Application of the Convergence and Divergence of two functions

Claude Ziad Bayeh

Abstract--- The Convergence and Divergence of two functions is an original study introduced and developed by the author in the mathematical domain, the main goal of introducing this study is to know the behavior of two functions in the Cartesian coordinate system. This study will allow us to determine if two functions are converging or diverging or running parallel to each other. This study can have many applications in mathematics, physics and engineering domains, for example in physics one can study the behavior of two objects in the space such as a rock and a satellite and see if they will collapse or not in the future by studying their functions, we can know also if they are converging, diverging or running parallel in every point in the space and this will allow us to control the satellite direction and to eliminate the possibility of collapse with the rock. In this paper, the author limited the study in developing the main concept in mathematics.

Key-words--- Convergence, Divergence, Parallel, Two functions, Variable functions, Mathematics.

I. INTRODUCTION

IN mathematics, the convergence and divergence terms are related directly to the integral of a function in the Cartesian coordinate system in which we can see if the integral is converging to a value or diverging to the infinite [1-9]. But till now, no one has introduced the study of the convergence and divergence of two functions in the Cartesian coordinate system, to see the behavior of the two curves if they are converging or diverging to each other. So that is why the author has developed this study in order to find more applications in mathematics, physics and engineering that may be helpful and useful.

In this paper, the convergence and divergence of two functions is an original study introduced by the author in mathematics, the main goal of introducing this study is to know the behavior of two functions in the Cartesian coordinate system. This study will allow us to determine if two functions are converging or diverging or running parallel to each other in each point in the Cartesian coordinate system.

In the second section, a definition of the convergence and divergence function is presented. In the third section, some examples are presented in order to give an idea about how the theoretical part will be applied in mathematics. And finally, in the section four, a conclusion is presented.

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II. DEFINITION OF THE CONVERGENCE AND DIVERGENCE FUNCTION

Let's consider that $CDF(x)$ is the Convergence and Divergence Function defined as:

$$CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} \qquad (1)$$

With f_1 is the first function in the Cartesian coordinate system.

And f_2 is the second function in the Cartesian coordinate system.

And ε is a very small value with $\varepsilon \to 0^+$

• If $CDF(x) > 1$, then we say that the two functions are Diverging.

• If $CDF(x) < 1$, then we say that the two functions are Converging.

• If $CDF(x) = 1$, then we say that the two functions are running parallel.

Whatever are the two functions, continuous or not, derivable or not, limited or not, we can study the Convergence and Divergence between them.

The study of the convergence and divergence between two functions is similar to the study of a function in the Cartesian coordinate system. In which we study the domain of definition of the function, then we study the asymptotes, then the derivative of the function, then we draw the table and see the locations of the convergence and divergence and finally we draw the curve of the $CDF(x)$.

• Firstly, we have to find the equation of $CDF(x)$ by applying the equation (1), for example:

$$CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} = \frac{2x+\varepsilon-1}{x-1}$$

• Secondly, we have to find the domain of definition of the function $CDF(x)$, for example:

$x \in]-\infty; +\infty[-\{2\}$

• Thirdly, we have to find if there exist asymptotes in the domain of definition in order to facilitate our study and our drawing of graph, for example:

$\lim_{x \to 1^+} CDF(x) = +\infty$ Vertical asymptote.

$\lim_{x \to -\infty} CDF(x) = 1$ Horizontal asymptote.

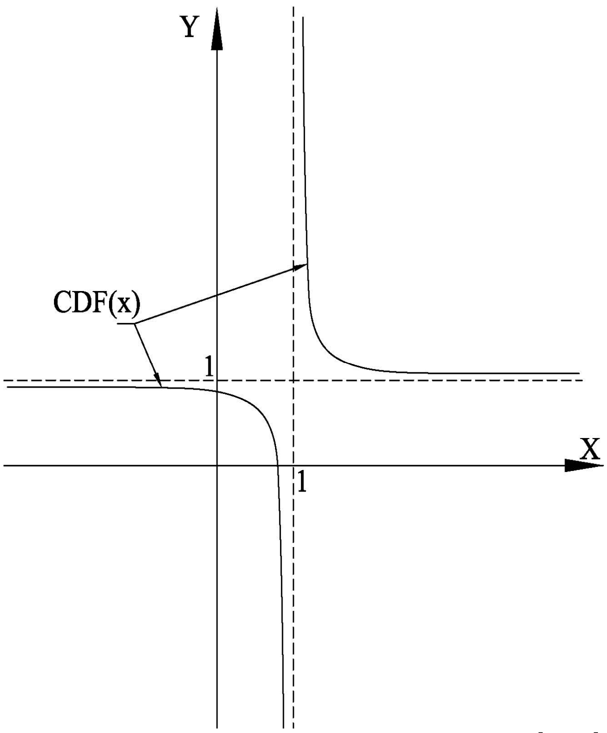

Fig. 1: represents two asymptotes, one vertical and one horizontal.

• Fourthly, we have to find the derivation of the function $CDF(x)$ in order to know how the curve is behaving, for example:
$CDF'(x) = \frac{-\varepsilon}{(x-1)^2} < 0$ Whatever is x

• Fifthly, we draw the table of study, for example:

x	$-\infty$	1	$+\infty$
CDF'(x)	-	‖	-
CDF(x)	1^- ↘ $-\infty$	‖	$+\infty$ ↘ 1^+
	Convergence	‖	Divergence

For $x \in]-\infty; 1[$ the two functions are converging
For $x \in]1; +\infty[$ the two functions are diverging

This table of study will allow us to know where the $CDF(x)$ function is converging, diverging or running parallel in the Cartesian coordinate system.

• Sixthly, we draw the graph of $CDF(x)$, for example:

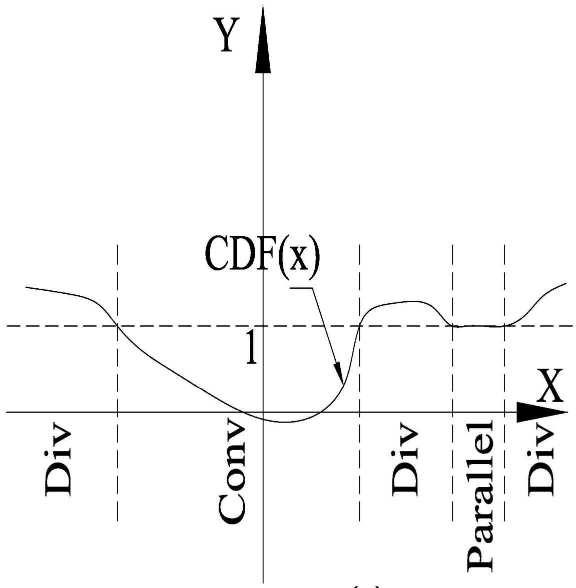

Fig. 2: represents the graph of $CDF(x)$.

In the drawn graph, we can see clearly the convergence, divergence and parallel regions in the Cartesian coordinate system.

A. Example of Convergence functions

$$CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} < 1 \qquad (2)$$

If the function $CDF(x) < 1$, then we say that the two curves are converging. That is mean they are approaching to each other.

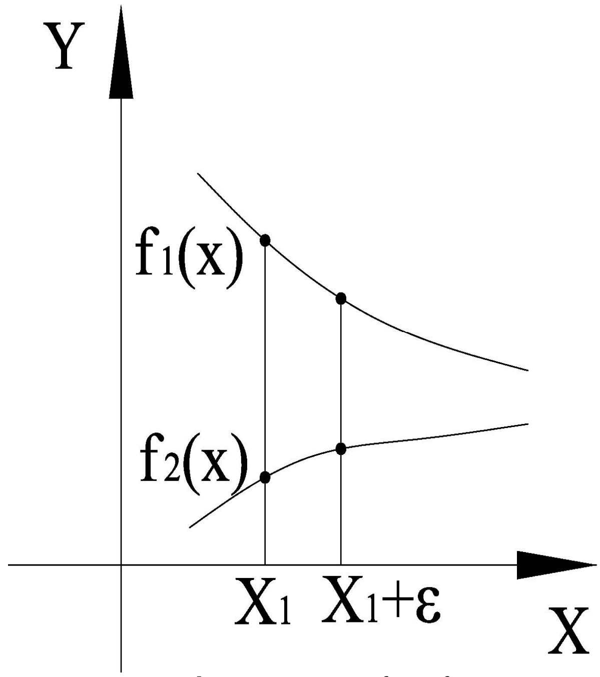

Fig. 3: represents the convergence of two functions.

B. Example of Divergence functions

$$CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} > 1 \qquad (3)$$

If the function $CDF(x) > 1$, then we say that the two curves are diverging. That is mean they are separating from each other.

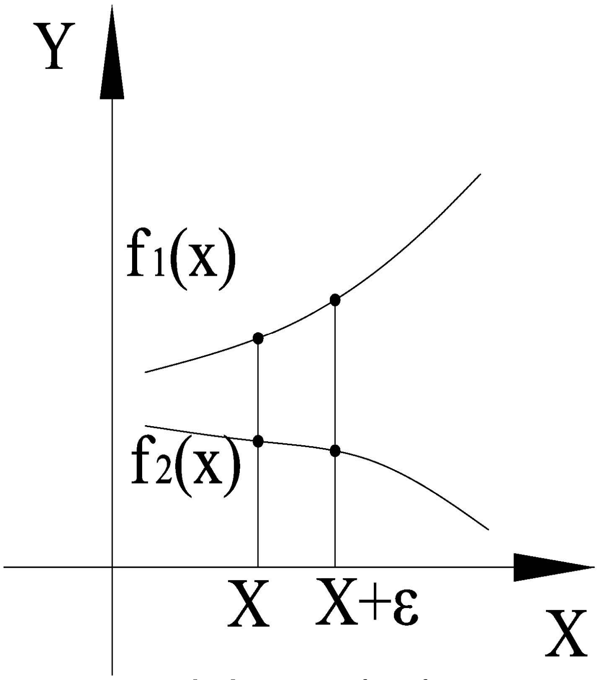

Fig. 4: represents the divergence of two functions.

C. Example of parallel functions

$$CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} = 1 \quad (4)$$

If the function $CDF(x) = 1$, then we say that the two curves are running parallel.

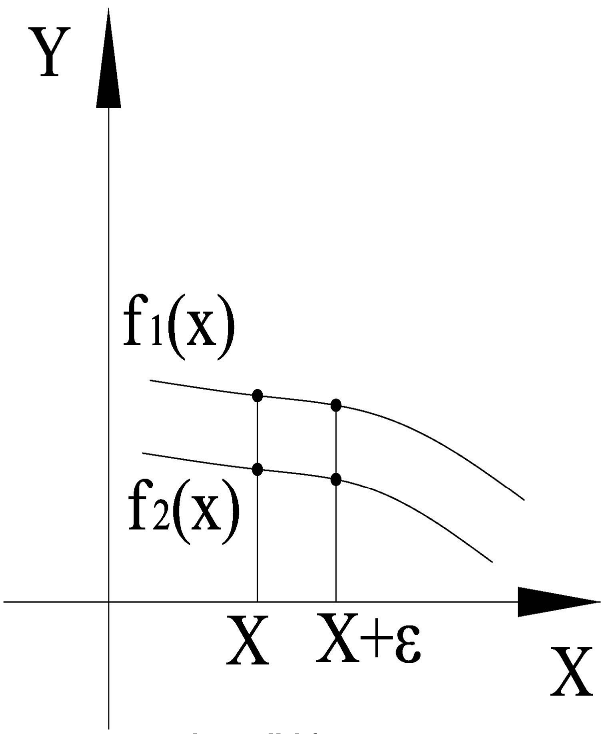

Fig. 5: represents the parallel functions.

D. Example of discontinuity of a function

In the case of discontinuity of a function in certain regions, we can't study the Convergence and Divergence in these particular regions because the two curves can't be compared. The discontinuity can be in a point or in a segment as shown in the figure 6 and 7.

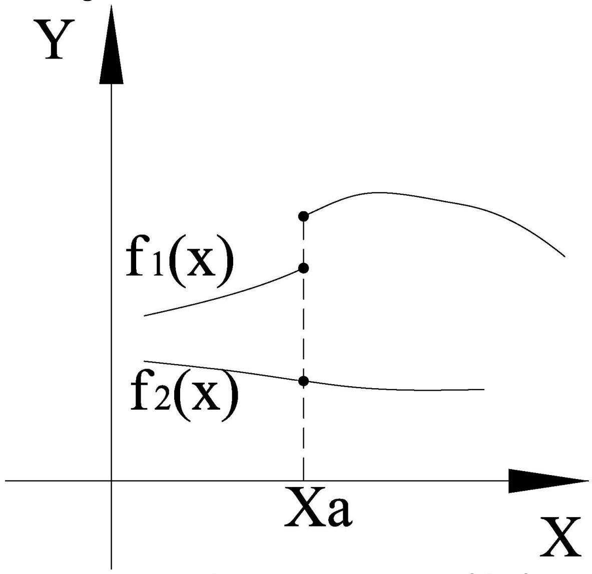

Fig. 6: represents a discontinuity in a point of the function f_1.

In figure 6, we can study the convergence and divergence of the two functions in whole domain except the point A.
So the studied domain is $x \in]-\infty; +\infty[- \{A\}$
Because we can't know the behavior of the curve in a discontinuous point.

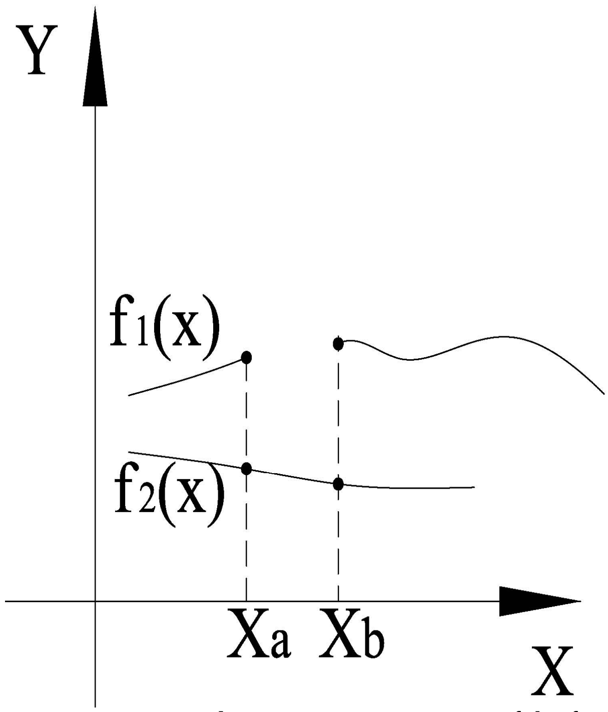

Fig. 7: represents a discontinuity in a segment of the function f_1.

In figure 7, we can study the convergence and divergence of the two functions in whole domain except the segment AB.
So the studied domain is $x \in]-\infty;+\infty[-[x_A;x_B]$
Because we can't know the behavior of the curve in a discontinuous segment that doesn't exist.

E. Example of formed curve CDF(x)

By applying the equation (1), the $CDF(x)$ curve is formed in the Cartesian coordinate system as shown in figure 8.
•If the curve $CDF(x) < 1$, therefore the two functions are converging.
•If the curve $CDF(x) > 1$, therefore the two functions are diverging.
•If the curve $CDF(x) = 1$, therefore the two functions are parallel.

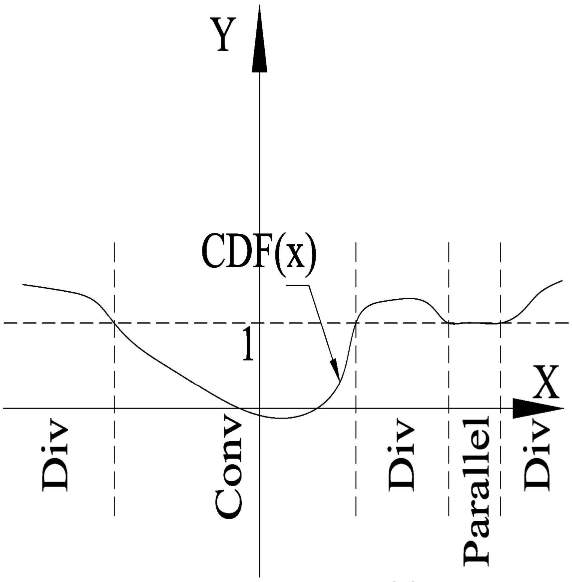

Fig. 8: represents the curve $CDF(x)$ in the Cartesian coordinate system with all possible cases of convergence, divergence and parallel functions.

III. PRACTICAL EXAMPLES

In this section, some practical examples in mathematics are given in order to let the reader know how to study curves and how to extract the convergence and divergence between two functions by knowing their equations.

A. Example 1

Let's consider the two curves:
$f_1(x) = x + 1$ and $f_2(x) = \frac{x}{2} + \frac{3}{2}$

We have to study the convergence and divergence of the two functions as following:
1- $CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} = \frac{x+\varepsilon-1}{x-1}$
The domain of definition is for
$x \in]-\infty;+\infty[-\{1\}$
Now we can study the function $CDF(x)$ as a normal function.

2- Study of the Asymptotes:
$\lim_{x \to 1^-} CDF(x) = -\infty$ Vertical asymptote
$\lim_{x \to 1^+} CDF(x) = +\infty$ Vertical asymptote
$\lim_{x \to -\infty} CDF(x) = 1$ Horizontal asymptote
$\lim_{x \to +\infty} CDF(x) = 1$ Horizontal asymptote

3- Derivation of the $CDF(x)$
$CDF'(x) = \frac{-\varepsilon}{(x-1)^2} < 0$ Whatever is x
We conclude that the curve is always descending.

4- Table

X	$-\infty$	1	$+\infty$
CDF'(x)	-	‖	-
CDF(x)	1^- ↘ $-\infty$	‖	$+\infty$ ↘ 1^+
	Convergence	‖	Divergence

For $x \in]-\infty;1[$ the two functions are converging
For $x \in]1;+\infty[$ the two functions are diverging
For $x = 1$ there is an intersection between the two curves.

5-Graph of $CDF(x)$

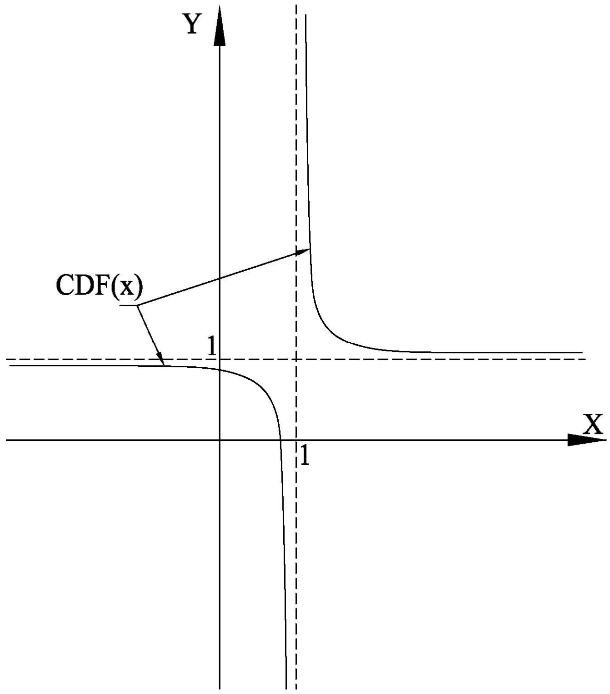

Fig. 9: represents the curve $CDF(x)$ in the Cartesian coordinate system.

6-Graph of the two functions

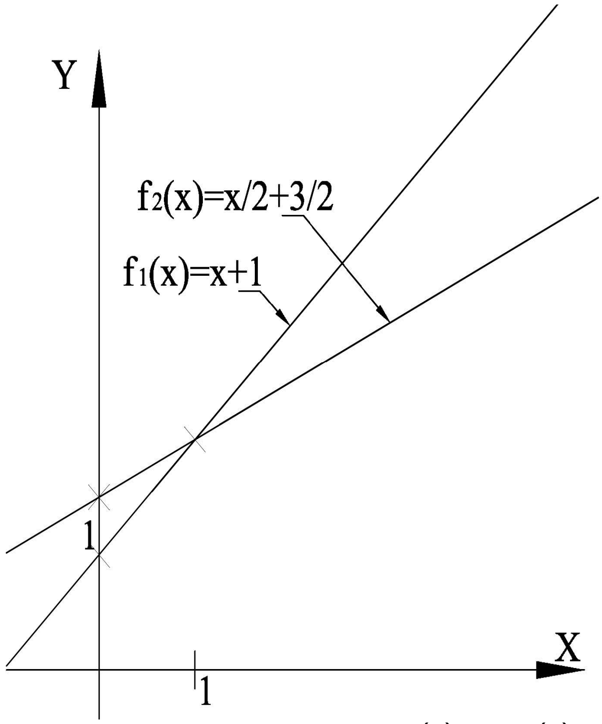

Fig. 10: represents the two curves $f_1(x)$ and $f_2(x)$ in the Cartesian coordinate system.

B. Example 2

Let's consider the two curves:

$f_1(x) = x + 1$ and $f_2(x) = x + 2$

We have to study the convergence and divergence of the two functions as following:

1- $CDF(x) = \lim_{\varepsilon \to 0^+} \frac{f_1(x+\varepsilon) - f_2(x+\varepsilon)}{f_1(x) - f_2(x)} = \frac{-1}{-1} = 1$

The domain of definition is for

$x \in]-\infty; +\infty[$

Now we can study the function $CDF(x)$ as a normal function.

2- Study of the Asymptote:

$\lim_{x \to -\infty} CDF(x) = 1$ Horizontal asymptote

$\lim_{x \to +\infty} CDF(x) = 1$ Horizontal asymptote

3- Derivation of the $CDF(x)$

$CDF'(x) = 0$ Whatever is x

We conclude that the curve is always constant.

4- Table

x	-∞	+∞
CDF'(x)	0	
CDF(x)	1 →	1
	Parallel	

For $x \in]-\infty; +\infty[$ the two functions are "Parallel" because $CDF(x)$ is always equal to 1.

5-Graph of $CDF(x)$

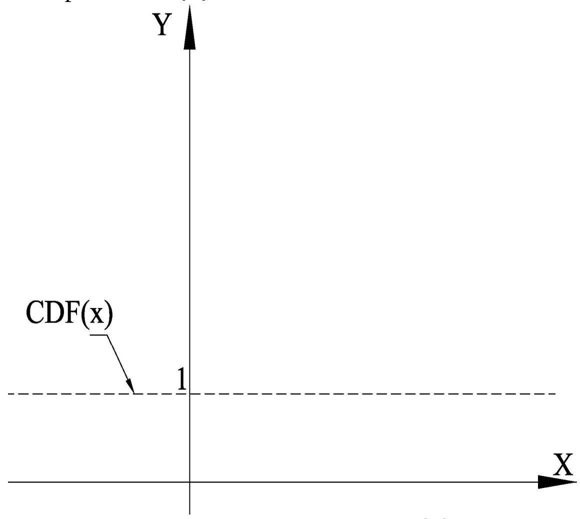

Fig. 11: represents the curve $CDF(x)$ in the Cartesian coordinate system.

6-Graph of the two functions

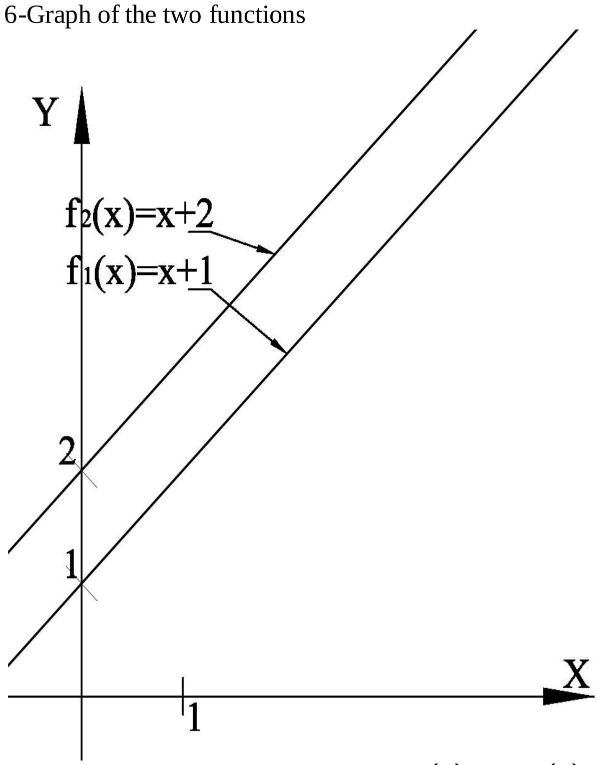

Fig. 12: represents the two curves $f_1(x)$ and $f_2(x)$ in the Cartesian coordinate system.

C. Example 3

Let's consider the two curves:

$f_1(x) = -x + 1$ and $f_2(x) = -4x + 3$

We have to study the convergence and divergence of the two functions as following:

1- $CDF(x) = \lim_{\varepsilon\to 0^+} \frac{f_1(x+\varepsilon)-f_2(x+\varepsilon)}{f_1(x)-f_2(x)} = \frac{3x+3\varepsilon-2}{3x-2}$

The domain of definition is for

$x \in]-\infty; +\infty[-\{\frac{2}{3}\}$

Now we can study the function $CDF(x)$ as a normal function.

2- Study of the Asymptotes:

$\lim_{x\to(\frac{2}{3})^-} CDF(x) = -\infty$ Vertical asymptote

$\lim_{x\to(\frac{2}{3})^+} CDF(x) = +\infty$ Vertical asymptote

$\lim_{x\to-\infty} CDF(x) = 1^-$ Horizontal asymptote

$\lim_{x\to+\infty} CDF(x) = 1^+$ Horizontal asymptote

3- Derivation of the $CDF(x)$

$CDF'(x) = \frac{-9\varepsilon}{(3x-2)^2} < 0$ Whatever is x

We conclude that the curve is always descending.

4- Table

x	-∞	2/3	+∞
CDF'(x)	-	‖	-
CDF(x)	1^- ↘ -∞	‖	+∞ ↘ 1^+
	Convergence	‖	Divergence

For $x \in]-\infty; 2/3[$ the two functions are converging

For $x \in]2/3; +\infty[$ the two functions are diverging

For $x = 2/3$ there is an intersection between the two curves.

5-Graph of $CDF(x)$

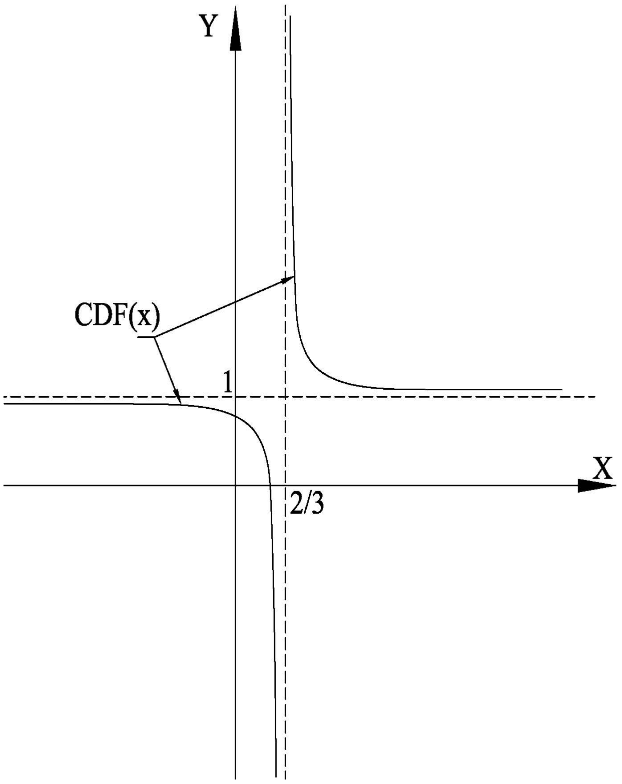

Fig. 13: represents the curve $CDF(x)$ in the Cartesian coordinate system.

6-Graph of the two functions

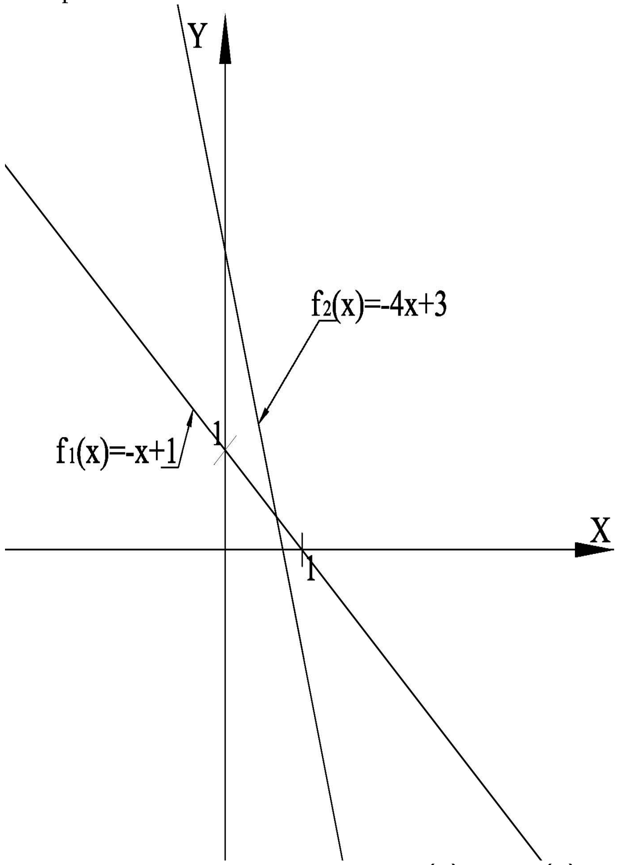

Fig. 14: represents the two curves $f_1(x)$ and $f_2(x)$ in the Cartesian coordinate system.

D. Example 4

Let's consider the two curves:

$f_1(x) = -2x^2 + 4x$ and $f_2(x) = 0$

We have to study the convergence and divergence of the two functions as following:

1- $CDF(x) = \lim_{\varepsilon\to 0^+} \frac{f_1(x+\varepsilon)-f_2(x+\varepsilon)}{f_1(x)-f_2(x)}$

$$= \frac{x^2 + \varepsilon^2 + 2\varepsilon x - 2x - 2\varepsilon}{x(x-2)}$$

$$= 1 + \frac{\varepsilon(\varepsilon + 2x - 2)}{x(x-2)}$$

The domain of definition is for

$x \in]-\infty; +\infty[-\{0; 2\}$

Now we can study the function $CDF(x)$ as a normal function.

2- Study of the Asymptotes:

$\lim_{x\to(0)^-} CDF(x) = -\infty$ Vertical asymptote

$\lim_{x\to(0)^+} CDF(x) = +\infty$ Vertical asymptote

$\lim_{x\to(2)^-} CDF(x) = -\infty$ Vertical asymptote

$\lim_{x\to(2)^+} CDF(x) = +\infty$ Vertical asymptote
$\lim_{x\to-\infty} CDF(x) = 1^-$ Horizontal asymptote
$\lim_{x\to+\infty} CDF(x) = 1^+$ Horizontal asymptote

3- Derivation of the $CDF(x)$

$$CDF'(x) = \frac{\varepsilon(-2x^2 - 2x\varepsilon + 2\varepsilon + 4x - 4)}{x^2(x-2)^2}$$

To study the derivative form of the function we should put the derivative equal to zero.
$CDF'(x) = 0$
$\Rightarrow CDF'(x) = \frac{\varepsilon(-2x^2-2x\varepsilon+2\varepsilon+4x-4)}{x^2(x-2)^2} = 0$
$\Rightarrow \varepsilon(-2x^2 - 2x\varepsilon + 2\varepsilon + 4x - 4) = 0$
$\Rightarrow (-x^2 + 2x - 2) = 0$ if we resolve this equation we find that the equation is always less than zero.
$\Rightarrow CDF'(x) < 0$

If we take $\left(f_1(x) - f_2(x)\right)'$ we find a summit
Therefore $\left(f_1(x) - f_2(x)\right)' = -4x + 4$
$-4x + 4 = 0 \Rightarrow x = 1$ a summit
In this summit the two curves are parallel therefore the $CDF(x)$ function will be equal to 1 for $x = 1$.

4- Table

x	$-\infty$ … 0	0 … 1	1 … 2	2 … $+\infty$
CDF'(x)	-	-	-	-
CDF(x)	1^- ↘ $-\infty$	$+\infty$ ↘ 1	↘ $-\infty$	$+\infty$ ↘ 1^+
	Convergence	Divergence	Convergence	Divergence

For $x \in]-\infty; 0[$ the two functions are converging
For $x \in]0; +1[$ the two functions are diverging
For $x = 1$ the two functions are running parallel
For $x \in]1; 2[$ the two functions are converging
For $x \in]2; +\infty[$ the two functions are diverging

5-Graph of $CDF(x)$

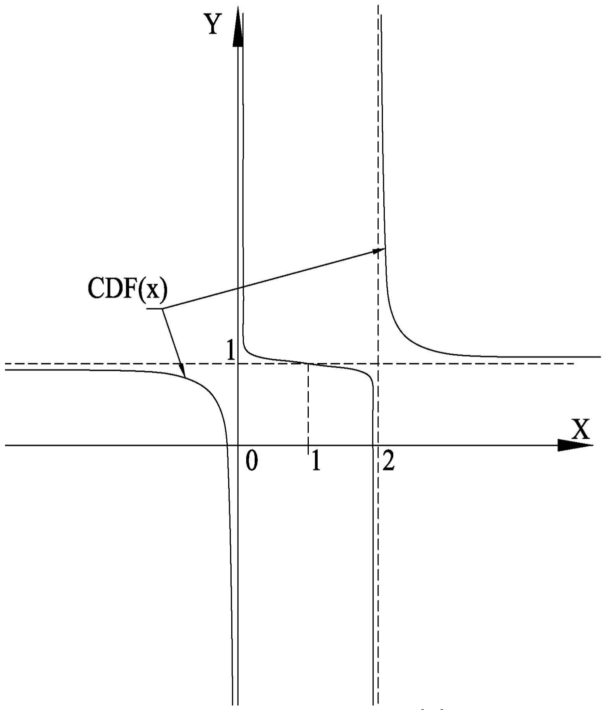

Fig. 15: represents the curve $CDF(x)$ in the Cartesian coordinate system.

6-Graph of the two functions

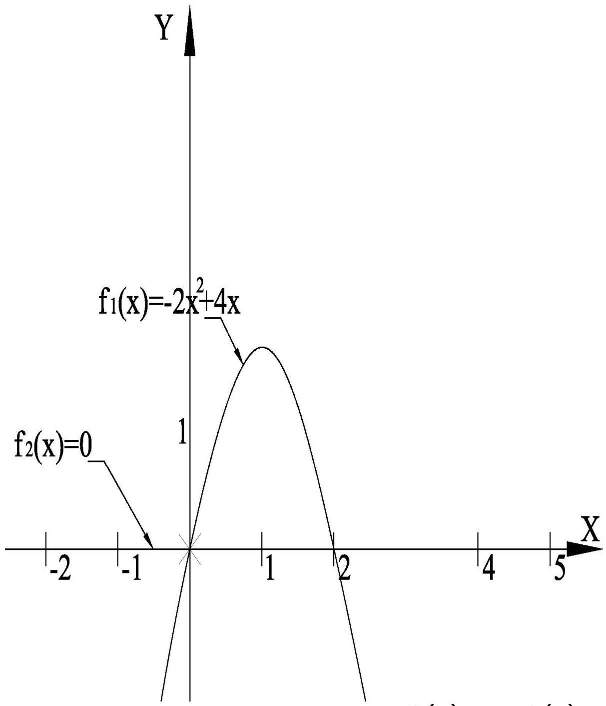

Fig. 16: represents the two curves $f_1(x)$ and $f_2(x)$ in the Cartesian coordinate system.

IV. CONCLUSION

The convergence and divergence of two functions is an original study introduced by the author in mathematics, the main goal of introducing this study is to know the behavior of two functions in the Cartesian coordinate system. This study will allow us to determine if two functions are converging or diverging or running parallel to each other. This can resolve many problems in physics and especially in Astronomy in which we can study the behavior of two objects that are running in the space and we can know if they are converging that is mean there is a crash in the space or if they are diverging or running parallel to each other. Many researches will follow this study in order to find more applications in several domains. The main concept of this study is presented in the section 2. Some examples in mathematics are presented and developed in section 3.

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