to the Origin (0), the more Height it gains. The Subject is the Centre of any Human Interest	**H = f (Tn)** **H = a / Tn**
Strenght "**S**" (in this Article: Structural Weight, "**SW**")	How easily can the human interest at stake be acknowledged?	The more the Mass (**n**) is near to the Forefront (**Tmax**), the more Strenght it Gains. The observer is located at the Forefront	**S = f (Tmax – Tn) = = f (dT)** **S = b / (dT + b)**

B. Gap Mass

Some delicate problems seem to emerge considering that inside any string (i.e. any Situation) the same element/concept

usually occurs more than once; moreover, the paths leading from the origin to the ultra-peripheral element are usually more than one; finally, the repeated element of a string, can frequently assume different values of T in the different paths. Indeed, the functioning of our model necessarily requires to univocally associate a value of Tn to any possible n element of CM. Is it possible to overcome this problem?

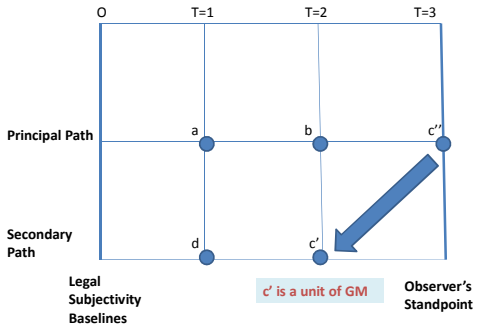
Yes, it is, in our humble opinions.

When a concept (say c) takes part of the structure of a Situation and is related with several different other Elements of the string, it is likely that it will assume more than one value of Tc (say Tc'=2 and Tc"= 3). So as to attribute to this concept one single value of Time, we shall propose the following reasoning.

The observer's standpoint is the privileged one. It is positioned at the forefront of the sequence. Seen from the periphery, it is easier to acknowledge c as positioned in Tc"= 3 than in Tc'= 2. The former is the value of T associated with the concept c. Nonetheless c occurs twice within the Situation in question, what affects the relations inside it, impacting on the axiological equilibrium of R. However, the observer's view is deviated, while the conceptual information concerning c is originated from Tc"= 3. The Mass Tc'= 2 is sort of obscured, it loses its capacity to deliver its conceptual information, though it continues to exist keeping its momentum and affecting the equilibrium of the Situation (H and S). The unit of Mass c placed at Tc'= 2 is converted into Gap Mass (GM).

Fig.3 shows an example of how Gap Mass emerges.

FIG. 3 - Gap Mass



GM creates uncertainty within the process of legal assessment of the axiological potential of the Situation, because the conversion of the unit of CM into a unit of GM, is to some extent non-deterministic: it is only the privileged consideration for the observer the criterion for the conversion of which CM into GM. This seems more a useful and reasonable convention, than a deterministic operation implied in the structural *Natur der Sache* of the Situation. It could be even suggested that GM is the mathematical logical basis for the separation of administrative and judiciary powers from legislative power.

Summing up all SWn and FWn associated with each unit of GM in a Situation, this quota of axiological potential is the total amount of W at disposal for the discretionary tactics of the operator. It is the non-deterministic Positive Law Value (PW). The operator bearing a legal power (e.g. the Constituent, the legislator, the administrative Bureau, the judge, the private contracting party etc.) can manage the process of legal assessment of the Situation within the boundaries of such margin. Of course the operator cannot deny the deterministic quota of axiological potential deriving from CM (Natural Law portion of potential: NW), but it can decide upon how to manage the portion of W emerging from GM, what leads the way to a managerial theory of Legal Rules and to a systemic theory of interactions among Legal Rules and Legal Values.

$$W_{tot} = NW + PW = (W_{tot}^{CM} + W_{tot}^{GM}) =$$
$$=[(FW_{tot}^{CM} + SW_{tot}^{CM}) + (FW_{tot}^{GM} + SW_{tot}^{GM})]$$

Should the operator try to manage even the deterministic quota of W (i.e. NW), the logics of the legal system would impose a remedy, in order to restore such unduly axiological operation.

VI. THE GOLDEN STRING

Of particular mathematical interest seems to be the relation between SW and FW in the easiest series of strings that is possible to conceive. Such strings do not exist in the reality of Positive Law into force. They might be named Golden String (GS).

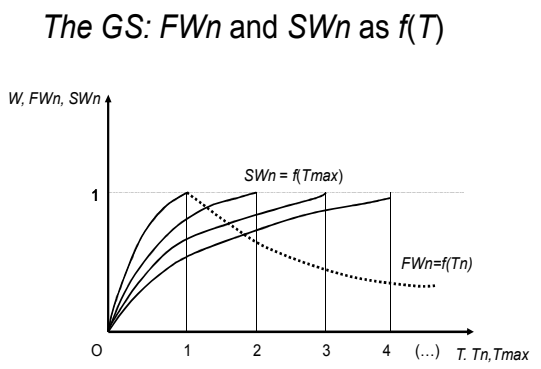
A GS is to be described as a string in which at any level of T is located just one unit of CM. The length of the string can increase from T=1 to Ti for any i€{N}.

In terms of legal general theory, the GS could be conceived as the universal lowest common denominator hidden inside Positive Law. That is to say that in any conceivable legal structure at each level of T corresponds at least one unit of Mass: usually more than one, but necessarily at least one. The GS is hence a hidden DNA of any legal Situation, therefore it appears useful to inquire into the ciphers of SW and FW associated to the GS for any level of complexity (i.e. for any level of Tmax).

It emerges that for each Tmax every n unit of Mass keeps unvaried its own FWn, independently of the value of Tmax, whilst the value of SWn does depend upon the value of Tmax.

This assertion can be easily understood pointing out that whilst FWn= f(Tn), instead SW(n)= f(Tmax- Tn) = f(ΔTn).

$$\Delta T_n = T_{max} - T_n$$
$$SW_n = f(\Delta T_n)$$
$$FW_n = f(T_n)$$
$$T_n = T_{max} \cdot FW_n^2$$



For this reason, increasing the complexity of the GS, SWn has to be reassessed, because the observer's standpoint has incrementally gone farther away.

VII. LEGAL ENTROPY

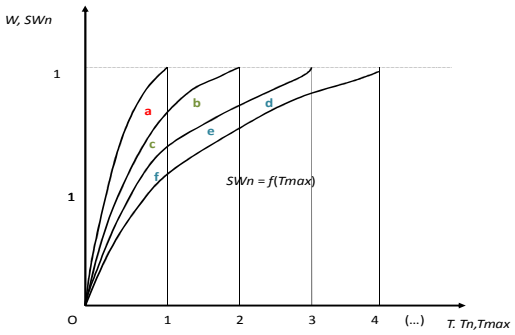
When in a GS Tmax↑, the variation in SWn before and after a marginal increase of Tmax expresses the balance, in terms of axiological potential, of the addition of one more level of complexity to R (i.e. of the increase of Tmax in the GS). Adding one more level of Tmax to GS, and putting one more Mass unit on the last ring of the chain, the cipher of SWtot on the one hand increases (the newly added Element at forefront of the string has its own SWn), but on the other hand a portion of SW for the other non-peripheral Elements units of Mass will be cancelled or, perhaps more suitably, it will just become latent inside the system.

Summing up such latent quotas of vague axiological potential, one could approximate the level of Legal Entropy (LE) within the String. It is a portion of axiological potential existing within the legal structure, though being no more active. It is latent and vague, due to the increase in complexity of the GS.

Finally it can be observed that if in a GS Tmax ↑, then initially Wtot↑ and finally it → to stabilize itself at a certain constant positive value. In our opinions, the mathematical interaction of the variables at issue will deserve to be more thoroughly investigated.

LE depends upon the trajectories of FWn and SWn, therefore upon α and β.

Thus, LE seems to have somewhat to do with the concept of Entropy developed within the Theory of Information [14]. Even LE depends upon the information capacity of Mass within a system.



In the graphical figure above, we can identify the different areas of legal entropy within the Golden String. To try and put it in the simplest words, when the level of complexity of the string reaches T=2, the S associated with the mass placed at T=1 has to be re-calculated, as the observer's peripheral standpoint has incrementally gone further away. In graphical terms the curve of S changes, and the area addressed to as "a" represents the legal entropy of the Golden String with complexity T=2.

If the golden string assumes a level of complexity T=3, the same recalculation of S has to occur in relation to the mass unit located at T=2; then new areas of legal entropy emerge, specifically the areas addressed to as "b" and "c".

When the level of complexity of the Golden String reaches T=4, then new entropy arises for the mass unit located at T=3: these areas are the ones addressed to as "d", "e" and "f" areas.

And so on for any incremental level of complexity of the string.

We can then hypothesize that more Legal Entropy denotes more Legal Welfare within the system. At this stage of our inquiry such result is not well established, but we can guess that higher level of latent axiological potential within the legal systems, can prove to be beneficial for all the subjects. Legal entropy is that part of existing axiological potential that is not active in the entitlement decisions and in the adjudication processes, that it to say that it forms an axiological asset that is not appraisable from any subject and cannot be directly challenged; yet, it does exist, it has been inoculated within the legal systems by its legal rules.

We hope that in the next future we can develop better understandings of the logics and the axiology of Legal Entropy. What we can express now is that the evaluation of axiological efficiency and equity of a legal (micro- and/or macro-) system seems to have much to do with the determination of the amount of Legal Entropy. Moreover the recent advances in relativistic thermodynamics [15] might prove to be beneficial for a thorough understanding of our theme.

For the time being we are not able to guess whether such construction might be combined with a *Quantum* interpretation of legal mechanics, though some elements can orientate in this direction, like the emergence of legal entropy and the constant

value of aggregate axiological potentials for the golden string, together with the identification of uncertainty potential associated to gap-mass. In such case the results of most recent *Quantum* mechanics could have a great impact in on our model [16].

VIII. GAME THEORY

At this point, it seems possible to adjudicate conflicts among legal Situations via a pattern of mathematical indicators. The interactions among legal entitlements have already been described by one of the Authors in previous works [2 – 3 – 4].

We would like to stress here that the conflicting interactions among the stakeholders can lead either to a static analysis of the equilibrium, or to dynamic and strategic equilibrium in the adjudication process. A rough version of this model has been applied to a case study, using the methods of Game Theory [4]. In this scenario the equations above formulated would provide for the payoffs of the Game Tree.

Sometimes the Game will be non-cooperative (e.g. in the adjudication of Tort Law: wrongdoer against hindered party); in other areas the Game is cooperative (e.g. when a Public Administration discretionary denies a permission it is axiologically allied with the counter-involved party against the demanding party; instead, when it decides to grant the permission, it is allied with the demanding party against the counter-involved party).

IX. ONE EXAMPLE

It seems to be the time to try and implement the above described model to one possible and significant example selected from positive law.

At this stage some preliminary remarks appear to be necessary, in order to explicit the basic criteria founding the selection of the situation to be described, given the specific seminal purposes of this article.

It seems to be wise to choose a legal situation applicable at the moment to the utmost number of national legal systems. Even though the model at issue can as well be implemented to the case law of any national legal system, it is the intent of this article what advises that we extrapolate a situation of international law, which is in principle applicable today all over the world.

Moreover, given the fact that this is the first formal attempt to implement our model, and the fact that in this article we have been able to describe nothing but the mere fundamental basics of our intended model, we will privilege a legal situation of little complexity, under a structural point of view, that is to say with not so many elements and temporal levels within its logical framework.

Keeping such preliminary remarks in mind, we will be discussing the elementary axiological equilibrium of the legal situation enshrined in art. 3 of Universal Declaration on Human Rights ("UDHR"), stating that "Everyone has the right to life".

It is apparent that the UDHR, for its truly legal nature, can satisfy the need for universal current applicability and low complexity of its pattern, in the sense meant by the variables of our model.

A. Methodological remarks about the implementation on Art. 3 UDHR.

As to the procedure we will be following in this introductory example, our protocol will be as simple and straightforward as possible, bearing in mind the objective of our attempt, i.e. demonstrating that our above described model is ready to be massively implemented to positive case-law, once it has been tested by the scientific community.

Hence, we will firstly describe the structure of the legal rule at stake, analytically determining the elements of the situations involved, both in the model fact situation and in the model effect situation.

Secondly, we will apply the basic equations above formulated to the legal rule already described, in order to obtain the ciphers of structural and functional weights for any element and situation involved, in order to be able to calculate the aggregate axiological numbers for the entire rule in question.

Finally, once identified the values of gap-mass and conceptual-mass in the legal rule concerned, we will complete the exercise by expressing some statements about the structure of the rule examined, both in descriptive and in normative terms.

B. Describing the elements of the legal rule under examination.

Interpreting a legal rule is not so easy as it may appear since the very beginning. In fact, the legal construction of the rules, both written and unwritten, involves some conventional and discretionary operations, to be conducted using lots of legal tools and, most of all, common and good sense. It is not surprising that the science of legal interpretation has got the dignity of an autonomous body of legal science [17].

In our example, the mere text of the simple rule at stake can be very differently interpreted. Moreover, any even slight different interpretation could trigger important consequences in legal terms. This is common sense!

However, what we would like to suggest right here since the very beginning of our procedure, is that our mathematical model can amplify any even tiny difference in the configuration of the rule. In other terms, assuming that - let's say - the legal capacity of a subject begins with procreation or with birth, or even within a specified time elapsed since birth, might prove to be not so relevant in most of the contingent cases, whereas a child after the birth has survived after the span of time in question. Instead, within the logics of our model, even if the contingent case has satisfied all the requirements at issue (procreation; birth; span of time after birth), the legal situation in the three cases is not the same by its truly axiological nature. The mere identification of a further

required concept and of a further required element within the structure of the legal rule has had a significant impact within the axiological equilibrium of the rule, because more mass has been inoculated in the situation, more temporal steps have been described, and all this has determined different axiological dimensions (Strength and Height). Last but not least, given that the relations between the elements are even relevant, the addition of more mass could affect the proportion between gap-mass and conceptual-mass, and consequently the conclusion in terms of positive-law-potential and natural-law-potential arising from the implementation of our model.

For these reasons, once the legal operator has identified some possible alternative options as to the interpretation of a rule, it should not be underestimated the potential impact, in legally axiological terms, of the options. This is why we believe that the interpret should analytically set out the logical structures of the alternatives, in order to be able to correctly apply the equations of the model, deriving from them the ciphers of the equilibrium and possibly test the comparative reasonableness of the options under scrutiny, both in terms of axiological efficiency and axiological equity for the entire legal (micro- or macro-) system.

To put it in other words, the discretion of the interpretative operations is not bad for our model, instead it can stress the virtual practical adding-value skills of a mathematical legal way of legal reasoning.

C. The interpretation of the model effect situation on Art. 3 UDHR.

In the case of the example in consideration (Art. 3 of UDHR), what might be reasonably controversial consists of the legal logical structure of its model fact situation.

In effect, the model effect situation of the rule at issue seems quite easy to be analyzed. The effect of art. 3 UDHR is the attribution of the right to life in favor of any child. Hence, putting the subjectivity at the origin of this string ("Subject" takes T=0), we can assume that the element "Child" takes the next temporal step (T=1) and finally the element "Life" takes the peripheral step of this chain (T=2).

Even in this case the structure of this model situation could be debated. Just to draw a possible example, one could argue that the "Child" of which we are discussing has to be any "Human" "Child". This question, within the logics of our model, is not trivial, given that the adoption of this further element would imply inoculating one more unit of mass and even one more temporal level of complexity within this structure. This is to say that such option may be set apart, once it has been explicated and assessed in legal terms: the elimination of this interpretative option is part of our model. The model can deliver the more added-value, the more extended number of reasonable options one is able to consider within its boundaries.

Thus, in our example we can decide to avoid the element "Humanity" in the model effect situation at stake, because we consider that "Humanity" is an element of the model fact situation of the same rule: we will later assume that "Everybody" in the sense of art 3 UDHR corresponds to "Any human Being". Once "Humanity" describes the structure of the model fact situation, concerning three human subjects (mother, father, child), the reference made within the model effect situation to any "Child" makes the element "Humanity" simply unnecessary and redundant in the structure of the model effect situation.

However, this way of reasoning could be challenged and the conclusions that one could get from the other construction might be compared to the one here proposes, in order to stress the benefits and the inconvenient of both theories.

D. The interpretation of the model fact situation of Art. 3 UDHR.

The alternatives seem to be even more variable as to the construction of the structure within the model fact situation of the legal rule set forth by art. 3 UDHR.

Who is "everybody" in the sense of art. 3 UDHR? Is any human being once procreated (alternative # 1), or any human procreated being once born (alternative # 2), or even any human being once procreated, born and survived after a certain span of time (alternative # 3)?

To translate such alternatives in more formal legal language, does the fact, from which the right to human life derives, coincide with procreation, birth or survival for a specified time?

The issue is not so simple, as its solution might involve enormous consequences in terms of regulation of phenomena such as abortion and medical procreation.

In effect, assuming that the right to human life begins with procreation, instead of birth, has certainly a great impact as to conditions and requirements for any legalization of abortion, as in this case the right to life of the child has to be compared against the right to life and/or to health and/or free determination of the mother (and eventually of the father).

The intended utility of our model is to provide axiological objective and transparent legal tools to assess and challenge the equilibrium fixed by statutory law, case-law and legal interpretations of the opposed values at stake.

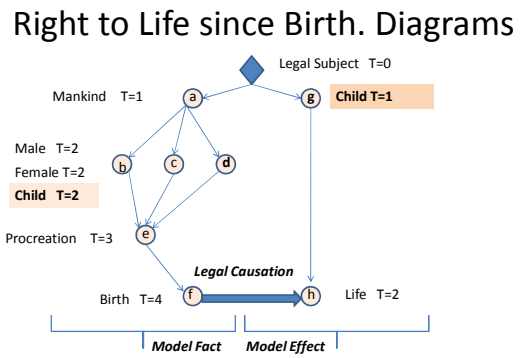
We will assume in this article that the right to life affirmed in art. 3 UDHR begins with birth, just for illustrative purposes, and at the same time we add the consideration that our model might already be easily applied to the other alternatives above itemized, so as to enrich the depth of its analysis.

Therefore, in our example the model fact situation includes the following elements:

The origin (Subjectivity) taking T=0;
The concept of Humanity (T=1);
The concepts of Father, Mother and Child (T=2);
The concept of Procreation (T=3);
The concept of Birth (T=4, i.e. Tmax in this model situation).

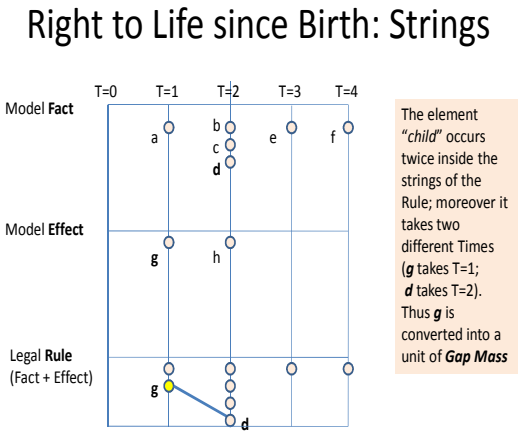
E. The logical structure of the legal rule stated in Art. 3 UDHR.

It is then possible to describe the logical structure of the legal rule at issue, once interpreted according to the alternative #2, in the following diagram.



It is possible to represent the same legal structure in a graphical form enabling to identify the logical sequences and strings of mass from the origin to the ultra-peripheral element of the chain.

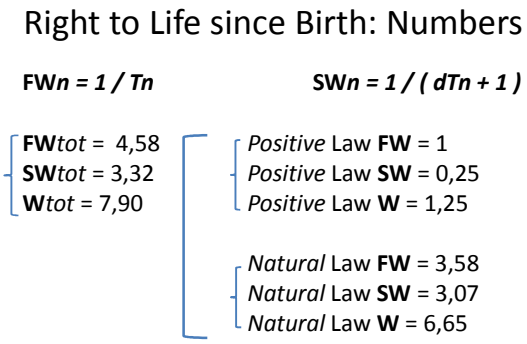
In particular, we will identify one unit of gap-mass, whose separate and distinguished consideration allows the interpret to assess the amount of axiological natural-law and positive-low potentials generated from the legal rule analyzed.



F. The mathematical equilibrium of the legal rule at stake.

After having described the legal rule in such an analytical way, through the position of each of its elements, it is time to apply the above basic equations to the legal rule at issue, in order to calculate the axiological legal potential deriving from this rule.

The results can be summed up as follows:



Such data can be considered as the inputs of a more complex decision-making process, aiming at assessing and governing the conflict between this legal situation and some other situations, eventually opposed against it.

X. Due Process Of Legal Axiology

We might apply the same protocol to describe the legal rule generating the right to property, and then compare the right to human life and right to property, which can conflict in a case of alleged legitimate self-defense of property causing the offender's death.

In any case of axiological conflicts, the comparison between the legal values at stake has not to be determined in abstracting ways, but instead in a concrete way. In effect, a comparison of this sort implies that we have first described a third model situation (*Tertium comparationis*), the one describing the pattern of the conflict. This is to say that the legal systems provides for different schemes of axiological conflicts (for instance: legitimate self-defense in Tort Law; legitimate self-defense in Criminal Law; the judgment of Injustice of the damage with some Tort-law provisions, as in the case of art. 2043 Italian Civil Code, etc.).

The pattern of *Tertium comparations* varies: it is possible that the elements of criminal-law relevant self-defense differ from those of tort-law self-defense. In this case, it can occur that the result of the contrast between the conflicting values at stake be different.

The decision-making criteria involved by using a certain *Tertium Comparationis*, imply that after the description of the three situations at stake (for ex., the situation generating the right to life; the situation generating right to property; the situation of our *Tertium comparationis*, such as Tort-law self-defense provision), we can identify the elements of mass shared by *Tertium* and any of the two conflicting situations. The weights of the shared quota of Mass consists of a fraction Q (0<Q<1) of the total amount of axiological potentials involved. This quota indicates the fraction of theoretical

potential that becomes active in this particular conflict, influencing the result of the adjudication process. The facts that the *Tertium* may differ, and that the quotas Q of any legal values depending on the circumstances of the conflict, give to the results a concrete and unforeseeable dimension. It is therefore admissible that the same values can prevail or instead be "defeated" while contrasting under different *Tertia comparationis* and under different circumstances.

This means that any legal entitlement can be challenged while conflicting with other entitlements, giving birth to an axiological adjudication process governed by the rules of a specified *Tertium*. Obviously, the description of the structure of the *Tertia* is a matter of positive law, depending on the provisions applicable to the conflict under scrutiny.

We want to add that the adjudication process ruling out the axiological conflicts among opposed legal values assumes a clear function of guarantee for any legal subject. The content of this legal guarantee might be stated as follows: a legal entitlement can be overruled by a different conflicting legal entitlement if and only if this occurs through an axiological adjudication process.

We propose to term this guarantee of legal entitlements as Due process of legal axiology.

XI. Conclusion

We have tried to describe the basics of a mathematical model for the quantitative analysis of law.

In our humble opinions such a method could prove to be beneficial for both legal theory and practice, once academically tested and validated, in order to foster more uniformity and objectivity within the legal assessments. One could build up a list of mathematical axiological indicators signaling the need for a legal remedy in a given situation. This would reinforce the legitimacy of the legal system and the transparency of legal decision-making processes. The legal rules would be shaped and challenged, so as to maximize the total output of the alternative feasible options in terms of axiological potential. A series of algorithms and protocols would support a newly construed normative engineering. Such mathematical tools would inform the legal governance of axiological assets. Every legal decision would have to be taken according to specified sets of axiological parameters.

Our model seems to be a suitable tool to the develop an axiologically-founded expert legal system to automatically govern algorithms for legal decision-making processes, within the boundaries of natural-law deterministic quota of potential; moreover one could construe indexes and parameters to objectively orientate event the discretionary decisions as to the positive-law quotas of axiological legal potentials.

Is this desirable?

Acknowledgment

We thank Prof. Siavash S. Sohrab (Northwestern University) for his useful feedback on our presentation at the American Conference on Applied Mathematics (American Math '12), with specific reference to his commentaries on the implications of the concept of Entropy in social science contexts.

One of the authors of this article had already studied entropy functions in the economic domain [18]; now it seems that the conditions of its application to legal science should be investigated.

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