Effects of changes in some parameters on the deterministic and stochastic dynamic economic model with wealth and human capital accumulation

Nicoleta Sîrghi, Mihaela Neamţu and Dumitru Opriș

***Abstract*—**This paper analyzes a deterministic and stochastic dynamic economic model with wealth and human capital accumulation. The deterministic model is described and using the numerical simulations we can notice that the stationary state is asymptotically stable. The stochastic model is built and the mean values of the linearized variables are proven to be asymptotically stabile. We also examine effects of changes in the propensity to receive education, efficiency of learning, and efficiency of education upon dynamic paths of the system.

***Keywords* —** deterministic dynamic economic model, economic growth, education product, human capital, stochastic dynamic economic model.

I. INTRODUCTION

Dynamic interdependence between economic growth and human capital is currently a main topic in economic theory. The deterministic model from the present paper was built by Wei-Bin Zhang [1] and it is based on the three main growth models – Solow's one-sector growth model [2], Arrow's learning by doing model [3], and the Uzawa-Lucas's growth model with education [4], [5], [6]– in the economic growth literature. The main mechanisms of economic growth in these three models are integrated into a single framework. This study models interaction between physical capital and human capital accumulation by taking into account Arrow's learning by doing, Uzawa-Lucas's learning through education, and Zhang's learning by consuming. The aim is to combine the economic mechanisms in the three key growth models - Solow's growth model, Arrow's learning by doing model, the Uzawa-Lucas education mode into a single comprehensive framework.

The paper is organized as follows. Section 2 introduces the deterministic model with wealth accumulation and human capital accumulation. The model describes a dynamic interdependence between wealth accumulation, human capital accumulation, and division of labor under perfect economic competition. The stationary state of the model is analyzed. Section 3 introduces the stochastic model with wealth accumulation and human capital accumulation. The square mean value of the stationary state is studied. In section 4, we simulate the model to show effects of changes in some parameters on the economic system. Section 5 concludes the study.

II. DETERMINISTIC BASIC MODEL

Economy has two sectors: the production sector and the education sector. It is assumed that there is only one (durable) commodity in the economy under consideration. Households own economic assets and distribute their incomes to consume and save. Production sectors of firms use inputs such as labor with varied levels of human capital, different kinds of capital, knowledge and natural resources to produce material goods or services. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership.

In what follows, a homogenous and fixed population denoted by N_0 is considered. The labor force is distributed between the two sectors. Commodity functions as numeraire,

Manuscript received September 9, 2011: Revised version received….This work was supported in part by the Post-doctoral studies in economics: continuous forming program of the leading researchers **(SPODE), POSDRU/89/1.5/S/61755,** Romanian Academy.

Nicoleta Sîrghi is with the West University of Timisoara, Faculty of Economics and Business Administration, Pestalozzi Str., 16, 300115 Timisoara, Romania (e-mail: nicoleta.sirghi@feaa.uvt.ro).

Mihaela Neamtu is with West University of Timisoara, Faculty of Economics and Business Administration, Pestalozzi Str., 16, 300115 Timisoara, Romania (corresponding author to provide phone: +40-(0)256-592505; fax: +40-(0)256-592500; e-mail: mihaela.neamtu@feaa.uvt.ro).

Dumitru Opris is with West University of Timisoara, Faculty of Mathematics and Informatics, Blvd. V. Parvan 16, 300223 Timisoara, Romania (e-mail: opris@math.uvt.ro).

taking into account that all the other prices being measured relative to its price. The wage rates are assumed to be identical among all professions.

The following notations are used [1]:
$F_1(t)$ – the output level of the production sector at time t,
$K(t)$ – the level of capital stocks of the firm,
$H(t)$ – the level of human capital of the population,
$N_1(t)$ and $K_1(t)$ – the labor force and the capital stocks employed by the production sector, respectively,
$N_2(t)$ and $K_2(t)$ – the labor force and the capital stocks employed by the education sector, respectively,
$T(t)$ and $T_2(t)$ – the work time and the study time, respectively,
$p(t)$ – the price of education (service) per unit of time, and
$w(t)$ and $r(t)$ – the wage rate and the rate of interest, respectively.

The total capital stock K(t) is allocated between the two sectors. The full employment of labor and capital are given by:

$$K_1(t)+K_2(t)=K(t),\quad N_1(t)+N_2(t)=N(t) \tag{1}$$

where $N(t)=T(t)N_0$ and N(t) represents the total work time of the population.

The relations (1) can be rewritten as

$$n_1(t)k_1(t)+n_2(t)k_2(t)=k(t),\quad n_1(t)+n_2(t)=1 \tag{2}$$

in which

$$k_1(t)=K_1(t)/N_1(t),\quad n_1(t)=N_1(t)/N(t),\quad k_2(t)=K_2(t)/N_2(t),\quad n_2(t)=N_2(t)/N(t). \tag{3}$$

In [1] production is assumed to combine 'qualified labor force' $H^m(t)N_1(t)$ and physical capital $K_1(t)$. The conventional production function is described by a relationship between inputs and output. The function $F_1(t)$ defines the flow of production at time t. The production process is defined by:

$$F_1(t)=A_1 K_1^{\alpha_1}(t)\,(H^m(t)N_1(t))^{\beta_1}, \tag{4}$$

where A_1, $\alpha_1,\beta_1>0$, $\alpha_1+\beta_1=1$.

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. Both the rate of interest and the wage rate are determined by markets. Therefore, r(t) and w(t) are given at each point of time for any individual firm.

The variables $K_1(t)$ and $N_1(t)$ are chosen to maximize the production sector's profit. The marginal conditions are given by:

$$r(t)+\delta_1=\alpha_1F_1(t)/K_1(t)=\alpha_1A_1H^{m\beta_1}k_1(t)^{-\beta_1} \tag{5}$$
$$w(t)=\beta_1F_1(t)/N_1(t)=\beta_1A_1H^{m\beta_1}k_1^{\alpha_1}(t)$$

where δ_1 is the depreciation rate of physical capital.

In [1] there are three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow first introduced learning by doing into growth theory [2]; Uzawa took account of the tradeoffs between investment in education and capital accumulation [3], and Zhang introduced the impact of consumption on human capital accumulation (via the so-called creative leisure) into economic growth theory [7]. The human capital dynamics is given by:

$$dH(t)/dt=v_2F_2^{a_2}(t)(H^m(t)T_2(t)N_0)^{b_2}/(H^{p_1}(t)N_0)+$$
$$+v_1F_1^{a_1}(t)/(H^{p_2}(t)N_0)+v_3c^{a_3}(t)/(H^{p_3}(t)N_0)-\delta_1H(t) \tag{6}$$

where $\delta_1>0$ is the depreciation rate of human capital, v_1, v_2, v_3, a_1, a_2, a_3, b_2, a_1 are non-negative parameters. The signs of the parameters p_2, p_1 and p_3 are not specified as they can be either negative or positive. The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation.

The education sector is also characterized by a perfect competition. Any government's financial support for education is neglected; however, it is important to introduce the government's intervention in education. The students pay the education fee p(t) per unit of time. The teachers and capital are paid with the market rates by the education sector.

The cost of the education sector is given by $w(t)N_2(t) + r(t)K_2(t)$.

The total education service is measured by the total education time received by the population, T_2N_0. The production function of the education sector depends on $K_2(t)$ and $N_2(t)$. The production function of the education sector is as follows

$$F_2(t)=A_2K_2^{\alpha_2}(t)(H^m(t)N_2(t))^{\beta_2} \tag{7}$$

$\alpha_2,\beta_2>0$, $\alpha_2+\beta_2=1$, A_2, α_2 and β_2 are positive parameters.

The education sector maximizes the following profit

$$\pi(t)=p(t)A_2K_2^{\alpha_2}(t)(H^m(t)N_2(t))^{\beta_2}-(r(t)+\delta_1)K_2(t)-$$
$$-w(t)N_2(t)$$

For given p(t), H(t), r(t) and w(t) the variables $K_2(t)$ and $N_2(t)$ are chosen to maximize the education sector's profit. The optimal solution is given by:

$$\begin{aligned} r(t)+\delta_1&=\alpha_2p(t)F_2(t)/K_2(t)=\\ &=\alpha_2A_2p(t)H^{m\beta_2}(t)k^{-\beta_2},\\ w(t)&=\beta_2p(t)F_2(t)/N_2(t)=\\ &=\beta_2A_2p(t)H^{m\beta_2}(t)k_2^{\alpha_2} \end{aligned} \tag{8}$$

The demand for labor force for the given price of education, wage rate and level of human capital is given by

$$N_2(t)=K_2(t)(\beta_2A_2H^{m\beta_2}(t)/w(t))^{1/\alpha_2} \tag{9}$$

Consumers make decisions on choice of consumption levels of services and commodities as well as on how much to save. Different from the optimal growth theory in which utility defined over future consumption streams is used, we assume that we can find preference structure of consumers over consumption and saving at the current state. We denote per capita wealth by $k_3(t)$, where $k_3(t)\equiv k(t)/N_0$. According to the definitions, we have $k_3(t) = T(t)k(t)$. Per capita current income from the interest payment $r(t)k_3(t)$ and the wage payment $T(t)w(t)$ are given by:

$$y(t)=r(t)k_3(t)+T(t)w(t). \tag{10}$$

In [1], y(t) is called the current income in the sense that it comes from consumers' daily toils (payment for human capital) and consumers' current earnings from ownership of wealth. The current income is equal to the total output. The total value of wealth that consumers can sell to purchase goods and to save is equal to $k_3(t)$. The per capita disposable income is given by

$$y_1(t) = y(t)+k_3(t)= =(1+r(t))k_3(t)+T(t)w(t). \quad (11)$$

The disposable income is used for saving, consumption, and education. At each point of time, a consumer would distribute the total available budget among saving $s(t)$, consumption of goods $c(t)$, and education $p(t)T_2(t)$. The budget constraint is given by

$$c(t)+s(t)+p(t)T_2(t)=y_1(t)= = (1+r(t))k_3(t)+T(t)w(t). \quad (12)$$

The consumer is faced with the following time constraint

$$T(t)+T_2(t)=T_0$$

where T_0 represents the total available time for work and study.

Thus, the budget constraint (12) becomes:

$$c(t)+s(t)+(p(t)+w(t))T_2(t)=y_4(t)= =(1+r(t))k_3(t)+T_0w(t) \quad (13)$$

The consumers' utility function is assumed to be a function of level of goods $c(t)$, level of saving $s(t)$ and education service $T_2(t)$. It is given by [7]:

$$U(t) = c^{\xi}(t)s^{\lambda}(t)T_2^{\eta} \quad (14)$$

$\xi, \lambda, \eta > 0, \xi + \lambda + \eta = 1$.

Maximizing $U(t)$ in (14) subject to the budget constraint (14) yields

$$c(t) = \xi y(t), s(t) = \lambda y(t), \\ (p(t) + w(t))T_2(t) = \eta y(t) \quad (15)$$

In what follows, the dynamics of the capital accumulation is found. According to the definition of $s(t)$, the change in the household's wealth is given by

$$dk_3(t)/dt = \lambda y(t)-k_3(t).$$

For the education sector, the demand and supply balance each other at any point of time

$$T_2(t)N_0=F_2(t).$$

Taking into account that the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have

$$C(t)+S(t)-K(t)+ \delta_1 K(t)=F_1(t)$$

where $C(t)$ is the total consumption, $S(t) - K(t) + \delta_1K(t)$ is the sum of the net saving and depreciation.
We have

$$C(t)=c(t)N_0,$$
$$S(t)=s(t)N_0.$$

Thus, the economic system can be described by the differential system:

$$dk_1(t)/dt=G_1(k_1(t), H(t)),$$
$$dH(t)/dt= G_2(k_1(t), H(t)) \quad (16)$$

where the functions $G_1(k_1(t), H(t))$ and $G_2(k_1(t), H(t))$ are functions of $k_1(t)$ and $H(t)$ given by:

$$G_1(k_1(t), H(t)) = (\lambda G(k_1(t), H(t)) - F_0(k_1(t), H(t)) \cdot \\ \cdot F(k_1(t), H(t)) - F_{02}(k_1(t), H(t)) \cdot F(k_1(t), H(t)) + \\ + F_2(k_1(t), H(t)) \cdot F_0(k_1(t), H(t))) \cdot G_2(k_1(t) H(t)) \cdot \\ \cdot (F_{01}(k_1(t), H(t)) \cdot F(k_1(t), H(t)) + F_1(k_1(t), H(t)) \cdot \\ \cdot F_0(k_1(t), H(t)))^{-1},$$

$$G_2(k_1(t), H(t)) = G_{21}(k\ (t), H(t)) + G_{22}(k\ (t), H(t)) + \\ + G_{23}(k\ (t), H(t)) - \delta_1 H(t),$$

$$F(k(t), H(t)) = k_1(t) \cdot ((\alpha_0\beta_1 - \alpha - Ak^{\alpha 2}(t)H^{\beta 2m}(t)) / \\ (\alpha - 1)\delta\eta k_1^{\beta_1}(t) / ((A_1H)^{\beta_1 m}(t)) - Ak_1^{\alpha 2 -1} + \alpha_0\alpha_1))$$

$$F_0(k_1(t), H(t)) = T_0(1 + ((\alpha^{\alpha 2} A_2 H(t))^{\beta 2m} - k_1(t) / \\ ((\alpha - 1)k_1^{\beta 2}))^{-1}$$

$$G(k_1(t), H(t)) = (\delta + \alpha_1 A_1 H^{\beta_1 m}(t)k_1^{-\beta_1}(t))F(k(t), H(t)) \cdot \\ \cdot F_0(k(t), H(t)) + T_0\beta_1 A_1 H^{\beta_1 m}(t)k_1^{\alpha_1}$$

$$G_{22}(k_1(t), H(t)) = (v_2 N_0^{a2+b2-1})A_2^{\alpha 2a2} F(k_1(t), H(t)) - \\ - k_1(t))^{a2}(T_0 - F_0(k_1(t), H(t))^{b2} F_0(k_1(t), H(t))^{a2} \cdot \\ \cdot k_1^{-a2\beta 2}(t)H^{mb2+a2\beta 2-p2}(t))/(\alpha - 1)^{a2},$$

$$G_{21}(k_1(t), H(t)) = (v_1 A_1^{\alpha_1} N_0^{\alpha_1 - 1}(\alpha k_1(t) - F(k(t), H(t)))^{\alpha_1} \cdot \\ \cdot F_0(k_1(t), H(t))k_1^{-\alpha_1\beta_1} H^{m\alpha_1\beta_1 - p_1}(t),$$

$$G_{23}(k_1(t), H(t)) = v_3\xi^{a_3} N_0^{a_3 - 1} G(k_1(t), H(t))^{a_3} H^{-p_3}(t).$$

We denote by

$$F_{01}(k_1(t), H(t)) = \frac{\partial F_0(k_1(t), H(t))}{\partial k_1},$$
$$F_{02}(k_1(t), H(t)) = \frac{\partial F_0(k_1(t), H(t))}{\partial H},$$
$$F_1(k_1(t), H(t)) = \frac{\partial F(k_1(t), H(t))}{\partial k_1}, \quad (17)$$
$$F_2(k_1(t), H(t)) = \frac{\partial F(k_1(t), H(t))}{\partial H}$$

and

$$\alpha = \alpha_2\beta_1 / \alpha_1\beta_2, \beta = \beta_2 - \beta_1, \alpha_0 = (\alpha - 1)(\xi + \lambda)$$
$$A = \alpha^{\alpha 2}\beta_1 A_2(\xi + \lambda).$$

The stationary states are obtained by solving the system of equations:

$$G_1(k_1, H)=0, \quad G_2(k_1, H)=0 \quad (18)$$

Let (k_{10}, H_0) be a solution of the system (18).

In this point, the linearized system (16) is:

$$du_1(t)/dt= a_{11}u_1(t) + a_{12}u_2(t),$$
$$du_2(t)/dt= a_{21}u_1(t) + a_{22}u_2(t), \quad (19)$$

where

$$a_{11} = \frac{\partial G_1(k_1, H)}{\partial k_1}\Big|_{(k_{10}, H_0)}, \ a_{12} = \frac{\partial G_1(k_1, H)}{\partial H}\Big|_{(k_{10}, H_0)}$$

$$a_{21} = \frac{\partial G_2(k_1, H)}{\partial k_1}\Big|_{(k_{10}, H_0)}, \ a_{22} = \frac{\partial G_2(k_1, H)}{\partial H}\Big|_{(k_{10}, H_0)}.$$

The characteristic equation is

$$x^2 - (a_{11} + a_{22})x + a_{11}a_{22} - a_{12}a_{21} = 0. \quad (20)$$

If the roots of the equation (20) have the real negative part, then the stationary state is asymptotically stable. The analysis of the stability is done in the Numerical simulation section.

III. STOCHASTIC BASIC MODEL

Let (Ω, F_t, P), be a given probability space and $B(t) \in R$ be a scale Wiener process defined on Ω having independent stationary Gauss increments with

$$B(0)=0,\ E(B(t)B(s))=\min(t, s).$$

The symbol E denotes the mathematical expectation. The sample trajectories of B(t) are continuous, nowhere differentiable and have infinite variation on any finite time interval [8]-[13].

What we are interested in the effect of the noise perturbation on the form the stochastic system:

$$dk_1(t) = G_1(k_1(t), H(t))dt + \sigma_1(k_1(t) - k_{10})dB(t)$$
$$dk_2(t) = G_2(k_1(t), H(t))dt + \sigma_2(H(t) - H_0)dB(t) \qquad (21)$$

where $k_1(t)= k_1(t, \omega)$, $H(t)= H(t, \omega)$, $\omega \in \Omega$

Linearizing (21) around the stationary state is:

$$du_1(t)=(a_{11}u_1(t) + a_{12}u_2(t))dt + \sigma_1 u_1(t)dB(t), \qquad (22)$$
$$du_2(t)=(a_{21}u_1(t) + a_{22}u_2(t))dt + \sigma_2 u_2(t)dB(t),$$

To examine the stability of the second moment of $u_1(t)$, $u_2(t)$ for the linear stochastic differential equation (22) we use Ito's rule.

We use the following notations:

$R_{11}(t,s) = E(u_1(t)u_1(s))$, $R_{12}(t,s) = E(u_1(t)u_2(s))$,

$R_{21}(t,s) = E(u_2(t)u_1(s))$, $R_{22}(t,s) = E(u_2(t)u_2(s))$.

From (22) we have:

$$dR_{11}(t,t)/dt = (2a_{11} + \sigma_1^2)R_{11}(t,t) + 2a_{12}R_{12}(t,t)$$
$$dR_{12}(t,t)/dt = (a_{11} + a_{22} + \sigma_1\sigma_2)R_{12}(t,t) + a_{12}R_{22}(t,t) + a_{21}R_{11}(t,t),$$
$$dR_{22}(t,t)/dt = (2a_{22} + \sigma_2^2)R_{22}(t,t) + 2a_{21}R_{12}(t,t). \qquad (23)$$

The characteristic equation of the system (23) is:

$$(2x-2a_{11} - \sigma_1^2)(2x-2a_{22} - \sigma_2^2)(2x- -a_{11} - a_{22} - \sigma_1\sigma_2) - 2a_{12}a_{21}(4x - 2a_{11} - 2a_{22} - \sigma_1^2 - \sigma_2^2) = 0 \qquad (24)$$

If the roots of the equation (24) have a negative real part, then the stationary state is asymptotically stable in the square mean sense. If $\sigma_1 = \sigma_2 = \sigma$, the equation (24) becomes:

$$(2x - a_{11} - a_{22} - \sigma^2)(4x^2 - 4(a_{11} + a_{22} + \sigma^2) + (2a_{11} + \sigma^2)\cdot \cdot(2a_{22} + \sigma^2) - 4a_{12}a_{21}) = 0 \qquad (25)$$

If $\sigma^2 < (-a_{11} - a_{22})$ then the characteristic equation has roots with a negative real part. Then, the square mean values of the variables are asymptotically stable.

IV. NUMERICAL SIMULATIONS

The numerical simulation was done using Maple14. In what follows we use the following parameters:

$T_0=1$, $\alpha_1=0.3$, $\alpha_2=0.34$, $\lambda=0.8$, $\eta=0.008$, $N_0=50000$, $m=0.6$, $v_1=2.5$, $v_2=0.8$, $v_3=0.7$, $p_1=0.7$, $p_2=0.2$, $p_3=0.1$, $a_1=0.4$, $a_2=0.3$, $a_3=0.1$, $b_2=0.5$, $A_1=0.9$, $A_2=0.9$, $\beta_1=1-\alpha_1$, $\beta_2=1-\alpha_2$, $\delta_1=0.05$, $\delta_2=0.04$, $\delta=1-\delta_1$. (26)

With these parameters we compute:

$\alpha=\alpha_2\beta_1/(\alpha_1\beta_2)$, $\beta=\beta_2-\beta_1$, $\xi=1-\lambda-\eta$, $\alpha_0=(\alpha-1)(\xi+\lambda)$, $A=\alpha^{\alpha}{}_2\beta_1 A_2(\xi+\lambda)$.

The graphics of the functions $G_1(k_1, H)=0$, $G_2(k_1, H)=0$ are given in Figure 1:

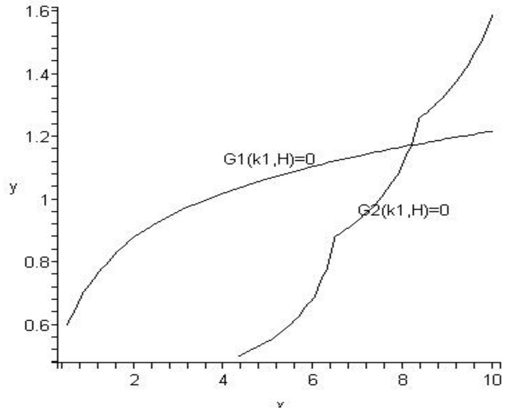

Figure 1. $G_1(k_1, H)=0$, $G_2(k_1, H)=0$

The stationary state is $k_{10} = 8.0783$, $H_0 = 1.1716$. The equation (20) has the roots $x_1=-0.017676$, $x_2=-0.170120$. Thus, the stationary state is asymptotically stabile. The orbits $(t, k_1(t))$ and $(t, H(t))$ are given by Figure 2 and Figure 3:

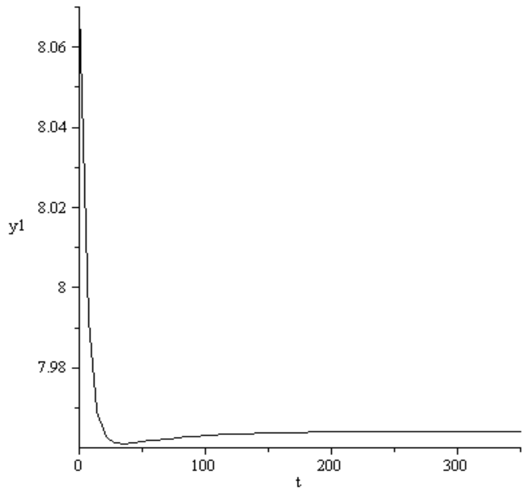

Figure 2 $(t, k_1(t))$

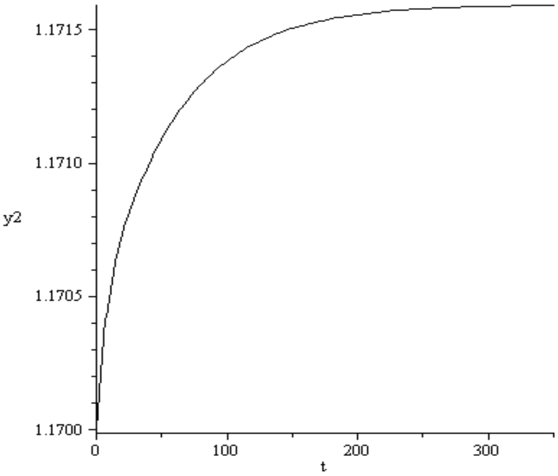

Figure 3 (t, H(t))

In what follows we examine the impact of changes on the dynamic processes of the system. Consider the case when all the parameters, except the education efficiency parameter, A_2, are the same as in (26). We increase the education efficiency parameter from $A_2 = 0.9$ to $A_{20} = 1.2$. The simulation results are showed in Figure 4 and Figure 5. In the plots, a variable Δx_j (t) stands for the change rate of the variable x_j (t), in percentage due to changes in the parameter value from $A_2 = 0.9$ (in this case) to $A_{20} = 1.20$ than

$$\Delta x_j (t) = ((x_j (t, A_{20}) - x_j (t, A_2))*100/ x_j (t, A_2).$$

(called relative estimation error).

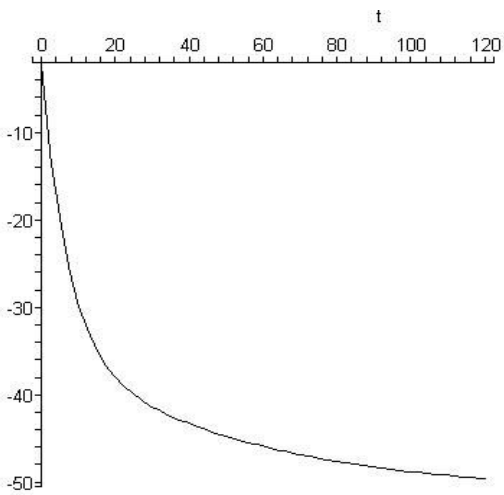

Figure 4 (t, Δk_1(t))

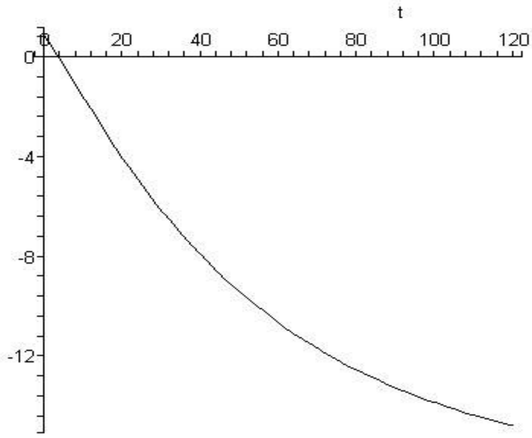

Figure 5 (t, ΔH(t))

Secondly, we increase the production sector's productivity from A_1=0.9 to A_{10}=1.2. The simulation results are displayed in Figure 6 and Figure 7

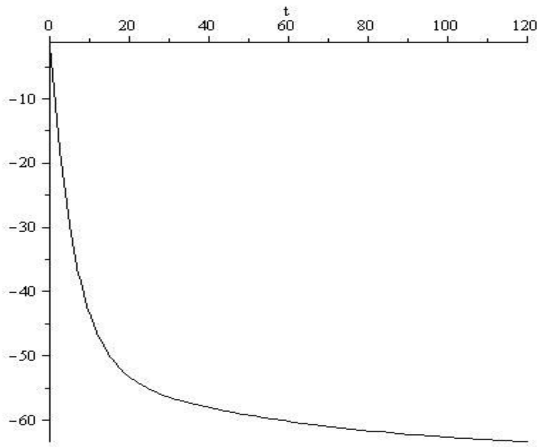

Figure 6 (t, Δk_1(t))

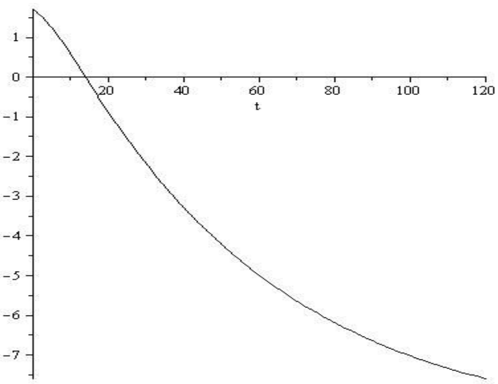

Figure 7 (t, ΔH(t))

It is important to examine effects of changes in the household's preference for education. We allow the propensity to receive education to increase from $\eta = 0.008$ to $\eta_0 = 0.014$. The simulation results are showed in Figure 8 and Figure 9

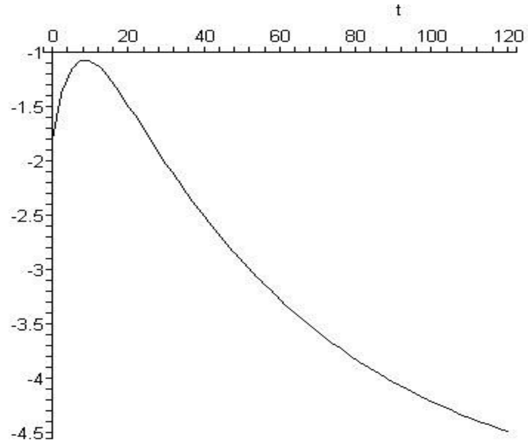

Figure 8 $(t, \Delta k_1(t))$

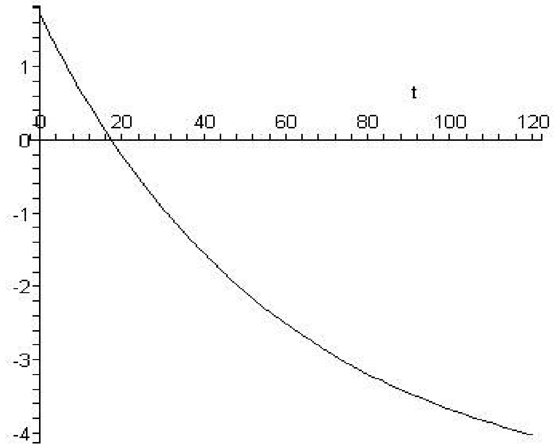

Figure 9 $(t, \Delta H(t))$

The effects of change in the population are showed in Figure 10 and Figure 11, where the population increases from $N_0 = 50000$ to $N_{00} = 60000$.

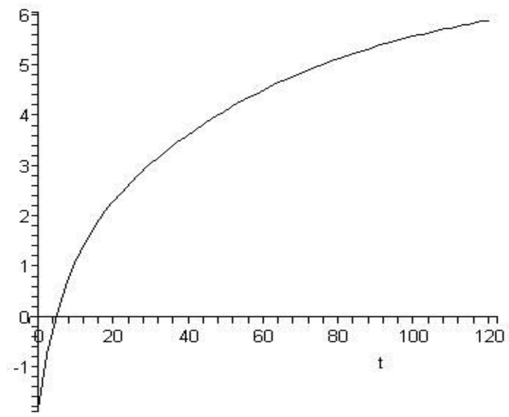

Figure 10 $(t, \Delta k_1(t))$

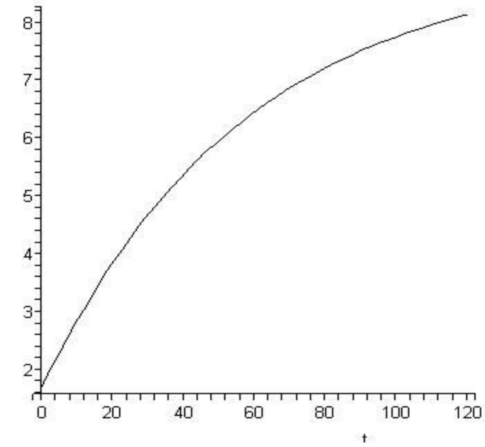

Figure 11 $(t, \Delta H(t))$

Using the Euler stochastic method, the orbits $(t, k_1(t, \omega))$, $(t, H(t, \omega))$ of the stochastic system (21) are obtained and they are given in Figure 12 and Figure 13.

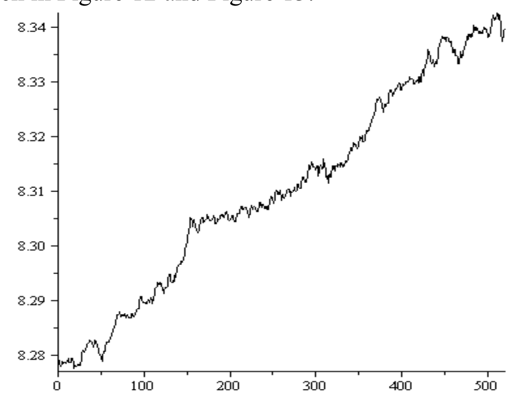

Figure 12 $(t, k_1(t, \omega))$

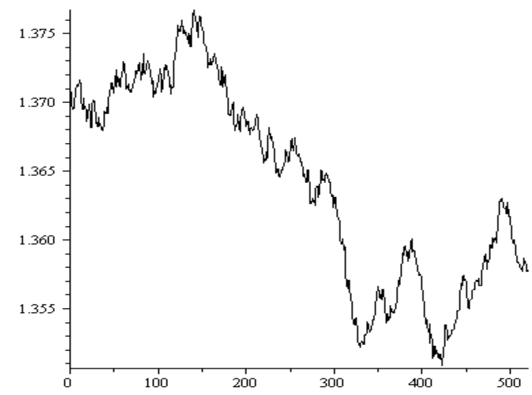

Figure 13 (t, H(t, ω))

The characteristic equation (24) with $\sigma_1 = \sigma_2 = 0.1$ has the roots x_1=-0.165120, x_2=-0.088898, x_3=-0.012676. The square mean values of the state variables k_1(t, ω), H(t, ω) are asymptotically stable.

In what follows, we consider the stochastic case. Firstly, we examine the case when all the parameters, except the education efficiency parameter, A_2, are the same as in (26). We increase the education efficiency parameter from A_2 = 0.9 to A_{20} = 1.2. The simulation results are showed in the figures 14 and 15. In the plots, a variable $\Delta x_j(n, \omega)$ stands for the change rate of the variable $x_j(n, \omega)$, in percentage due to changes in the parameter value from A_2 =0.9 (in this case) to A_{20} =1.20 than

$\Delta x_j(n, \omega) = ((x_j(n, \omega, A_{20}) - x_j(n, \omega, A_2))*100/ x_j(n, \omega, A_2)$.

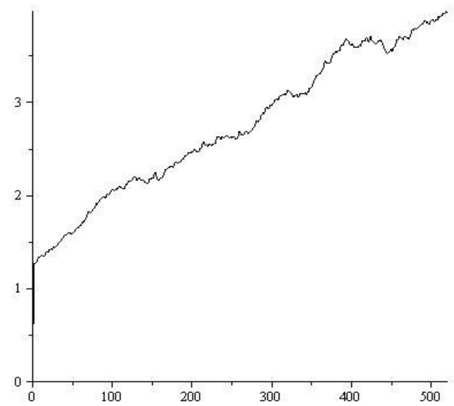

Figure 14 (n, Δk_1(n, ω))

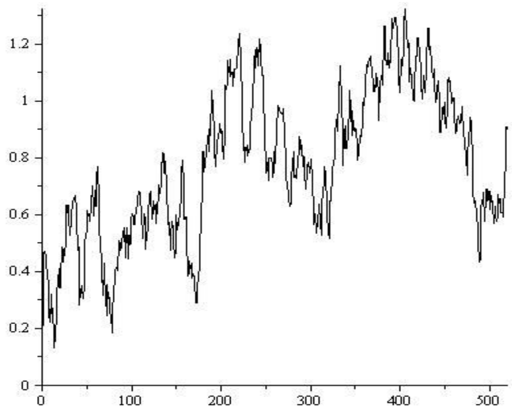

Figure 15 (n, ΔH(n, ω))

We now increase the production sector's productivity from A_1=0.9 to A_{10}=1.2. The simulation results are displayed in Figure 16 and Figure 17

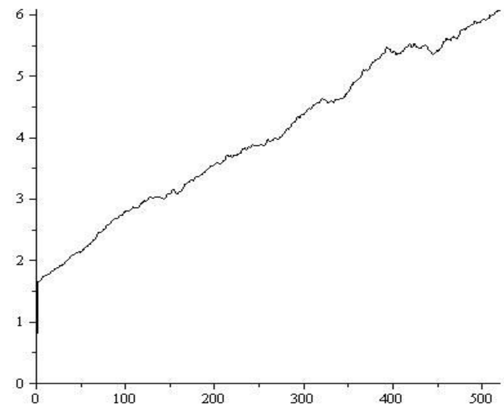

Figure 16 (n, Δk_1(n, ω))

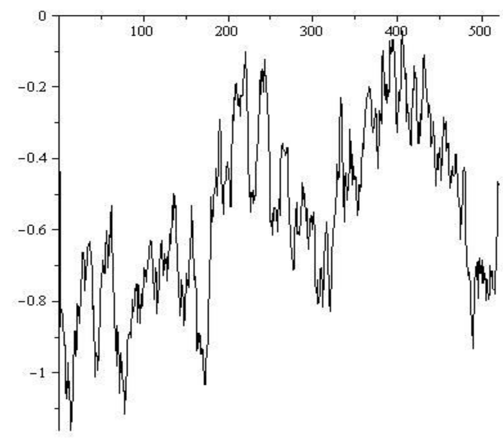

Figure 17 (n, ΔH(n, ω))

The effects of changes in the household's preference for education are displayed in Figure 18 and Figure 19. In this case the propensity to receive education increases from η = 0.008 to η_0 = 0.014.

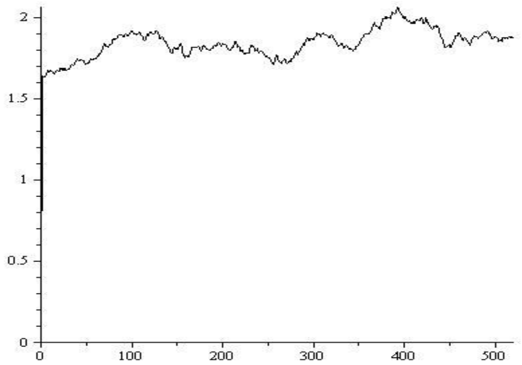

Figure 18 (n, Δ k_1(n, ω))

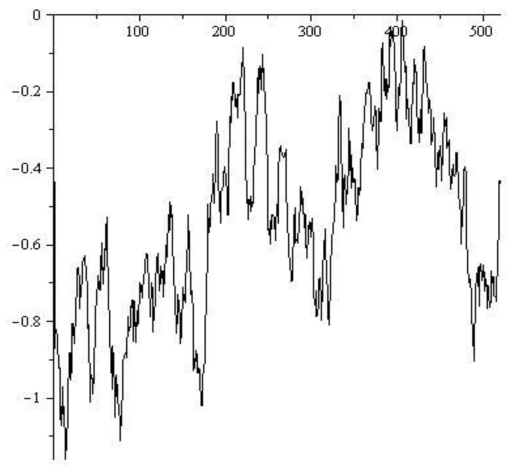

Figure 19 (n, ΔH(n, ω))

The effects of change in the population have negative effects on the living conditions, as proved in Figure 20 and Figure 21 where the population increases from N_0 = 50000 to N_{00} = 60000.

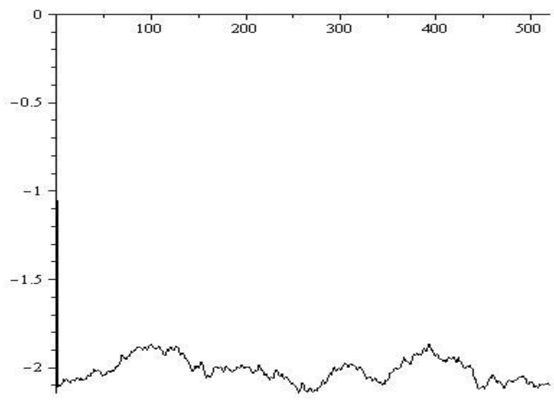

Figure 20 (n, Δk_1(n, ω))

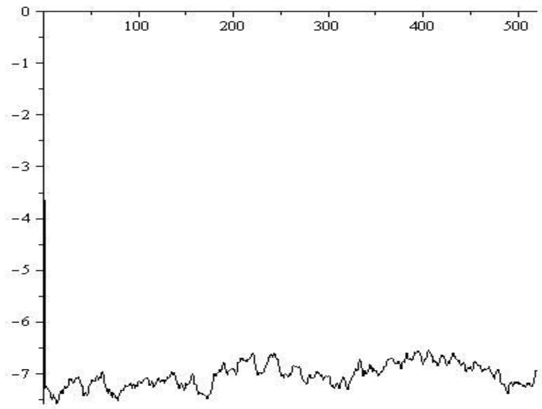

Figure 21 (n, ΔH(n, ω))

III. CONCLUSIONS

This paper proposes a dynamic firm model with wealth and human capital accumulation in the deterministic and stochastic cases.

For the deterministic model the stationary state is analyzed. Using the stationary state, for the stochastic model, the square mean values of the system variables are studied. In the deterministic case, for different given values of the system parameters we can notice that the stationary state is asymptotically stable.

In the stochastic case, the square mean values of the state variables of the linearized system are asymptotically stable.

The numerical simulation enables the finding of the orbits and other elements which characterize the system. We also examined effects of changes in the propensity to receive education, efficiency of learning and efficiency of education upon dynamic paths of the system. Using the method from this paper a similar analysis was carried out in [14]. Also, it can be applied for discrete economic models with delay [15].

REFERENCES

[1] W.B. Zhang, "Economic growth with learning by producing, learning by education and learning by consuming", *Interdisciplinary Description of Complex Systems* 5(1), 2007, pp. 21-38.

[2] R. Solow, "Growth Theory – An Exposition". *Oxford University Press,* New York, 2000.

[3] K.J. Arrow, *The Economic Implications of Learning by Doing,* Review of Economic Studies 29, pp. 155-173, 1962;

[4] H. Uzawa, "Optimal Technical Change in an Aggregative Model of Economic Growth", *International Economic Review* 6, 1965, pp. 18-31.

[5] R.E. Lucas, *On the Mechanics of Economic Development*, Journal of Monetary Economics 22, pp. 3-42, 1986;

[6] Y. Hu, K. Mino, "Schooling, Working Experiences, and Human Capital Formation", *Economics Bulletin* 2005 vol 15, pp. 1-8.

[7] W.B. Zhang, "Preference, Structure and Economic Growth*", Structural Change and Economic Dynamics* 7, 1996, pp. 207-221.

[8] P.E. Kloeden, E. Platen, "Numerical Solution of Stochastic Differential Equations," *Springer–Verlag*, Berlin, 1995.

[9] J. Lei, M.C. Mackey, “Stochastic Differential Delay Equation, Moment Stability and Application to Hematopoietic Stem Cell Regulation System,” *SIAM J. Appl. Math*., 67(2), 2007, pp. 387–407

[10] G. Mircea, M. Neamtu, D. Opris, ”Deterministic, uncertainty and stochastic models of Kaldor-Kalecki model of business cycle”, *WSEAS Transactions on Mathematics*, 8(9), 2010, pp. 638-647.

[11] G. Mircea, M. Neamţu, D. Opriş, “Uncertain, stochastic and fractional dynamical systems with delay. Applications.*” Lambert Academic Publishing*, 2011.

[12] G. Mircea, M. Pirtea, M.Neamtu, D.Opris, “The Stochastic Monetary Model with Delay”, *WSEAS Proceedings of 2nd World Multiconference on Applied Economics, Business and Development*, Kantaoui, Sousse, Tunisia, may 3-6, 2010, pp. 41-47.

[13] M. Neamtu, “Deterministic and stochastic Cournot duopoly games with tax evasion”, *WSEAS Transactions on Mathematics*, 8(9), 2010, pp. 618-627.

[14] L. I. Dobrescu_ ,M. Neamtu, D. Opris, “The Deterministic and Stochastic Three-Sector Growth Model. Effects of changes in some parameters on the economic system*.”, in press*, 2011.

[15] M. Neamtu, N. Sırghi, C. Babaita, R. Antonie-Nitu, ”Discrete-time deterministic and stochastic triopoly game with heterogeneous players and delay”, *NAUM International journal of mathematical models and methods in applied sciences,* 2(5), 2011, pp. 343-350.

Nicoleta Sirghi is Associate Professor Ph.D at the Faculty of Economics and Business Administration, West University of Timisoara, Romania. She has been a visiting Professor at the Institut of Business Adminstration (IAE) of University Paul Verlaine Metz (France). Ph.D. Sirghi Nicoleta has over 55 articles published in Journals and Proceedings of the International Conferences and 6 books; she has been a regular referee of papers for International Business Information Management Conference (IBIMA). She has been participating in 4 multiannual grants , in 3 as a member and in 1 as a director. Her main academic interests are in game theory with applications in economics.

Mihaela Neamtu is Professor Ph.D at the Faculty of Economics and Business Administration, West University of Timisoara, Romania. She has been a visiting Professor for short periods of time at The Nottingham Trent University, Economics & Politics (Great Britain) and Faculty of Mathematics, Bonn (Germany). Professor Mihaela Neamtu has over 60 articles published in Journals and Proceedings of the International Conferences and 5 monographs; she has been a regular referee of papers for several International Journals and a reviewer of Mathematical Reviews (MathSciNet). She has been participating in 10 multiannual grants (1 of them is international), in 8 as a member and in 2 as a director. Her main academic interests are in dynamical systems and applications in biology and economy.

Dumitru Opris is Professor Ph.D at the Faculty of Mathematics and Informatics, West University of Timisoara, Romania. He has over 100 articles published in Journals and Proceedings of the International Conferences and 15 monographs; he has been a regular referee of papers for several International Journals and a reviewer of Mathematical Reviews (MathSciNet). He has been participating in 15 multiannual grants. His main academic interests are in dynamical systems with applications in biology and economy, geometry.