Mathematical aspects of acceptance sampling procedure

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***Abstract*—**A mathematical background generally important for the usage of acceptance sampling procedure is shown in this paper. A theoretical base of the Lot Acceptance Sampling Plans used to control large lots of different components purchased and installed during maintenance and overhaul is elaborated. The differences between main types of the Lot Acceptance Sampling Plans are explained. Considering the costs in the integral production−assembly chain, an economic way of items' inspection is chosen, and an acceptance scheme of the Philips sampling plan for the considered case is drawn.

***Keywords*—**Acceptable Quality Level, Dodge-Romig Plan, Lot Acceptance Sampling Plan, MIL-STD 105E, Philips Plan, Type I Error, Type II Error.

I. Introduction

IN general, a very complex energetic structure characterizes each large company in both the gas and electricity sectors. Hence, the processes of maintenance and overhaul are very complex in these companies. For example, in the electric–power industry there are different mechanisms used by specialists during these processes (many control techniques and methods are used for quality control and functionality diagnostics of the equipments). Periodical overhauls of thermal power plants, hydro power plants and cogeneration facilities represent the highest level of maintenance in electricity generation. During maintenance and overhaul, a number of provided components must be controlled by the quality control division in order to assure the safety of the plants. All this calls for selecting the category of maintenance strategy and engages considerable resources, manpower and time [1]. To ensure safe functioning of the electric power supply system, an expert approach to maintenance and reliability is crucial [2]. Only high-quality components should be used during maintenance and overhaul to ensure the reliability of the power system. Hence, it is necessary to select a suitable number of items (an adequate sample size) that have to be checked before installation regardless of using the items for maintenance, overhaul or common operation.

The procedure of acceptance sampling should be used in case of incoming large lots to determine whether to accept or reject a specific quantity of goods or materials [3], [4] because it ensures that customer's risk of receiving a bad lot is minimal, as well as supplier's risk of rejecting a good lot. Some useful computer supported designs, such as Visual Basic program for designing single-sample acceptance sampling plans, are being developed during time [5].

The Lot Acceptance Sampling Plan (LASP) or Acceptance Sampling Plan is an efficient method for acceptance of large lots. A sample is picked at random from the lot in the LASP's method, and a decision is made regarding the disposition of the lot based on information that was yielded by the sample. To choose a too large lot for inspection is a costly approach. Furthermore, testing can be destructive or 100% inspection can take too long in some cases. So, the LASP is developed as a method for checking a statistical reliable number of components that represent an incoming lot well.

A large number of components comes in lots in different industrial branches. Therefore, it is not possible for quality control experts to carry out 100% inspection of such lots. Because of this reason, the acceptance sampling has been often used in practice. Consequently, quality control experts must choose an appropriate LASP, such as Philips, Dodge-Romig or MIL-STD 105E.

There are not many quality control experts who know the mathematical background of the implementation of the LASPs well, even though the quality control divisions use the sampling procedure for testing the incoming lots routinely due to the costs of 100% inspection being high and 100% inspection taking too long [6]. Therefore, quality control experts have to study the LASPs theory completely, regardless of its complex mathematical base.

It should be accentuated that there are remarkable differences between LASPs and control charts as the common used quality control tool. LASPs focus on the product, in fact they are used for making decisions on products (accept or reject), and the activities of the quality control divisions were carried out towards the produced lots to ensure delivered products' quality. The control charts focuses on the process because they are used to inspect a process running (to regulate or not) and for making decisions on process improvements to achieve non-defective products. In general, the following

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question intrudes: how much time and resources are spent on the LASPs' implementation relating to the process improvements? There is no clear answer in literature to this question. It should be pointed out that there are significant benefits from LASPs if the quality control experts in a company know all advantages and disadvantages. Usage of LASPs cannot ensure deliveries of only good products. An optimal approach encompasses focus on prevention to avoid defectives and on the continuous process improvements at the supplier side, as well as the use of LASPs at the customer side can enable the realization of desired quality. Furthermore, collecting the results obtained by usage of LASPs during time can be very useful for the evaluation of suppliers.

II. Acceptance Sampling Formulation

The aim of acceptance sampling is to decide if the lot is likely to be acceptable, not to estimate the quality of the lot. Acceptance sampling is used in quality control practice when the cost of 100% inspection is very high, 100% inspection takes too long, and/or testing is destructive. The main advantages of using LASPs are:

- The risk of a bad lot acceptance is very small if a well equipped incoming control division with high professional control staff is organized in a company.
- The plans are relatively cheap and fast because only a small part of the lot (i.e. sample) must be inspected instead of the whole lot.

The controller should make a decision towards the status of the lot based on information that was provided from the sample n_i picked at random from lot.

There are two classifications of sampling in the theory of LASPs:

- Sampling by variables when the item inspection leads to a continuous measurement. The sampling by variables plans encompass LOT-PLOT and BENDIX that have a set of rules for making the decision whether the lot should be accepted or rejected [7].
- Sampling by attributes is used when the control leads to a binary result − either the item is conforming or nonconforming − or the number of nonconformities in an item is counted. The sampling by attributes plans encompass Philips, Dodge-Romig (after the authors Harold F. Dodge and Harry G. Romig) and MIL-STD 105E (replaced by ANSI/ASQC Z1.4 and ISO 2859).

The attribute case is the most common for acceptance sampling, wherefore the plans for sampling by attributes will be further considered in the paper. These plans are based on the binomial and Poisson distributions. A decision on acceptance or rejection of a lot is based on the number of defectives or nonconformities.

The sampling by variables plans are based on the Gaussian and Student distributions and the inspection result is a measured data which implicates the measurement and calculation. A decision on acceptance or rejection of a lot is based on the arithmetic mean and standard deviation of the sample.

Single, double and multiple sampling plans are three basic categories of LASPs. Single sampling plans are the easiest plans to use between those categories of the LASPs that use sampling by attributes principle. Single sampling plans are denoted as (n, c) plans for a sample size n. The lot is unacceptable if the number of defectives is larger than the acceptance number c. The decision on acceptance of a lot depends on counting the number of defectives in a sample n. Single sampling plans are the most common although not the most efficient in terms of average number of samples needed.

The double sampling plan is used if the result of the first sample n_1 taken from the lot is not informative enough, i.e. conclusive with regard to rejecting or accepting [8]. Consequently, the second sample n_2 has to be taken. Using a double sampling plan, the decision on acceptance or rejection is made as follows:

- The lot is rejected if the number of defectives in the first sample n_1 is bigger than the acceptance number c_2.
- If the number of defectives is between c_1 and c_2, the second sample n_2 has to be taken from the lot to establish the total number of defectives in both samples (n_1+n_2) and to compare it with the acceptance number c_2.

More samples are needed to reach a conclusion in the case of multiple sampling. In fact, additional samples can be drawn after the second sample n_2. Smaller sample sizes characterize this category of LASPs. There are k stages in the multiple sampling procedure which starts with taking a random sample of size n_1 from a large lot N and counting the number of defectives d_1. The number of defectives d_i is compared with the acceptance number a_i and the rejection number r_i for each i-th stage of multiple sampling until a decision is made. The main advantages of multiple sampling plans (MSPs) are a smaller number of total inspection items and an additional opportunity for acceptance of a lot. However, there are some difficulties with MSPs that are complicated to use, the possibility of error is greater, some problems with time resources might occur, and there is the uncertainty of not knowing how much sampling and inspection will be done on a daily basis.

In addition, the Skip Lot Sampling Plan and Sequential Sampling Plan are in use in quality control practice. In the Skip Lot Sampling Plan's procedure only a fraction of the submitted lots is inspected. The Sequential Sampling Plan represents the ultimate extension of multiple sampling.

III. Calculating the Average Sample Number and Average Total Inspection

A. Average Sample Number

The Average Sample Number (ASN) is an important characteristic of LASPs. Assuming all lots come in with a defect level of p, a long term average sample number can be calculated for any given double, multiple or sequential sampling plan. The number of samples for multiple LASPs can depend on the lots. Hence, the ASN represents the average of what can happen in many cases that include a constant level of incoming quality (constant level of defectives in the incoming lots). A plot of the ASN vs. the incoming defect level p describes the sampling efficiency of a given LASP scheme by the ASN curve.

The equation for an ASN curve of a double sampling plan is:

$$ASN = n_1 P_1 + (n_1 + n_2) \cdot (1 - P_1). \quad (1)$$

After some alterations, equation (1) is settled up as follows:

$$ASN = n_1 + n_2 \cdot (1 - P_1). \quad (2)$$

B. Average Total Inspection

The Average Total Inspection (ATI) is the next important term in relation to LASPs. ATI denotes the average amount of inspection per lot. In an ideal case, all the good lots will be accepted and all the bad ones will be rejected by using any LASP. In quality control practice, since the decision whether to accept or reject the lot depends on the state of a sample taken from the lot, there is always a possibility to make the wrong decision.

Naturally, no lot will be rejected if there are zero defectives in all inspected samples n_i taken from the lots N_i. When all inspected items are defective all lots will be inspected, and the amount to be inspected is N. The average number of inspected items per lot will vary between the sample size n and the lot size N if the lot quality is $0 < p < 1$.

If lots come consistently with a defect level of p and rejected lots are 100% inspected, the ATI can be calculated as follows:

$$ATI = n + (1 - p_a) \cdot (N - n), \quad (3)$$

where p_a is the probability of accepting a lot, N is the lot size and p is the defect level, for a LASP (n,c).

The equation for ATI if defectives are not replaced is as follows:

$$ATI = n + (1 - P_{a(e)}) \cdot (N - n). \quad (4)$$

Furthermore, the equation for ATI, after replacing the defectives and if during the replacement procedure some type of error occurs, is as follows:

$$ATI = \frac{n + (1 - P_{a(e)}) \cdot (N - n)}{1 - p_e}. \quad (5)$$

Example: The sampling plan with $n = 52$, $c = 3$, $p = 0.03$ for the lot $N = 10,000$ is under consideration. The calculation of ATI will include an appropriate value of p_a. For the considered case $p_a = 0.93$ for given $p = 0.03$, as it follows from the OC-table in which the range of p_a versus p is given.

p_a	0.998	0.98	**0.93**	0.845	0.739	0.62	0.502
p	0.01	0.02	**0.03**	0.04	0.05	0.06	0.07
p_a	0.394	0.3	etc.				
p	0.08	0.09					

By using (3) the Average Total Inspection value is calculated, i.e.:

$$ATI = 52 + (1 - 0.93) \cdot (10,000 - 52) = 748.$$

Obviously, this ATI would result in high costs because a total of 748 items was examined, i.e. 7.48% from the whole lot N = 10,000. Hence, the considered p = 0.03 shouldn't be accepted. It would be suitable to use p = 0.02 for which a more acceptable ATI = 251 is calculated (2.51% of the lot size).

The Incoming Lot Quality based on the series of fourteen *'p –ATI'* pairs is shown in Fig. 1.

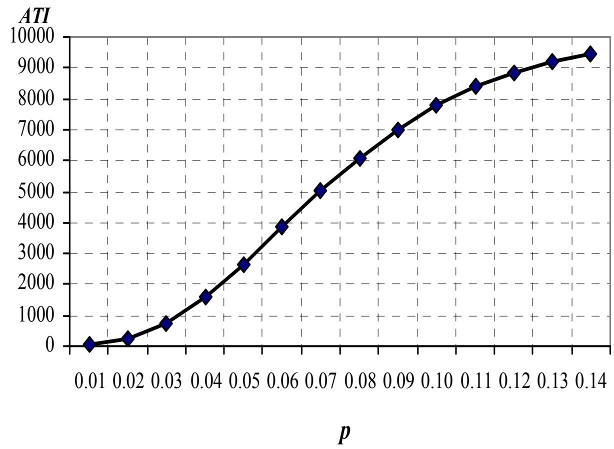

Fig. 1 Incoming Lot Quality

IV. The Operating Characteristic Curve

Generally, the incoming inspection of the lot is done by the customer, after the lot was received from the supplier. If many lots have been inspected, the following is certain: the lots will not always contain the same percent of defectives and this is the reason for using the Operating Characteristic Curve (OC curve). The OC curve plots the probability of accepting the lot on the Y-axis versus the lot fraction or percent defectives on the X-axis (Fig. 2). Hence, the OC curve shows the probability of accepting the lot depending on the percent of defectives, with the precondition that the lot contains a certain number of defectives.

The samples are taken at random from the lot. The lot size compared to the sample size is large (for example, n_1 = 45 and $n_2 = 2n_1$ = 90 for the Philips double sampling plan with the lot size N = 1,100 and p_a = 1%). Hence, removing the sample doesn't significantly change the state of the lot, no matter how many defectives are in the sample. However, the sampling procedure doesn't guarantee that all accepted lots will be good, and there is also a possibility that some of the delivered good lots can be rejected.

Considering the shapes of OC curves shown in Fig. 2, it can be concluded that the round OC curve is appropriate for real (smaller) values of n and c, while the ideal OC curve is appropriate for bigger values of n and c.

The Average Outgoing Quality (AOQ) is the expected average quality level of the outgoing component for a given value of incoming component quality. The equation for calculation of the AOQ for the single LASP when there is no error is as follows:

$$\text{AOQ} = \frac{p_a p \cdot (N - n)}{N}. \tag{6}$$

If all lots come with a defect level of exactly p, the OC curve for the chosen LASP (n, c) indicates a probability p_a of accepting such a lot, in the long run.

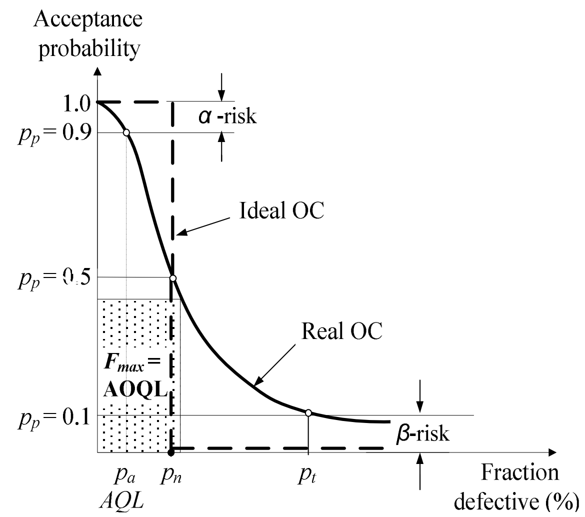

Fig. 2 Shapes of the ideal and real OC curves

Equation (6) is significantly simplified if $N >> n$, i.e.:

$$\text{AOQ} \approx p_a p. \tag{7}$$

A plot of the incoming quality p (X-axis) versus the AOQ (Y-axis) starts at $p = 0$ for AQL = 0. The AOQ returns to 0 for the incoming quality $p = 1$. In between these extremes, the AOQ rises, reaches a maximum (the AOQL), and then drops.

The AOQ is good if there is a small fraction of defectives in the lots. The outgoing quality is also good because the fraction of defectives in the outgoing lots is small. The incoming lots' quality is bad when the lots come in with a high-defect level. In such a case, the rejected items in lots are eliminated, rectified or replaced by good ones, so that the quality of the outgoing lots (the AOQ) becomes good. It is necessary to accentuate that errors affect the AOQ as follows:

- The AOQ increases if the lot with unacceptable quality is accepted, because the lot will probably not be additionally inspected.
- The AOQ decreases if the lot with acceptable quality is rejected, because an additional inspection is necessary.

Each LASP guarantees a certain average quality as a result of the received lots. A plot of the AOQ of the sampling plan (n = 52, c = 3), with N = 10,000 and the quality of incoming lots p = 0.05 is drawn in Fig. 3. When sampling and testing is non-destructive there is a common procedure, i.e. to inspect rejected lots totally (100% inspection) and replace all defectives with good items. The rejected lots become no defective. Hence, the only defects left are those in lots that were accepted. AOQs refer to the long term defect level for this combined LASP and 100% inspection of rejected lots process.

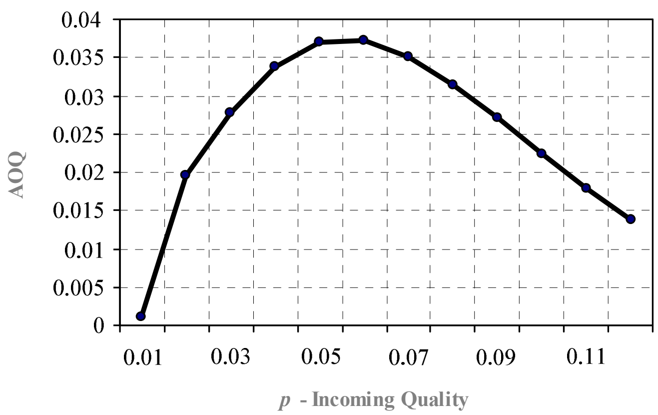

Fig. 3 Plot of the AOQ

Furthermore, the Average Outgoing Quality Level (AOQL) is an important characteristic of the LASP because it represents the worst possible long term AOQ.

The AOQL is the maximal ordinate on the AOQ curve which represents the worst possible quality that results from the rectifying inspection program. For example, AOQL = 0.0372 at p = 0.06 − taken from the plot of the AOQ versus p, for N = 10,000 and sampling plan (n = 52, c = 3). Graphically, the AOQL is a maximum rectangle that can be placed below the OC curve (as it is shown in Fig. 2).

Besides the AOQL there are two important points on the X-axis of the shown OC curve. The Neutral Quality is an indifference quality level (p_n). It points out that there is a 50:50% balance between two opposite possibilities, i.e. between the chances that the bad lot can be accepted and that the good lot can be rejected. The Acceptable Quality Level – AQL (p_a) is the maximal number of nonconformities per 100 items (or the maximal percent of nonconforming items) which is considered for inspection purposes. As a satisfying process mean, the AQL is the main criterion for the maximal percentage of nonconformities (defectives) that is acceptable both for the customer and for the producer in quality control practice.

Considering the ideal OC curve in Fig. 2, it can be noticed that the probability of lot N acceptance, based on the sample n, is p_p = 1.0 (100%). It is valid up to the Neutral Quality (indifference quality level p_n). The probability of lot acceptance is $p_p = 0$ if the percent of defectives is bigger than the Neutral Quality.

When sampling and testing is non-destructive, there is a possibility in quality control practice to inspect all rejected lots and replace all defectives with good items. Hence, all rejected lots become perfect and the only defects left are those in lots that were accepted.

A. Calculating the AQL and LTPD

The probability of observing exactly d defectives in a random sample of n items is given by the formula for the binomial distribution characterized by parameters n and p:

$$P_d = f(d) = \frac{n!}{d!(n-d)!} \cdot p^d (1-p)^{n-d} . \tag{8}$$

The probability that the number of defectives is less than or equal to the acceptance number c is done by the following equation:

$$P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} \cdot p^d (1-p)^{n-d} . \tag{9}$$

Obviously, the equations for calculating a sampling plan with a given OC curve are complex. If a sampling plan with 1-α probability of acceptance for lots with fraction defective p_1 and with β probability of acceptance for lots with fraction defective p_2 is considered, the AQL is p_1 and the LTPD (Lot Tolerance Percent Defective) is p_2. The LTPD is a designated high-defect level unacceptable to the customer. It is an important criterion for the LASP. Generally, the customer prefers the sampling plan to have a low probability of accepting a lot with a defect level as high as the LTPD.

The possibility of lot acceptance with defectives p_1, for the sample size n and the acceptance number c, and binomial sampling, is:

$$P_a = 1-\alpha = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} \cdot {p_1}^d (1-p_1)^{n-d} . \tag{10}$$

The possibility of lot acceptance with defectives p_2 is:

$$P_a = \beta = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} \cdot {p_2}^d (1-p_2)^{n-d} , \tag{11}$$

where α is the supplier's risk that a good lot is rejected (type I error) and β is the customer's risk that a bad lot is accepted (type II error).

The OC curve is designed in such a way that it passes through two designated points, usually the ones corresponding to the AQL and LTPD. It is possible to correct the procedure of acceptance in case of an error to ensure that the OC curve passes through the designated points. This can be shown, but only if the OC curve passes through (AQL, 1−α) and (LTPD, β).

The equations for calculating the AQL_e and LTPD_e are as follows:

$$\mathrm{AQL}_e = \mathrm{AQL} \cdot (1-e_2) + (1-\mathrm{AQL}) \cdot e_1 , \tag{12}$$

and

$$\mathrm{LTPD}_e = \mathrm{LTPD} \cdot (1-e_2) + (1-\mathrm{LTPD}) \cdot e_1 . \tag{13}$$

Hence, an OC curve is generally summarized by the AQL which describes what the sampling plan generally accepts and LTPD which describes what the sampling plan generally rejects.

In general, the customers prefer the plan with low probability of acceptance of the lots that have the same defective level as the LTPD. The Lot Tolerance Percent Defective is an important criterion of a sampling plan, so a special procedure for its usage is developed which is especially useful in the case of requesting a minimal sample size due to the limited resources of the customer's incoming control division. By implementing such a procedure, the chosen sampling plans ensure rejection of the lot if there is any defect in the sample.

It is important to accentuate that sampling plans are based on the defined correlation with the size of incoming lots. The procedure defines the percentage of the lot which must be inspected to guarantee with a probability P = 0.9 that a proportion of defectives in the lot is smaller than the prescribed level.

The sizes of the samples are based on the Hyper-geometry distribution. The Schilling table which is named after E. G. Schilling who developed this specific table [9], [10] can be used for carrying out the procedure. If there is a chosen sampling plan with the acceptance number $c = 0$, the production process should be managed on an average quality level less than 5% of LTPD to achieve a reasonably small probability of rejecting a good lot. But, the aforementioned average of the production process could not be ensured at all events. Hence, any other plan should be chosen in such a case because the sampling plan with the acceptance number c = 0 is not a good choice. Thus, a sampling plan with larger samples is chosen in control practice.

If some type of error occurs, the probability of acceptance is:

$$P_{a(e)} = \sum_{d=0}^{c} \binom{n}{d} p_e^d (1-p_e)^{n-d} \,. \tag{14}$$

The equation to calculate the AOQ is more complex than (6) if some type of error occurs during additional testing and if defectives were replaced with good items, i.e.:

$$\begin{aligned} \text{AOQ} = &\frac{npe_2 + p(N-n)(1-p_e)\cdot P_{a(e)}}{N(1-p_e)} \\ &+ \frac{p(N-n)(1-P_{a(e)})\cdot e_2}{N(1-p_e)} \end{aligned} \,. \tag{15}$$

The equation to calculate the AOQ if defectives are not replaced is:

$$\begin{aligned} \text{AOQ} = &\frac{npe_2 + p(N-n)\cdot P_{a(e)}}{N - np_e - (1-P_{(e)})(N-n)\cdot p_e} \\ &+ \frac{p(N-n)(1-P_{a(e)})\cdot e_2}{N - np_e - (1-P_{(e)})(N-n)\cdot p_e} \end{aligned} \,. \tag{16}$$

There is a simple rule to choose between the LTPD and AQL:

- LTPD should be used when producing smaller number of lots.
- AQL should be used when producing a lot of lots of a certain product.

B. *Maximum Percent Defective for Which Acceptance is Desired*

The sampling plans can be used for a variety of purposes depending on specific circumstances. For example, an AQL of 1% can be specified for inspections of major defects. This given AQL is not necessarily equal to the sampling plan AQL. Hence, it will be specified as Alt-AQL. The Alt-AQL should be interpreted as the maximum percent defective for which acceptance is desired, but should not be interpreted as a permission to produce defects. The Alt-AQL represents the break-even quality between acceptance and rejection because lots above the Alt-AQL are best rejected, and lots below the Alt-AQL are best accepted. All lots should be 100% inspected if a process is known to consistently produce lots with percent defectives above the Alt-AQL. The QC division should use a sampling plan to screen out lots not requiring 100% inspection if some lots are below the Alt-AQL. A sampling plan with LTPD = Alt-AQL can be used to reject the lots worse than the Alt-AQL, but at the risk of rejecting some acceptable lots. The single sampling plan (n = 230, c = 0) with an LTPD of 1% is appropriate if the Alt-AQL is 1%.

Assuming c = 0 and two alternative desired confidence levels, the required sample size is:

$$n = \frac{230}{\text{Alt-AQL}} \tag{17}$$

for a confidence level of 90%, and

$$n = \frac{300}{\text{Alt-AQL}} \tag{18}$$

for a confidence level of 95%.

C. *Type I and Type II Errors*

An important task of quality control staff is to avoid making errors during implementation of any LASP. Considering LASPs, it is basically assumed that the sampling procedure is free of error. However, each process involves a certain level of errors and the quality control staff must ensure that these errors cannot misrepresent the LASPs during complex sampling procedures. There are two types of errors with single LASPs: type I error when a lot with acceptable quality is rejected and type II error when a lot with unacceptable quality is accepted. The ATI depends on type I and type II errors so that it increases if a type I error occurs during testing and it decreases in the case of a type II error occurrence. The following equation could be used for the possible percent of defectives $P(B)$:

$$P(B) = P(A)P(\overline{E}_2) + P(\overline{A})P(\overline{E}_1) \,, \tag{19}$$

where A is the defective, B are the items classified as defective, E_1 is the good item being rejected as defective, and E_2 is the defective item being accepted.

The percentage of defectives is shown as:

$$p_e = p(1-e_2) + e_1(1-p) \,, \tag{20}$$

where $p = P(A)$ is the real percentage of defectives, $p_e = P(B)$ is the possible percentage of defectives, $e_1 = P(E_1)$ is the probability of a type I error, and $e_2 = P(E_2)$ is the probability of a type II error.

V. Comparison of Different Acceptance Sampling Plans

The main types of the LASPs are Philips [11], Dodge-Romig [12]–[14] and MIL-STD 105E [15]. MIL-STD 105E was used until 1995 and then replaced by ANSI/ASQC Z1.4 (ISO 2859). Considering the OC curves of these sampling plans, it can be concluded that the Dodge-Romig plan protects the customer, MIL-STD 105E protects the producer, while the Philips plan is somewhere in between. For example, if the MIL-STD 105E, Dodge-Romig and Philips double LASPs are compared, for lot N = 3,500, it can be concluded based on $n_1 + n_2$ that:

- MIL-STD 105E plan is the least demanding (125 + 125 = 250, for p_a as the basic criterion),

- Philips is the most demanding plan (135 + 270 = 405, for p_n as the basic criterion), and
- Dodge-Romig plan is in the midle (140 + 230 = 370, for p_t as the basic criterion, or 145 + 240 = 385, for AOQL as the basic criterion).

Regarding the AOQL value the situation is as follows:

- AOQL is 0.62% for MIL-STD 105E.
- AOQL is practically the same for the Philips (0.5%) and Dodge-Romig plans (0.5% or 0.52% depending on p_t or AOQL that was chosen for the basic criterion).

VI. Considering Some Aspects of Sampling Plans Implementation

Interesting recent papers on acceptance sampling in quality control are [16]–[21]. Furthermore, some efficient computer-aided acceptance sampling procedures are developed to minimise the time and effort to design the sampling plan, and the risk of accepting the bad lots and rejecting the good ones [22], [23].

Some significant aspects of sampling plans implementation using a mathematical background described previously will be considered in this section.

Example 1: The incoming lots should be inspected in a quality control division by using the Philips double sampling plan. Let each lot hold 1,100 items, and let p_{a} = 1%. A rejection probability of a lot that holds just 2% defectives should be researched.

For the defined Philips double sampling plan, the supplier's and customer's risks are equal: α-risk = 10% and β-risk = 10%. For N = 1,100 items and p_{a} = 1%, it is as follows:

$n_1 = 45$,
$n_2 = 2n_1 = 90$,
$c_1 = 0$,
$c_2 = 3$,
$p_n = 3\%$,
$p_t = 5.8\%$.

Furthermore, AOQL = 1.6% is defined in the Philips sampling plan table.

The OC curve for the considered case is shown in Fig. 4.

Considering this OC curve, it is obvious that the acceptance probability of a lot that holds 2% defectives is 0.71 (p = 0.71). Hence, an opposite probability, i.e. a rejection probability of a lot that holds 2% defectives is as follows:

$q = 1 - p = 1 - 0.71 = 0.29$.

The following ordinates should be calculated in case of drawing the AOQ curve:

$p_a \cdot 0.9 = 1 \cdot 0.9 = 0.9$

$p_n \cdot 0.5 = 3 \cdot 0.5 = 1.5$

$p_t \cdot 0.1 = 5.8 \cdot 0.1 = 0.58$.

Example 2: Let production line 1 produce 2–3% defective items in the lots, and let production line 2 produce 5–6% defective items in the lots. There is a problem of mounting the produced defective item. The problem causes the additional cost of \$5 per each defective item found during mounting in the assembly plant. The cost of inspection of an item is \$0.2. Let each lot which comes from both production lines hold 15,000 items. Considering the costs in the integral production–assembly chain, an economic way of items' inspection should be chosen, and an acceptance scheme of the Philips sampling plan for this case should be drawn.

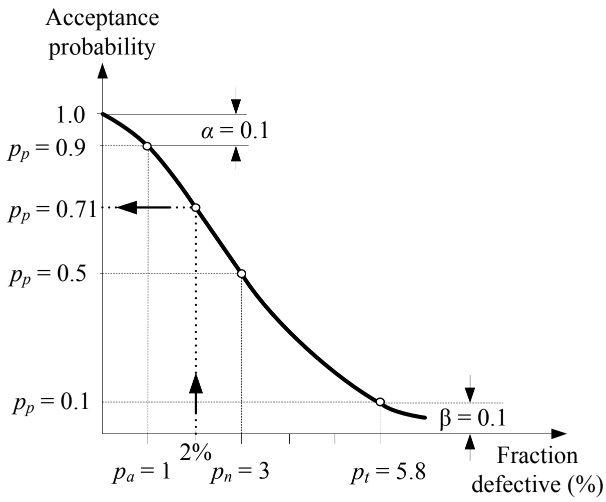

Fig. 4 OC curve for the Philips double sampling plan (N = 1,100 and p_{a} = 1%)

The marginal fraction defective p_b can be calculated using the following equation:

$$p_b = \frac{T_{\mathrm{item}}}{L}, \tag{21}$$

where T_{item} is the cost of inspection of an item and L is the cost caused by each defective item in the assembly plant.

The marginal fraction defective p_b by using (21) is:

$$p_b = \frac{0.2}{5} = 0.04 \ (4\%).$$

The calculated marginal fraction defective of 4% is an important criterion for using the Philips sampling plan in the considered case. The usage of the Philips sampling plan in the case of production line 1 which produces 2–3% defective items in the lots is obviously a correct choice. However, the usage of the Philips sampling plan in the case of production line 2 in which the percent of defective items is bigger than 4% is not economical. Therefore, the usage of 100% inspection is a better solution in such a case.

The following number of values of T_{item}/p is calculated by varying the lot's fraction defective p:

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
T_{item}/p	20	10	6.7	5	4	3.3	2.8	2.5

The curve which passes through the marginal fraction defective point is drawn in Fig. 5. The diagram in Fig. 5 shows that the inspection in the X-axis area below 1% is not reasonable due to high costs.

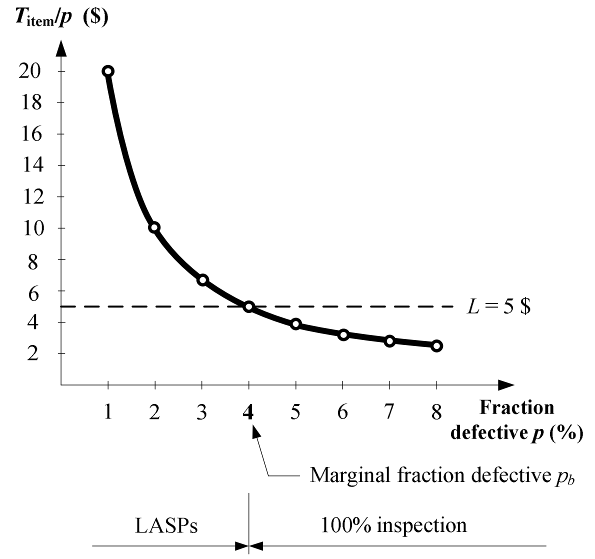

Fig. 5 The curve which passes through the marginal fraction defective point

The following data is taken from the sampling table (for the Philips sampling plan and production line 1, taking into consideration that $N = 15{,}000$):

$p_n = 3\%$ (because the fraction defective for production line 1 is 2–3%), $p_a = 1.9\%$, $p_t = 4.1\%$, and AOQL = 2%.

Furthermore, the following data is taken from the second sampling table: $n_1 = 180$, $n_2 = 2n_1 = 360$, $c_1 = 3$, and $c_2 = 15$.

The acceptance scheme of the Philips sampling plan for the considered case (production line 1) is drawn (Fig. 6).

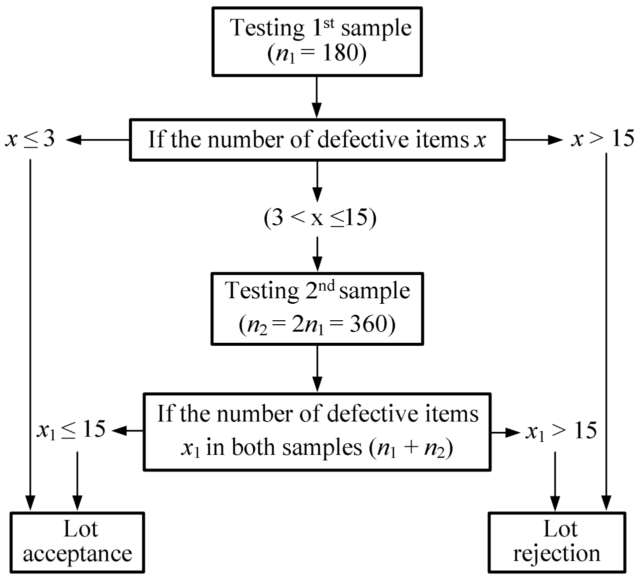

Fig.6 Philips sampling plan scheme for production line 1 (N = 15,000 and p_n = 3%)

VII. Conclusion

A mathematical background of acceptance sampling procedure analyzed in this paper demonstrates the Lot Acceptance Sampling Plans (LASPs) by attributes as advanced statistic and high-reliable quality control tools used by educated staffs of the incoming control divisions. Implementation of LASPs ensures high reliability of acceptance of a large number of different high-quality items (components/parts).

Using 100% inspection in quality control divisions of large enterprises where large lots must be inspected daily is practically impossible because it is too expensive and extremely time consuming. Hence, implementation of LASPs by attributes (Dodge-Romig, Philips and MIL-STD 105E replaced by ANSI/ASQC Z1.4 and ISO 2859) to accept large lots of various items in large enterprises (such as many companies in both gas and electricity sectors) is important for saving money and time without influencing the Average Outgoing Quality (AOQ) of the lots.

In general, a protection of companies in the energetic sector from accepting a number of defectives that can endanger their reliable and safety operation must be ensured due to their importance for the whole economy.

References

[1] N. Svarc, M. Jezidzic, and G. Vrtodusic, "Criterions for maintenance of transmission grid in market conditions (Kriteriji za odrzavanje prijenosne mreze u trzisnim uvjetima)," in *Proc. 7th Symposium HRO CIGRÉ (C5-10)*, Cavtat, Croatia, 2005, pp. 1−11.

[2] CIGRE Study Committees 23 and 39 Joint Working Group on Maintenance and Reliability (2000). Document: An international survey of maintenance and reliability, 152, Paris, France, February 2000.

[3] L. J. Krajewski and L. P. Ritzman, *Operations Management, Processes and Value Chains*. New Jersey: Prentice Hall, 2005.

[4] E. Banovac and I. Kuzle, "Applicability of the LASPs in the electric-power industry," in *Proc. IEEE Conf. Eurocon 2009*, Saint Petersburg, Russia, 2009, pp. 1152−1157.

[5] W. A. Levinson, How to design attribute sample plans on a computer. Avaliable: http://www.qualitydigest.com/july99/html/body_samplan.html

[6] E. Banovac and D. Kozak, "An analytic review of the characteristics of the Lot Acceptance Sampling Plans used for acceptance of large lots," *Int. Rev. Electr. Engi.*, vol. 3, no. 6, pp. 1070−1076, Dec. 2008.

[7] J. M. Juran and A. B. Godfrey, *Juran's Quality Handbook*. New York: McGraw-Hill, 1999.

[8] H. C. Hamaker and R. Van Strik, "The efficiency of double sampling for attributes," *J. Am. Stat. Assoc.*, vol. 50, no. 271, pp. 830−849, 1955.

[9] E. G. Schilling, "A lot sensitive sampling plan for compliance testing and acceptance inspection," *J. Qual. Technol.*, vol. 10, no. 2, pp. 47−51, Apr. 1978.

[10] E. G. Schilling, *Acceptance Sampling in Quality Control*. New York: Marcel Dekker, 1982.

[11] H. C. Hamaker, "Some basic principles of sampling inspection by attributes," *J. Appl. Stat.*, vol. 7, no. 3, pp. 149−159, Nov. 1958.

[12] H. F. Dodge and H. G. Romig, *Sampling Inspection Tables, Single and Double Sampling*. New York: John Wiley and Sons, 1959.

[13] C. H. Chen and C. Y. Chou, "Economic design of Dodge-Romig lot tolerance percent defective single sampling plans for variables under Taguchi's quality loss function," *Total Qual. Manage.*, vol. 12, no. 1, pp. 5−11, Jan. 2001.

[14] C. H. Chen, "Economic design of Dodge-Romig AOQL single sampling plans by variables with the quadratic loss function," *Tamkang J. of Science and Engineering*, vol. 8, no. 4, pp. 313−318, Dec. 2005.

[15] R. A. Banzhoof and R. M. Brugger, "Reviews of standards and specifications MIL-STD-1235 (ORD), single and multi-level continuous sampling procedures and tables for inspection by attributes," *J. Qual. Technol.*, vol. 2, no. 1, pp. 41−53, Jan. 1970.

[16] J. Shade, "Acceptance sampling in quality control," *J. Roy. Stat. Soc. A Sta.*, vol. 174, no. 4, pp. 1185−1186, Oct. 2011.

[17] G. S. Rao, M. E. Ghitany, and R. R. L. Kantam, "An economic reliability test plan for Marshall-Olkin extended exponential distribution," *App. Math. Sciences*, vol. 5, no. 3, pp. 103−112, 2011.

[18] K. Dumicic, V. Bahovec, and N. K. Zivadinovic, "Studying an OC curve of an acceptance sampling: a statistical quality control tool," in *Proc. 7th WSEAS Intern. Conf. on Mathematics & Computers in Business & Economics*, Cavtat, Croatia, 2006, pp. 1−6.

[19] K. Dumicic, V. Bahovec, and N. K. Zivadinovic, "Analysing the shape of an OC curve for an acceptance sampling plan: a quality management tool," *WSEAS Trans. on Business and Economics*, vol. 3, no. 3, pp. 169−177, Mar. 2006.

[20] W. Q. Qiang, L. Hua, and L. K. Hong, "Mixed-sampling approach to unbalanced data distributions: a case study involving leukemia's document profiling," *WSEAS Trans. on Information Science and Applications*, vol. 8, no. 9, pp. 356−379, Sept. 2011.

[21] C.-H. Jun, S. Balamurali, and S.-H. Lee, "Variables sampling plans for Weibull distributed lifetimes under sudden death testing," *IEEE T. Reliab.*, vol. 55, no. 1, pp. 53−58, March 2006.

[22] H. M. Judi, R. Jenal, and C. L. Chun, "Product inspection using computer-aided acceptance sampling for SMEs," *Eur. J. Sci. Research*, vol. 27, no. 1, pp. 6−15, Feb. 2009.

[23] E. Nicolae, "Computer program for the sequential-sampling plan for attributes," in *Proc. 5th WSEAS Intern. Conf. on Applied and Theoretical Mechanics*, Tenerife, Spain, 2009, pp. 124−127.