Stochastic Conception of Input-Output Model: Theoretical and Practical Aspects

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***Abstract*—**Complexity is immanent to economic-spatial structures and relations. There are not many models that allow for capturing the economic qualities of territories in a satisfactory manner. Input-output model can in spite of its simplicity be treated as one of success stories. Original version of input-output model is of deterministic character and reckons with deterministic inputs and outputs. In order to overcome this deterministic limitation when analyzing and forecasting the possible scenarios of territorial development, it is useful to introduce the interval estimates. Thus, the objective of the paper is to provide the insight into the stochastic version of input-output model. This innovative conception of input-output model will be examined from both theoretical and practical perspectives. Empirical utilization of the model is shown on the case of iron metallurgy sector in the Czech Republic.

***Keywords*—**Input–output model, Monte Carlo simulation, probability distribution, territorial development, economy of the Czech Republic, iron metallurgy sector.

I. Introduction

The description of economic – spatial structures and relations represents rather uneasy task. Complexity of economic – spatial structures constitutes only hardly surmountable hindrance for virtually all relevant approaches to the economic dimension of space, for more details see [12] or [15]. Notoriously known structural input – output model can in spite of its relative simplicity serve as one of success stories.

A structural input-output model was developed by V. Leontief. The model was not implemented for the sake of ideological reasons. When Leontief immigrated to the USA, a so-called closed model was implemented to analyze input-output relations of the US national economy. Since the economies of world powers, mainly the USA and the USSR, were at that time nearly autarchic, closed input – output model turned out to be pertinent for the analyses of these economies. Smaller economic entities represented by economies of small countries and namely by subnational regions have mostly more open economies. Moreover, in the current period of disappearing borders the exogenous relations are becoming increasingly important; see [5] or [14]. Put succinctly, in contemporary conditions a so-called open input – output model finds its rationale. At the same time, it has to be added that input – output model is far from perfect as it allows us to express neither agglomeration nor scale economies, see for instance [8] or [9].

A structural input-output model is an instrument that makes possible to capture an interface between components of economic system as well as the structure of relations with the surroundings. The model in a way represents the quantitative illustration of the production process. Structural input-output analysis is thus one of the most suitable instruments for the analysis of cross-sectoral relations in economics.

Usually we are dealing with input-output model for one territory. In case that the analysis is performed for more regions, we are entitled to talk either about interregional or multiregional input-output models. The difference between these two types of models consists in the way of modeling the connections among individual regions.

In multiregional input-output models, the connections among individual input-output models are not accomplished directly. Demand for the import from individual regions is directed into the special place located out of these models. Goods are differentiated according to particular economic sectors but not according to the territory of origin. The demanded goods are subsequently divided according to territorial principle and hence they can be assigned to concrete regions (for more details see [8]).

Contrary to multiregional input-output models that concentrate on the flow of the goods among regions, interregional input-output models emphasize the principle of interregional trade. This interregional trade is captured not only according to source and final regions but also according to source and final economic sector. In that way, all fundamental relations among economic sectors and regions are expressed and altogether they create a matrix depicting the relations of interregional input-output model (see also [8]).

Leontief type structural models use relatively simple mathematical apparatus and also interpretation of the results and their causes is easily understandable. On the other hand, in case of the nonlinear relations in an economic system the results obtained from the linear model might be distorted,

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especially if the required values are not sufficiently close to the initial state of the system in both static and dynamic sense. In such a case it is possible to consider the description of the system and its solutions using the system of linear and nonlinear differential equations. Moreover, in case of a strong stochastic character of the reciprocal relations in a system the result of the simple deterministic structural model analysis could be significantly inaccurate. The usage of other methods when solving the input-output models can resist this inaccuracy.

The traditional, basic version of input – output model is of deterministic character and reckons with deterministic inputs and outputs. Input values and results obtained from the calculations of the deterministic version of the original Leontief structural model are naturally only deterministic and hence they can be considered to be at most the point estimates in the sense of probability theory.

In order to overcome this deterministic restriction when forecasting and analyzing the possible scenarios of development, it is necessary to estimate from the interval estimates, which provides us with much better informational value, instead of the deterministic point estimates when entering the input values. In this case, we enter the lower and upper interval limit and the mode of a given probability distribution. This interval can be derived on the basis of the expert estimations in combination with existing statistical data.

The output of the model is represented by the amount having a probability distribution with its mean and standard deviation, which implies also wider utilization in the sphere of forecasting the future scenarios of the development of analyzed regions as well as retrospective analysis of the possible past alternative developments. Indeed, the reconsideration of traditional input – output model can provide us with surprisingly useful results, for details see [11].

In our paper, we attempt to show the nature as well as possible applications of the stochastic version of input – output model. In contrast to traditional, deterministic version of input – output, its stochastic variant is not commonly used. Nonetheless, the nature of stochastic input – output creates a true challenge for its possible applications in spatial sciences. Empirical dimension of our paper is embodied by stochastic input-output model of the iron metallurgy sector of the Czech economy.

II. Input – Output Essentials

In contrast to the export base model, see [3] or [13], the input-output model emphasizes the relations between the individual sectors of the regional economy. The grounds of the model are the relations (flows) between the individual branches, inputs (imports) and outputs (exports). The table structure of the input-output model is shown in Fig. 1. Let us assume that the economy of a region is formed by n sectors the interrelations of which are expressed by the n-th grade square matrix (n x n) of inter-branch relations V. Furthermore, l inputs comes into the region; these inputs get divided among all the n branches, which is expressed by the primary inputs matrix E of the grade l x n. Similarly, m export demands are the output of the regional economy. All n branches participate on these n demands as it is expressed in the final demand (consumption) Y matrix of the grade n x m. By summing all the rows or columns of the matrices we obtain the vector of gross production X of the grade n which represents the value of the total annual production of the region's all individual vectors. The number of sectors n determines the detail fineness of the entire model.

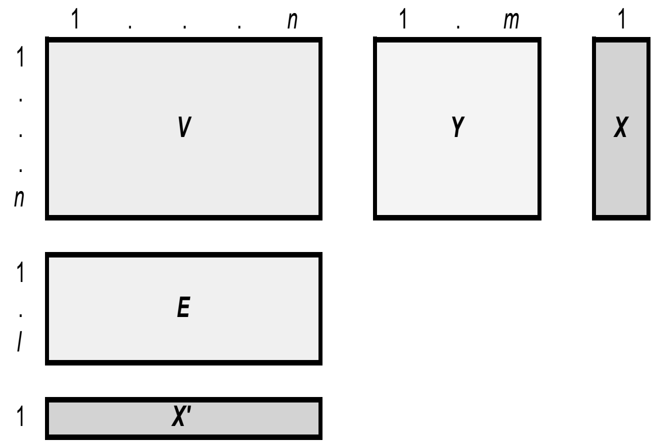

Fig. 1: Table structure of the regional input-output model

The simplified models make do even with a few sectors (e.g. agriculture, heavy industry, light industry, others), one input and one output. For simplicity we will assume that we have only one type of output and one primary input. Hence, the matrix Y and E change into vectors; see Fig. 2.

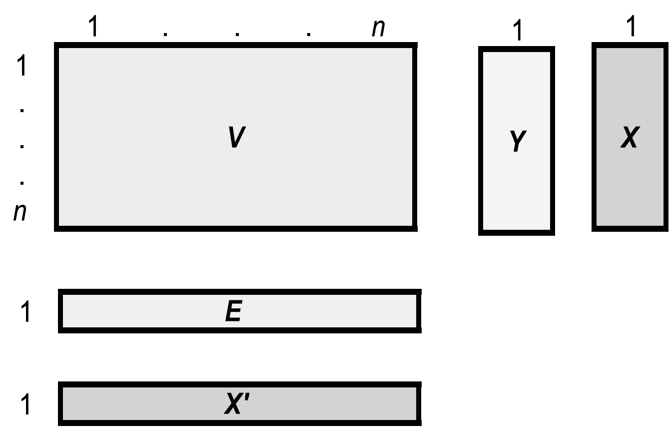

Fig. 2: Structure of the simplified input-output model

If we add up the values v_{ij} in the i-th row of the matrix V in this scheme and add the value y_i of the of the vector Y, we will obtain the appropriate value of gross production x_i:

$$x_i = \sum_{j=1}^{n} v_{ij} + y_i . \tag{1}$$

Thus we can trace in what volume the i-th sector supplies the other sectors with its products (the values v_{ij}). The value of the final consumption comprises the consumption of private consumers, the state sector and export. If we wish to make a clear distinction between these individual final demand components, we ought to go over to the complete model with the final consumption matrix Y, see Fig. 1.

Similarly, it is possible to add up the elements of the matrix V in its individual columns where after adding the primary inputs e_j – we get the value of the gross production x_j again:

$$y_j = \sum_{i=1}^{n} v_{ij} + e_i. \quad (2)$$

Thus we can find how much production the j-th sector takes from the other sectors for its own production needs (values v_{ij}) and how much of all the inputs are comprised by the primary inputs e_j – the outputs of households, state subsidies, import, etc.

Instead of the absolute inter-branch relations (flows) matrix V, the relative (technological) direct consumption A coefficients matrix is often used. The values of this matrix can be obtained by:

$$a_{ij} = \frac{v_{ij}}{x_j}. \quad (3)$$

Thus equation (1) changes to get the following form:

$$x_i = \sum_{j=1}^{n} a_{ij} x_j + y_i. \quad (4)$$

The equation above can be transformed to the matrix form of an equation for all the sectors (i = 1 through n):

$$X = AX + Y. \quad (5)$$

When isolating the gross production vector X, we obtain:

$$X = (I - A)^{-1} Y, \quad (6)$$

where I is the unit square matrix of the grade n. The matrix $B = (I - A)^{-1}$ is called the Leontief inverse matrix. It represents the multiplier of the model, its values b_{ij} are called the full material consumption coefficients. They state by how much the sector i has to produce more in order to increase the production for the sector j by one unit.

The three basic types of problems can be solved using the structural input-output model expressed by the (5), and (6):

- We know the total production X and we are looking for the final consumption Y;
- We know the final consumption Y and we are looking for the total production X;
- We know some vector X components, some vector Y components and we are searching for the other ones.

On the basis of the linear algebra knowledge we can deduce when a problem leads to the unambiguous solution. We will ignore the combined problem in this paper. The first type problem is trivial and always gives an unambiguous final solution for the matrix A and vector X. The second one has an unambiguous solution only when the matrix $I - A$ is regular.

There is also a need to require all the values of the vector X and Y be positive so that the solution is meaningful. Although these conditions can be mathematically specified, we will not introduce them in this paper as they are rather complicated.

III. Towards the Stochastic Version of Input – Output Model

An imperfection of input-output models consists in their static character. These models describe the relations among the individual resorts and relevant inputs and outputs in one specific moment – usually based on data coming from the system of national accounts for the appropriate year. Models allow analyzing the links among the particular industries, to identify key and crisis resorts and to find an answer to the question what would happen if a model component changed (e.g. export demand would decrease). Input-output models do not enable to describe economy the behavior of the economy in a dynamic way. It limits their usage in the field of prediction and prevention.

All the elements of the matrix of direct consumption coefficients A and vectors of the total consumption Y are deterministic in the model, i.e. we can consider them to be dot estimations in the context of a statistical theory. To overcome this deterministic limitation of the acquired results for the preparation of prognoses and analyses of the possible future scenarios it is desirable to take into account the interval estimations for input data. Contrary to the deterministic dot estimations, these interval estimations are more predicative.

One of the possible approaches towards the stochastic dynamisation of input-output models can be considering all basic parameters of the concrete model as random (stochastic) values with a certain probability distribution. Being random values we consider both the vector elements of final consumption Y, total production X or primary inputs E but matrix elements of the direct consumption coefficients A.

Such a model dynamisation puts high-level requirements on mathematical knowledge and probability theory knowledge. Responding a question how will a distribution of the output model quantities look like (e.g. vector X) when we know the parameters and distribution of the other values (vector Y) and structure matrix of the model A itself it means to pursue quite a complex transformation of these distributions. If we replaced the individual statistical parameters of the model by random quantities with a defined probability distribution the model complexity would increase in a way that it would not have solutions using traditional mathematical and analytical tools.

A linear transformation of a random vector whose elements have a normal distribution can be accomplished analytically (task solution $Y = (I - A)\ X$), for different distributions it would be more difficult. But matrix inversion containing the random quantities (task solution $X = (I - A)^{-1} Y$) cannot be solved in such a way.

In this case we can use numerical methods, particularly Monte Carlo method. The term Monte Carlo represents all the proceedings of the numeric task solutions based on repeated task simulation that has random test character, see for details [7], [10] or [16].

Generally, this term means all procedures to solve mathematical, physical and other problems realized by multiply repeated random tests. The quantity estimations are gained statistically and they have probability character. The method of Monte Carlo enables through repeated model simulations done as random tests to create an empirical distribution of output random vector of the model (e.g. final consumption vector) that can be tested by means of standard statistical tools for a shape and distribution parameters or to compare these parameter values for different simulations in the same or different initial conditions, see for details [1], [2] or [6].

For the effective calculation it is necessary to accomplish a huge number of these random experiments. Random experiment is not realized as a real experiment but it is simulated in a computer with the help of random figures. The practical creation of the random figures is very complex, time-consuming and has partial imperfections. The practice prefers a creation of the pseudo-random figures. These figures are formed in a computer and they are the result of the usage the proper algorithms and have the required qualities – they are random and independent.

The Monte Carlo method was formulated and for the first time used in 1940-ies in the US. The author was John von Neumann who used a roulette technique to generate random figures with the usage of which he simulated different processes (for instance neutron movement in the water). Generation itself proceeded in a computer. With regard to the computer speediness it was possible to get relatively reliable and precious behavior estimation of the simulated activity in a real time.

Over time, it became evident that it can be used for the solution of other complex technical, economic, transport or stocking tasks. Generally, the above mentioned method can be used and applied wherever the solution is dependent on probability and at the same time the different solution is difficult or even impossible.

The benefit of the Monte Carlo method is the fact that it is not necessary to know the internal relations between the results of the tests and task solutions we are looking for. It means the method can be used for the solutions of different tasks. Another important fact is that only one probability model can be applied for the solution of different tasks and vice versa for one concrete task solution it is possible to create a few various simulation schemes.

The success of the calculation using Monte Carlo method is generally determined by three basic factors:

- Quality of the random (or quasi-random) number generator,
- Selection of the rational algorithm of the calculation,
- Checking the accuracy of the acquired result.

The next advantage is timing of the simulation. The simulation time increases linearly depending on the number of variables for Monte Carlo method while for different procedures usually increases exponentially.

IV. Used Probability Distributions

In our approach, the input data designated for the calculation of the structural input-output model are a random variable. It is possible to use some of the appropriate probability distributions for the expert estimations of these input data. In our stochastic model we accept a hypothesis that the expert estimates shall respect one of the commonly used probability distributions and that the concrete type of the used distribution shall be conveniently chosen by an expert, see for details [4].

A. Uniform Distribution

The uniform distribution R (a; b) represents the simplest form of the probability distribution for the stochastic estimation of input-output model's parameters. It concerns the distribution with the constant probability density on the interval (a; b):

$$f(x) = \begin{cases} 0 & if\, x \leq a \\ \dfrac{1}{b - a} & if\, a < x \leq b. \\ 0 & if\, x > b \end{cases} \qquad (7)$$

The values a, b are the parameters of this distribution. These values can be interpreted as the minimum and maximum possible value of a given quantity being modeled. An independent expert should also understand these values in this way when carrying out the appropriate estimations.

The disadvantage of the interval distribution is the sensitivity to the extreme values as well as the fact that all the values from the interval (a; b) are considered be of the same probability.

Instead of uniform distribution the suitable one-mode probability distribution defined on the finite interval of a random variable (e.g. the triangular distribution or the Beta distribution) especially comes into consideration. Similarly, even other practically usable probability distributions are possible to be used when generating the expert estimates (e.g. the Gamma distribution or the Gaussian distribution).

B. Triangular Distribution

The interval estimation with the optimum value which uses the triangular distribution TR (a, b, c) is the refinement of the interval estimation based on the uniform distribution R (a, b). In this distribution, the probability density is not constant any more on the given interval, instead it has the shape of a triangle, i.e. at first, it linearly increases until it reaches the optimum value (the mode of a given distribution) and then it linearly decreases until it reaches zero again, see Fig. 3.

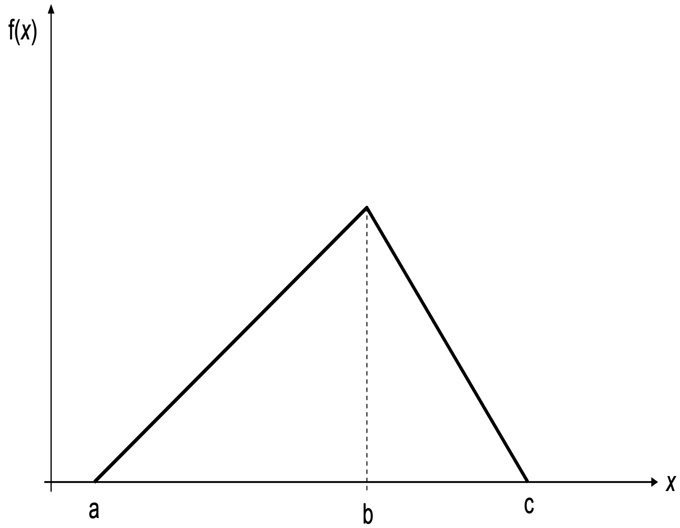

Fig. 3: Probability density of the triangular distribution *TR* (*a*, *b, c*)

Analytically, the probability density function for the triangular distribution *TR* (*a*, *b*, *c*) can be written as:

$$f(x)=\begin{cases}0 & if\, x\le a\\ \dfrac{2}{(b\text{-}a)(c\text{-}a)} & if\, a<x\le b\\ \dfrac{2}{(c\text{-}a)(c\text{-}b)} & if\, b<x\le c\\ 0 & if\, x>c\end{cases}. \tag{8}$$

If we want to enter input values of input-output model in an interval way we need to set interval limits and the mode with a help of experts.

From the expert estimate's point of view, the parameters of the triangular distribution have the following meaning:

- Parameter *a* means the minimum value, the lower ("pessimistic") expert estimate,
- Parameter *b* represents the optimal (expected) value, the mode of the distribution,
- Parameter *c* is the maximum value, upper ("optimistic") expert estimation.

The mode's value *b* is usually equal to the point estimate of a given parameter being modeled.

Fig. 4 shows the generated triangular distribution histogram of a given parameter using the Monte Carlo Method. The Random Numbers Generator from the MS Excel (the function RANDOMNUM ()) was used as the initial generator. This MS Excel function generates random numbers from the interval (0; 1) with the uniform distribution. The uniform distribution was subsequently transformed to the needed triangular distribution.

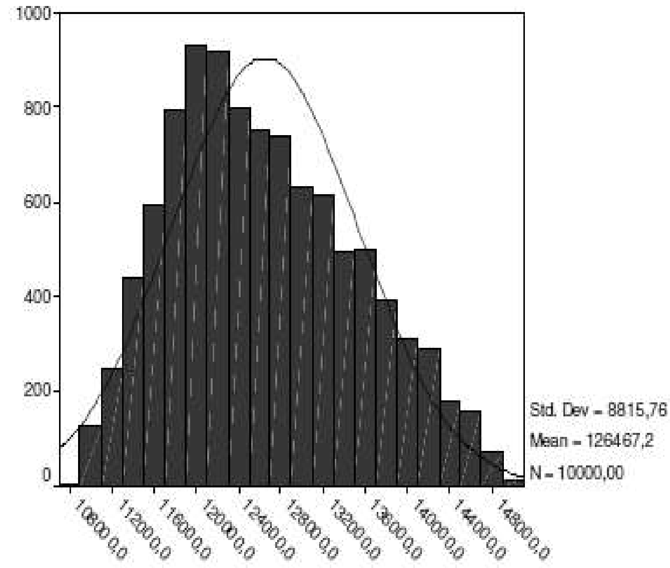

Fig. 4: Simulated triangular distribution histogram using the Monte Carlo method

Due to the linearity of the used input-output model, all the outputs of the model have the triangular distribution too, i.e. if we use triangular distribution for vector *X*, the values of final consumption or the vector *Y* elements have also the triangular distribution. As an example the histograms of 9 elements of the vector *Y* distribution are shown in Fig. 5.

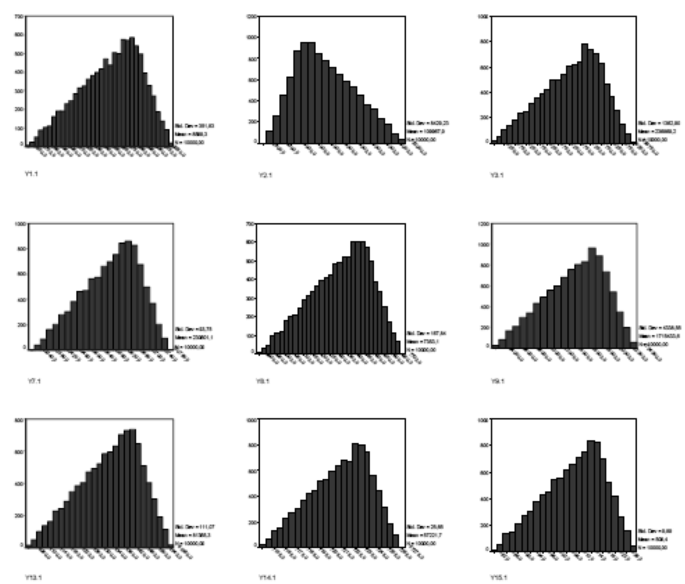

Fig. 5: Triangular distribution of 9 output vector Y components

When using more stochastic elements in the model or, pertinently, when using the inverse model (on the basis of the final consumption we would estimate the total production), some of the model's output parameters would not of course have the triangular distribution. In accordance with the central limit theorem, the output distribution should gradually be more and more similar to the normal distribution as the number of stochastic inputs increases.

For the model's further development we considered replacing the present triangular distribution by another one with the smooth course, i.e. without the breaking point at the mode area which could also be obtained by the three-point estimation with the help of the value triplet (minimum, optimum, maximum).

C. Beta Distribution

The expert may have a good reason for keeping the mode slightly vague or, possibly, for the requirement that the density function should not decrease too fast within a certain neighborhood of the supposed mode. Similarly he can require that the density function should not increase (decrease) too fast within a certain neighborhood of the boundaries of the permissible values range and that the majority of the generated values should remain clustered around the mode. In other words, the expert can prefer some of the smooth nonlinear density functions.

The beta distribution appears to be suitable for this purpose. Since the expert will not be able to estimate the necessary parameters α and β in advance, it appears to be quite reasonable to offer him the graphic interface with the density function chart being displayed and with some controls, whereby he can change these parameters so long until the displayed chart meets his wishes. This interactive procedure will help him to eliminate one unpleasant problem, namely that these parameters simultaneously influence the skewness, the kurtosis and the mode. Hence the determining of these parameters in the way such that they fulfill the preset criteria presents a rather complicated task. And undoubtedly, it is even more difficult for the expert to determine these criteria.

The Euler's beta distribution (a, b) with the probability density seems to be one of the possible candidates:

$$f(x) = \frac{x^{a-1}\left(1-x\right)^{b-1}}{B(a,b)}, \tag{9}$$

where:

$$B(a,b) = \int_0^1 t^{a-1}\left(1-t\right)^{b-1}\,dt, \tag{10}$$

is the so-called Euler's beta function.

The visualization of the "density level" can serve him as the helpful instrument. It can show him such an interval the boundaries of which have the same probability density and within which the sought out mode lies with the probability equal to his "degree of assurance". This evaluation is necessary to handle via the iterative operation that can slightly slow this process down. Nevertheless it is possible to expect that this way of thinking is very natural. This can help us to make the procedure of the estimation seeking and the data input more efficient.

The beta distribution enables to create relatively different shapes of the course of the probability density function, symmetrical and asymmetrical, including the convexity and concavity – see Fig. 6.

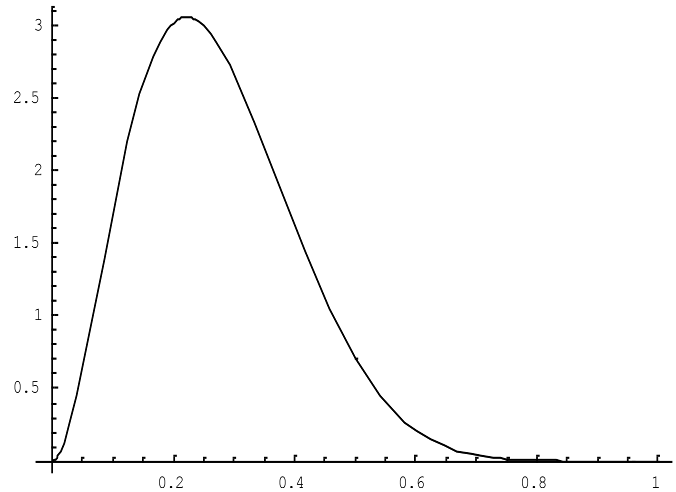

Fig. 6: Possible course of beta distribution probability density (Beta (3, 2))

Our task will especially be to transform the triplet of expert estimates (minimum, optimum, maximum) to the parameters a and b of the Beta (a, b). The input values generator itself with the corresponding beta distribution will be similar as the one in the case of the triangular distribution.

V. Stochastic Input – Output Model of the Economy of the Czech Republic

Our aim is to record the structure and behavior of a region, especially with regard to the medium-term Juglar business cycles and thus to create the medium-term input-output of the region's structure and behavior.

We have created and verified simulation input-output model of the Czech economy which has two basic versions:

- Deterministic version, which works with the deterministic inputs and outputs that have only limited information value; it only concerns the point estimates in the sense of the statistical theory;
- Stochastic version, which enables to enter the input values in an interval way; we enter the lower and upper interval limit and the mode of a given probability distribution; the quantity having a probability distribution with its mean and standard deviation is the output of the model.

The stochastic simulation input-output model of the Czech economy is able to process the outputs obtained by a combination two following basic input data fields:

- Statistical data obtained from the Czech Statistical Bureau; the input-output balance tables from 2009 were used in order to obtain the input data for the structural model creation;
- Data obtained from the expert estimates, which were projected into the possible development of the selected sector; in our case it concerns the use of experts' professional interval estimates in the field of iron metallurgy.

• One of the basic goals was to verify practically the possibilities of these two large extent independent input data sets within one common stochastic model.

The updated stochastic version of the input-output model of the Czech economy was created in the Visual Basic Language and it is equipped with the possibilities of parametric entering the required input and output information.

Simulation procedure algorithm is performed according to the flowchart, see Fig. 7.

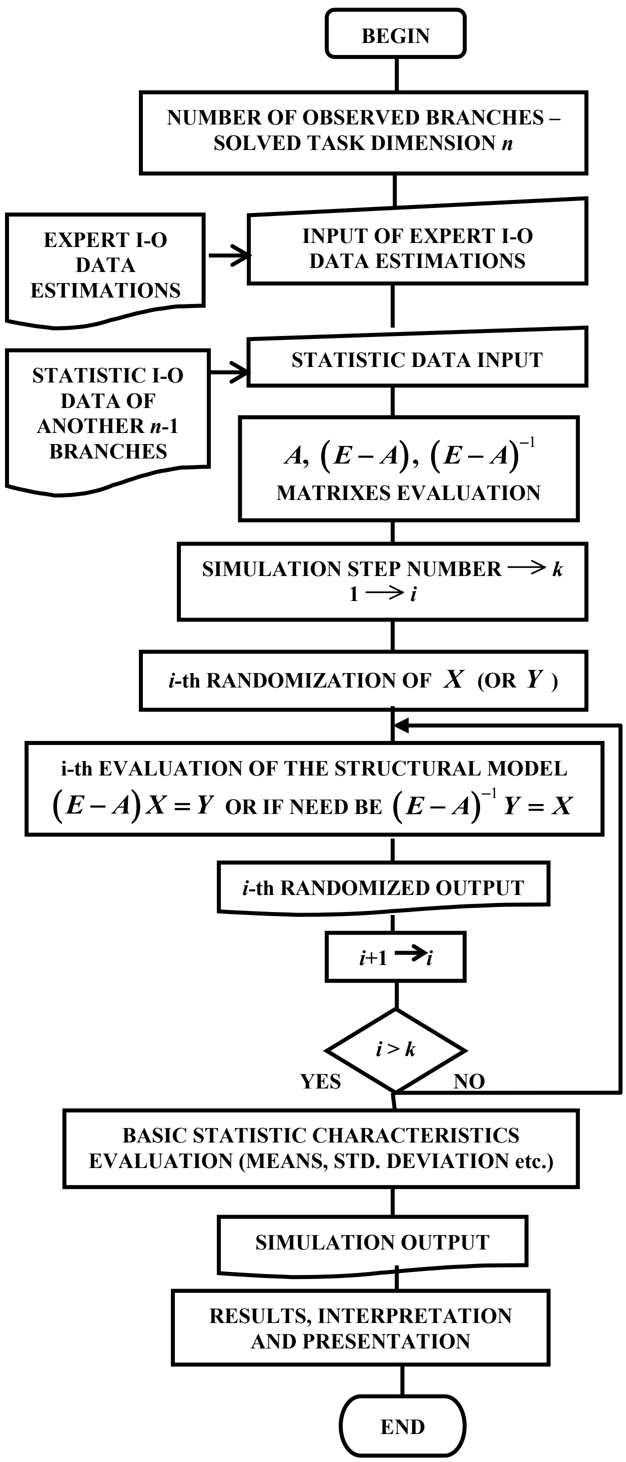

Fig. 7: Simulation procedure algorithm

The traditional procedures used during the calculations within the Leontief structural input-output models are used in our model. It is necessary to accept a premise that input data designated for the calculation of the structural model are a random variable. In our stochastic model we accept a hypothesis that expert estimates shall respect one of the commonly used probability distributions as defined in the previous chapter.

In order to verify the method of stochastic simulations, the input-output model of the 9 aggregated sectors of the Czech economy and the import solely into these sectors has been created by us.

When verifying the results of the simulation, we deliberately chose a small amount of sectors, hence the relatively high degree of aggregation and simplification, in order to be able to find easily and synoptically possible influences and impacts of the individual expert estimates on the possible development of the Czech economy's individual sectors.

VI. Empirical Dimension of Input-Output Model: Case Study on the Czech Republic with an Emphasis on Iron Metallurgy sector

We took into consideration expert estimates, furthermore the evaluation of older time series as well as current studies concerning the Czech Republic's iron metallurgy restructuring. As a result, determination of the percentage lower and upper limits of the possible alternatives of the iron metallurgy sector's total production development with regard to the reality of 2009 was achieved.

In the context of our research, an emphasis was put on the effect of the foreign trade (import and export of metallurgical products) and also on domestic consumption of metallurgical products in 2009. However, it should be stated that afore mentioned items constitute only a part of problems concerning the whole iron metallurgy sector development.

Nonetheless, since the significantly pro-export character of the sector determines its economy's development to the great extent, it is possible to consider this handicap not so important.

We did not purposely use the catastrophic alternatives, i.e. the assumptions that the iron metallurgy as an industrial branch disappears in the Czech Republic or that the entire domestic consumption of the metallurgical products in the economy of the Czech Republic will be imported to the country.

The specification of the iron metallurgy development is given by the following limits:

• The lower limit of the total production was set at the level of -15% from point estimate of the metallurgical production in the Czech economy in 2009,

• The upper limit of the total production was estimated at the level of +25% from mentioned point estimate of the metallurgical products in the Czech economy in 2009.

These limits capture two possible developmental alternatives with regard to the real manoeuvring space of the metallurgy in Czech Republic. Existing experience of sector's development during last four years was taken into consideration too.

Table I presents the results of the stochastic version of the Czech economy's simulation input output model after 10, 000 iterations. The first nine rows in Table I represent the nine aggregated sectors of the Czech economy; the last nine rows represent the exclusive imports into these sectors. The 2nd sector is just the iron metallurgy. The basic parameters of vector *Y* (mean, variance and standard deviation) are presented in the right part of the table.

Table I: Example of the results of stochastic simulation

	A	B	C	D	H	I	J
1	Stochastic Matrix X→Y	Lower X	Mode X	Upper X	Mean Y	Variance	Standard Deviation
2	1st sector	68672	68672	68672	10225,68	4811,89	69,37
3	2nd sector	108436	120484	150605	112397,51	96546042,75	9825,78
4	3rd sector	347741	347741	347741	245409,55	72206,85	268,71
5	4th sector	264518	264518	264518	199283,50	66,45	8,15
6	5th sector	253441	253441	253441	202852,72	1,69	1,30
7	6th sector	265357	265357	265357	71813,79	17083,76	130,70
8	7th sector	468966	468966	468966	224553,79	341,69	18,48
9	8th sector	41075	41075	41075	7210,72	1095,08	33,09
10	9th sector	2996090	2996090	2996090	1688388,85	731715,17	855,40
11	10th sector	51515	51515	51515	-208,96	3559,63	59,66
12	11th sector	39951	39951	39951	13717,98	118,76	10,90
13	12th sector	164188	164188	164188	71550,90	27195,16	164,91
14	13th sector	187169	187169	187169	63063,90	479,60	21,90
15	14th sector	97263	97263	97263	50464,21	25,57	5,06
16	15th sector	1970	1970	1970	509,92	3,80	1,95
17	16th sector	7478	7478	7478	-21,37	6,14	2,48
18	17th sector	2012	2012	2012	-44,16	4,43	2,10
19	18th sector	596145	596145	596145	100050,47	231035,17	480,66
20			Start simulation				

Results in Table I present the model that calculates the estimations of total consumption *X* for total production *Y* according to the equation $Y = (I - A)X$. The production of this key resort is based on expertise with the use of above mentioned triangle distribution; other resorts are estimated through dots based on Czech Statistical Office data, as well as coefficients of the structure matrix *A*.

In this simple model the individual components y_j of the final consumption vector *Y* will be set out as linear combination of the components x_i of the total production vector *X*. With regard to the fact that 17 components of the vector *X* are deterministic and only one is random quantity with a triangle probability distribution then the individual components y_j of the final consumption vector will have the triangle probability distribution.

If all components of the input vector *X* or coefficients of the matrix *A* had a stochastic character then during calculation the components of vector *Y* the triangle distributions would make up according to the central limit theorem and the final distribution of the components y_j should be close to the normal distribution. To verify this statement it is necessary to use some statistical normality tests.

For further model development we can consider a replacement of the triangle distribution with another distribution with a fluent running, i.e. beta distribution where the three-dot values *a*, *b*, *c* will be three selected quantiles of this distribution (e.g. 5%, 50% a 95%).

Concrete simulation results can be interpreted as the impacts of the possible stochastic influences caused by the value of the iron metallurgy sector‘s total consumption on the other aggregated sectors of the Czech economy. The results of these simulations and estimates cannot be considered sufficiently representative because of the chosen relatively high and asymmetrical degree of aggregation and simplification of real problem. The results should be considered to be only illustrative for the used method.

By aggregating the values of the vector *Y* it is possible to obtain the estimate of the magnitude of the Czech economy's total final use which represents the approximation of the economy's GDP.

VII. Conclusions

Traditional version of input-output model is deterministic and static. In order to capture the structure of the economy in its spatial dimension, this model requires large amounts of information. In spite of this demanding gathering the information, the model cannot express changes in economic structures, production technologies, quality and quantity of production factors etc.

Created stochastic input-output model utilizing Monte Carlo simulation in Microsoft Excel programme demonstrated the possibility of dynamisation of originally static Leontief's structural input-output model related to territorial economies. The model enables to understand all the components of input-output model, i.e. elements of the vectors of final consumption *Y*, total production *X* and elements of matrix of the direct consumption coefficients *A* as random quantity with given distribution and parameters. In case of the unknown real value of the concrete model parameter we can replace it by the expert estimation as an interval from – to or random quantity with a certain distribution.

This paper dealt with a calculation of the vector values of the final consumption *Y* for given economy model with a defined structure matrix *A* and total production vector *X*. One of the vector *X* elements wasn't entered deterministically (dots) but stochastically with the assistance of triangle distribution that was gained on three-point expert estimation (minimum, optimum, maximum).

Stochastic character of this item was reflected in the calculation because all the elements of calculated final consumption *Y* had random quantity character with triangle probability distribution.

Despite the fact that stochastic input – output model verification has not been accomplished yet, it is possible to state that for the successful use of this model, the following conditions and assumptions should be fulfilled:

- Determining the precisely-defined purpose, object and goal of the simulation; the choice of the type and size of the investigated region, the choice of the time horizon and periods of development, the choice of aggregation degrees of the individual sectors etc.;

- Obtaining sufficiently accurate and undistorted statistical data on the region's sector economy and the decision concerning their purposeful aggregation;
- Obtaining the serious expert estimates of the state or a development of the individual selected region's sectors, namely from the individual experts or the teams of experts;
- The use of the statistical evaluation and comparison of the individual simulations results and their distinction according to the degree of the results' significance.

Stochastic input – output model enables both qualitative and quantitative analyses of economic situation in concrete regions; however, the model can be applied also in the realm of forecasting the alternative developmental scenarios in particular regions as well as for the purposes of retrospective analysis of possible past alternative developments. The simulation technique renders a broad spectrum of possibilities to project the consequences of territorial economic policies as well as the consequences of different trends or megatrends. It also substantially facilitates the evaluation of measures related to economic programs of different political parties.

The question of possibilities of an access to the reachable data sets penetrates entire paper. This information is needed for ensuring the structural models smooth functioning. Without having required data, the structural models would fulfill their function only to the very restricted and simplified extent. In many cases, when reliable statistical data are not available, it is necessary to replenish them with a sample statistical census and also with qualified expert estimates. Case study that depicted iron metallurgy in the Czech Republic offered some possible answers to these complex and finely structured problems.

Structural models cannot be understood in the way that is isolated from the other model approaches. We have only attempted to outline the elementary links between macroeconomic models on one hand and structural models on the other.

Apparently, it will be necessary to deal also with other special econometric models, e.g. the models of the individual sector developments, multi-territorial models, models considering the distance among regions and others of that ilk. These different model approaches can sooner or later constitute a reciprocally interconnected unit so in the future we will be indeed entitled to talk about the system of regional models.

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