# Algoritmization of the Information Concept of the Complex Logistic System

Robert Bucki, Bronislav Chramcov, Roman Jašek

***Abstract*—**The paper highlights the vital problem of information mathematical modelling of the logistic system. The complex system itself consists of identical parallel manufacturing subsystems in which there is a manufacturing route arranged in a series of stands. Each stand is equipped with a machine with the dedicated tool. There are interoperation buffer stores between subsequent production stands. After getting worn out, certain tools require regeneration. Used tools from the identical production stands share the same regeneration plant. Irreplaceable tools need to be exchanged for new ones. The replaceable tool can be regenerated a certain number of times. The production process is optimized by means of the stated criteria respecting defined bounds. There is a set of control approaches of which the most effective one is to be chosen in order to either maximize the production output or minimize the lost flow capacity or, finally, minimize the total tool replacement time. The logistic system is controlled by a determined heuristic algorithm. There are also given sub-line heuristic algorithms. Equations of state illustrate the flow of charge material and changes of the order vector elements. Manufacturing strategies allow us to decide which approach will be implemented. Moreover, optimization issues are discussed by means of introducing the multi-stage process model.

***Keywords*—** Heuristic algorithm, logistic system, mathematical modelling, optimization criteria, manufacturing strategies, multi-stage process model.

## I. Introduction

In the contemporary times, manufacturing systems are understood as highly complex logistic plants where control meets pre-defined requirements in accordance with the assumed criteria. The flow of charge material is tunneled precisely so that the production output is maximal within the minimal possible time at the lowest allowable costs. However, the main goals remain manufacturing customers' orders by the assumed time as well as meeting their quality needs. Processes in logistic systems are planned and controlled operatively. The computer simulation of discrete events, the so-called Discrete Event Simulation (DES), is becoming an essential support instrument in making the operation of production and logistics systems more effective. Computer simulation is a widely used analytical tool which permits the study of complex systems which cannot be modelled by other mathematical and statistical methods. This simulation can be used to determine the state of certain controllable inputs to a system that will cause system outputs to be at their most favourable or optimal conditions. This is the principle of simulation optimization [1]. Unlike traditional simulation experimentation answering "what-if" questions, simulation optimization seeks to answer "how-to" questions by identifying the most effective system design or course of action that can maximize system performance.

Over the past three decades a large amount of research has been devoted to the analysis and modelling of production line systems or logistics systems. Papadopoulos and Heavey present a comprehensive literature review of related papers in [2]. One of the critical design factors is the allocation of buffer storage structures with certain capacities between stations. For example, a simulation model which was proposed in [3] defines the optimal dimension of the buffer with regard to the maintenance policy. A generalized meta-model is developed in the work [4]. It incorporates simulation and neural network modelling applications in order to determine the optimum buffer storage capacities between the stages of a serial production flow line. The procedure is based on generating a set of representative buffer storage capacities from all possible combinations; simulating the line with selected capacities; using the simulation output to train a neural network model; and evaluating all possible capacity combinations to select the best capacities available. It is also possible to use the Markovian production system model with a bottleneck [5]. Another work compares the performances of push, pull, and hybrid production control systems for a single line of the multi-stage and continuous process using simulation as a tool [6]. The study is inspired by a production scheduling problem in a large aluminum rolling and processing factory in Istanbul.

Simulation optimization is another vital issue as it is a highly timely topic in the field of manufacturing and supply chain management. Simulation optimization is an extremely valuable technique for investigating the behaviour of many business processes. The high abstraction level of the concept of discrete event simulation means that its application potential is extremely wide-ranging. Some common application areas of discrete event simulation or simulation optimization are service stations such as airports [7], call centers and supermarkets; road and rail traffic; industrial

Manuscript received October 26, 2011. This work was supported by the Ministry of Education of the Czech Republic under grant No. MEB 051024 and No. MEB051031.

R. Bucki is with the College of Informatics and Management, ul. Legionów 81, 43-300 Bielsko-Biała, Poland (e-mail: rbucki@wsi.edu.pl)

B. Chramcov and R. Jašek are with the Tomas Bata University in Zlin, Faculty of Applied Informatics, nam. T.G. Masaryka 5555, 760 01 Zlin, Czech Republic (e-mail: chramcov@fai.utb.cz, jasek@fai.utb.cz).

production lines [8] or technological process [9] and logistical operations like warehousing and distribution [10,11]. The possibilities and limits of simulation employed to create optimal order sequences for flow-shop production systems are outlined as well as discussed and some examples are emphasized in the work [12]. However, this kind of approach requires using sophisticated methods supported by validated tools created on the basis of thoroughly analyzed background. Even if the final output of the company can be achieved with the use of the traditional methods, complex data analysis may result in finding proper solution to the cost cutting issue [31].

Optimal control of a substitutable inventory system, structured assemble-to-order systems and the impact of advance demand information on various production-inventory control mechanisms are the key factors which must be taken into account while planning order realization procedures [13]. Deterministic systems used for these purposes do not involve any randomness in the development of subsequent states of the logistic system. Therefore, such a model will always produce the same output from a given initial state. Stochastic ordering is a fundamental guide for decision making under uncertainty. It is also an essential tool in the study of structural properties of complex stochastic systems [14]. Developing solutions with heuristic tools offers two major advantages: shortened development time and more robust systems [15]. Evolutionary design of intelligent systems is gaining much popularity due to its capabilities in handling several real world problems involving optimization, complexity, noisy and non-stationary environment, imprecision, uncertainty and vagueness [16]. Computational techniques can be used to solve complex problems, simulate nature, explain natural phenomena, and possibly allow the development of new computing technologies [17]. Proper implementation of algorithmic approach can solve many of today's intransigent problems by adapting to new manufacturing challenges [18]. Moreover, many problems can be solved by means of adequate multi-criteria decision-making using modern heuristics [19].

DES and simulation optimization are highly complex fields of research that have the potential of having a considerable impact on the practice - and particularly, when computers become significantly faster. Therefore, at present, a wide range of commercial products are available on the market which are intended for the Windows and UNIX platforms, and which offer an extremely wide spectrum of possibilities for the modelling and simulation of manufacturing, logistical and other queuing systems [20,21]. Currently, nearly every commercial discrete event simulation software contains a module (package) that performs some sort of "optimization" rather than just pure statistical estimation. The goal of an "optimization" package is to orchestrate the simulation of a sequence of system configurations so that a system configuration is eventually obtained providing an optimal or near optimal solution. The work [22] surveys the most prominent simulation optimization software packages (either plug-ins or integrated) currently available and their vendors and the simulation software product that they support and the search techniques used. Our workplace is equipped with a Witness environment in which, in close cooperation with industrial partners, we have conducted a number of simulation studies that have led – at least in part, to increases in the productivity of manufacturing, queuing and logistical systems. This environment especially Witness Optimizer package can also be used for the solution to the problem stated hereby. The Witness environment was used for the optimization of manufacturing, logistics and queuing systems in a whole range of simulation studies. Process analysis using Witness has been conducted, for instance, in the lens manufacturing process flow of at firm in order to identify improvement-prone areas and improvement alternative solutions were proposed [23]. Another work illustrates the use of Witness computer simulation in order to design the production of a manufacturing company that produces snow-melting modules. The analysis presented here describes the production design process and compares the performance of the new design with the existing system's performance [24]. The Witness environment was also used for the simulation of the ophthalmology service of the Regional Military and University Hospital of Oran in Algeria [25] or for analysis of the best layout for an industrial plant [26]. The results that were obtained from applying Witness Optimizer to a manufacturing example with seven decision variables are presented in [27]. Witness's applications in simulation solution deployment have been illustrated in [28].

## II. General assumptions

### A. Mathematical description of the system

Let us model a sample logistic structure which is controlled by means of decisions made in a deterministic way. The problem itself consists in determining the sequence of elements of the order vector which are to be realized subsequently. The proposed heuristic algorithms choose the required element on which certain operations are carried out. We assume that every decision about production, replacement or regeneration is made at the stage $k$-1 after passing the identical time intervals. The state of orders decreases after each production decision which influences the state of the whole logistic system at each stage $k$, $k$=1,...,$K$. The criteria of either production maximization or the lost flow capacity or the minimal tool replacement time are proposed on condition that they are associated by adequate bounds. Assuming that the results of calculations which are made for a chosen heuristic algorithm do not deliver a satisfactory solution, there arises a need to test other algorithms.

Let us introduce the vector of charges $W=[w_l]$, where $w_l$ is the $l$th charge material as well as the vector of orders $Z=[z_n]$, where $z_n$ is the $n$th production order (given in units).

Now, we propose the assignment matrix of products to charges in the form (1), where $\omega_{l,n}$ is the assignment of the $n$th product to the $l$th charge material.

$$\Omega = \left[\omega_{l,n}\right], \; l = 1,...,L, \; n = 1,...,N \tag{1}$$

Elements of the assignment matrix take the values according to the form (2).

$$\omega_{l,n} = \begin{cases} 1 & \text{if the } n\text{th product is realized from the } l\text{th charge,} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Manufacturing of the $n$th product becomes possible on condition in the form (3).

$$\sum_{l=1}^{L} \omega_{l,n} \geq 1, \; n = 1,...,N \tag{3}$$

However, we can distinguish three different types of charge:
1) dedicated (if the condition in the form (4) is valid)
2) alternate (if the condition in the form (5) is valid)
3) universal (if the condition in the form (6) is valid)

$$\forall_n \sum_{l=1}^{L} \omega_{l,n} = 1, \; n = 1,...,N \tag{4}$$

$$\forall_n \sum_{l=1}^{L} \omega_{l,n} \geq 1, \; n = 1,...,N \tag{5}$$

$$\forall_l \sum_{n=1}^{N} \omega_{l,n} \geq 1, \; l = 1,...,L \tag{6}$$

The charge is dedicated if only a specified product can be made from it. In case of an alternate charge, a product can be obtained from some types of it. The universal charge enables realization of a given product from each type. We also assume that used charge vector elements are immediately supplemented. However, for simplicity reasons, we assume that each $n$th product is made from the universal charge. It results in realization the $n$th product from any $l$th charge.

The logistic system presented hereby consists of $I$ parallel manufacturing subsystems. Each $i$th subsystem consists of $J$ production stands arranged in series. Realized products are passed subsequently through each stand in the $i$th subsystem. There are buffer stores between subsequent production stands.

Let us assume that $b_j^k(n)$ is the capacity of the $j$th buffer store in case of storing the $n$th semi-product. It is assumed that the capacity of each $j$th buffer store is limited and equals $b_j(n)_{max}$, $j = 1,...,J-1$. It is assumed that the condition in the form (7) is valid.

$$b_n^k(n) \leq b_j(n)_{max} \tag{7}$$

Moreover, we assume that all stands in the $j$th column share only one $j$th interoperation buffer store. Each $j$th production stand located in the $i$th row of the logistic system can carry out an operation on the $n$th product. It is assumed that each production stand placed in the $j$th column of any manufacturing subsystem realizes the same operation with the use of the identical tool. Moreover, it must also be assumed that $I<N$.

Let us introduce the vector of regeneration plants $R = \left[r_j\right]$, $j = 1,...,J$. The $j$th regeneration plant regenerates tools which are used in each manufacturing stand placed in the $j$th column of the discussed logistic system. The elements of the vector $R$ take the values according to the form (8).

$$r_j = \begin{cases} 1 & \text{if active} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

If tools of the $j$th production stand are to be replaced with new ones, there is no need for their regeneration to be carried out.

Minimizing the total production time by means of minimizing the regeneration time of tools as well as finding the optimal sequence of production decisions which are meant to send totally worn out tools to the $j$th regeneration plant remain the most important goal of the problem stated hereby in the paper. The given tool can be regenerated only a pre-defined number of times. If this number is exceeded, the tool is excluded from the production process and must be replaced by a new one. However, in certain predefined columns there are stands whose tools cannot be regenerated and must be replaced by new ones ($r_j=0$).

Let $E = \left[e_{i,j}\right]$ be the matrix of stands in the discussed logistic system. The elements of this matrix at the $k$th stage take the values in accordance with the form (9).

$$e_{i,j}^k = \begin{cases} 1 & \text{if active} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The elements of the vector of buffer stores $B = \left[b_j\right]$ at the $k$th stage take the values in accordance with the form (10) and at the same time $\forall_k b_J^k = 0$, $k=1,...,K$.

$$b_j^k = \begin{cases} 1 & \text{if the buffer store is necessary} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

If $b_j^k = 0$, then the state of the state of this buffer store is not considered. If $b_j(n) = b_j(n)_{\max}$, then no $n$th product can leave any production stand in the $j$th column unless the equation (11) is valid.

$$b_j(n) < b_j(n)_{\max} \tag{11}$$

The flow of the order vector elements in the manufacturing logistic system formed on the basis of the discussed assumptions is shown at the $k$th stage in the form (12).

$$\omega_{n,l} \Rightarrow \begin{Bmatrix} e^k_{1,1} \\ r_1 \hookleftarrow \\ \vdots \\ e^k_{i,1} \\ r_1 \hookleftarrow \\ \vdots \\ e^k_{I,1} \\ r_1 \hookleftarrow \end{Bmatrix} \rightarrow b^k_1 \ldots \rightarrow \begin{Bmatrix} e^k_{1,j} \\ r_j \hookleftarrow \\ \vdots \\ e^k_{i,j} \\ r_j \hookleftarrow \\ \vdots \\ e^k_{I,j} \\ r_j \hookleftarrow \end{Bmatrix} \rightarrow b^k_j \ldots \rightarrow \begin{Bmatrix} e^k_{1,J} \\ r_J \hookleftarrow \\ \vdots \\ e^k_{i,J} \\ r_J \hookleftarrow \\ \vdots \\ e^k_{I,J} \\ r_J \hookleftarrow \end{Bmatrix} \Rightarrow \begin{bmatrix} z^k_1 \\ \ldots \\ z^k_n \\ \ldots \\ z^k_N \end{bmatrix} \tag{12}$$

The life matrix of the logistic system for a brand new set of tools is defined as $G = \left[g_{n,j}\right]$, where $g_{n,j}$ is the number of units of the $n$th product which can be realized in any production stand in the $j$th column before its tool is completely worn out and requires an immediate replacement with either a regenerated or new tool.

### B. Manufacturing time of the order

Let us define the matrix (13) of production times for the $n$th product in the production stand in the $j$th column. If the $n$th product is not realized in the production stand in the $j$th column, then $\tau^{pr}_{n,j} = 0$.

$$T^{pr} = \left[\tau^{pr}_{n,j}\right],\ n = 1,\ldots,N,\ j = 1,\ldots,J \tag{13}$$

On the basis of the above, the equations in the form (14) and (15) must be introduced, where $y^k_{i,j}$ take the values specified in the form (16).

$$\left(I \cdot \tau^{pr}_{n,j} + \sum_{i=1}^{I} y^k_{i,j} \cdot \tau^{repl}_j\right) \le \left(I \cdot \tau^{pr}_{n,j+1} + \sum_{i=1}^{I} y^k_{i,j+1} \cdot \tau^{repl}_{j+1}\right) \Rightarrow b^k_j = 1 \tag{14}$$

$$\left(I \cdot \tau^{pr}_{n,j} + \sum_{i=1}^{I} y^k_{i,j} \cdot \tau^{repl}_j\right) > \left(I \cdot \tau^{pr}_{n,j+1} + \sum_{i=1}^{I} y^k_{i,j+1} \cdot \tau^{repl}_{j+1}\right) \Rightarrow b^k_j = 0 \tag{15}$$

$$y^k_{i,j} = \begin{cases} 1 & \text{if the replacement of the tool in the } i\text{th stand of the } j\text{th column is carried out,} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

For the above to be valid, it is assumed that the replacing tool is available on demand.

Let us define the vector (17) of replacement times for the tools in the logistic system, where $\tau^{repl}_j$ is the defined replacement time of the tool in the production stand in the $j$th column.

$$T^{repl} = \left[\tau^{repl}_j\right] \tag{17}$$

Let us introduce the production rate vector $V = \left[v_n\right]$. Its element $v_n$ represents the number of units of the $n$th product made in the defined time unit in the $i$th production line.

In order to calculate the total manufacturing time of all elements of the vector $Z$, it is necessary to take into account the production time, the replacement time and, finally, the regeneration time of used tools. The order realization time can be optimized by either employing more production lines at the same time to realize the $n$th element or replacing tools only then when they are fully worn or optimizing the regeneration process so that the tool after regeneration is available on demand. The total order realization time $T$ is calculated beginning with the moment when the first chosen $n$th element enters the logistic system till the moment when the last element of the order vector leaves any stand in the $J$th column.

### C. Equations of state

The state of the discussed parallel logistic system changes in case of manufacturing the $n$th product according to the scheme shown in (18).

$$S^0_n \rightarrow S^1_n \rightarrow \ldots \rightarrow S^k_n \rightarrow \ldots \rightarrow S^K_n \tag{18}$$

The state of the production stand in case of production the $n$th product changes according to the form (19). This state can be written in the form (20).

$$s(n)^0_{i,j} \rightarrow s(n)^1_{i,j} \rightarrow \ldots \rightarrow s(n)^k_{i,j} \rightarrow \ldots \rightarrow s(n)^K_{i,j} \tag{19}$$

$$s(n)^k_{i,j} = \begin{cases} s(n)^{k-1}_{i,j} & \text{if no } n\text{th product is realized in the } i\text{th stand of the } j\text{th column at the } k\text{-1 stage,} \\ s(n)^{k-1}_{i,j} + x^k_n & \text{otherwise} \end{cases} \tag{20}$$

The state of the production stand in case of replacement of the tool changes according to the form (21).

$$s(n)^k_{i,j} = \begin{cases} s(n)^{k-1}_{i,j} & \text{if the tool is not replaced in the ith stand of the jth column at the stage k - 1,} \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

Let $S^{k-1}_n = \left[s(n)^{k-1}_{i,j}\right]$ be the matrix of state of the logistic system for the $n$th product realization at the stage $k$-1 where $s(n)^{k-1}_{i,j}$ is the number of units of the $n$th product already realized in the stand in the $i$th row of the $j$th column with the use of the installed tool. The state of the $j$th interoperation buffer store is verified according to the form (22) after every decision which is made in the system.

$$b_j^k(n) = \begin{cases} b_j^{k-1}(n) + x_j^k(n) - x_{j+1}^k(n) & \text{in case of activity in the buffer store,} \\ b_j^{k-1}(n) & \text{otherwise} \end{cases} \tag{22}$$

The variable $x_j^k(n)$ represents the number of units of the $n$th semi-product leaving production stands in the $j$th column and $x_{j+1}^k(n)$ is the number of units of the nth semi-product entering the production stand located in the column $j+1$.

The order vector changes after every production decision according to the scheme (23).

$$Z^0 \rightarrow Z^1 \rightarrow \ldots \rightarrow Z^k \rightarrow \ldots \rightarrow Z^K \tag{23}$$

The order vector is modified after every decision about production accordance with the specification (24).

$$z_n^k = \begin{cases} z_n^{k-1} - x_n^k & \text{if the number of units } x_n^k \text{ of the } n\text{th order is realized at the } k\text{th stage,} \\ z_n^{k-1} & \text{otherwise} \end{cases} \tag{24}$$

### D. *The flow capacity of the system*

Let $P_n^{k-1} = \left[ p(n)_{i,j}^{k-1} \right]$ be the matrix of the flow capacity of the logistic system for the $n$th product realization at the stage $k$-1 where $p(n)_{i,j}^{k-1}$ is the number of units of the $n$th product which still can be realized in the stand in the $i$th row of the $j$th column. If the condition in the form (25) is valid, then the $n$th order awaits for completing the regeneration process and installing a new tool to enter the production system.

$$\underset{1 \le i \le I}{\forall} p_{i,1}^k = 0 \tag{25}$$

On the basis of the above assumptions we can determine the flow capacity of the production stand in the $i$th row of the $j$th column for the $n$th element of the order vector $Z$ at the stage $k$-1 in the form (26).

$$p(n)_{i,j}^{k-1} = g_{n,j} - s(n)_{i,j}^{k-1} \tag{26}$$

The manufacturing procedure consists in realizing orders in parallel production routes in sequence. It is assumed that manufacturing another order element in a route can begin when the previously realized one leaves the route. Its disadvantage consists in the need of waiting for completing the manufacturing process of a certain product in this route before resuming it again for the next one. This results in not using the available flow capacity of the whole production system. Moreover, during the production course tools must be replaced. The state of the system has to be recalculated when any decision is made in the system.

## III. Heuristic algorithms to control the choice of the order

In order to control the choice of the order vector elements we need to implement heuristics which determine elements from the vector $Z$ for the production process. The control algorithms for production are put forward.

### A. *The algorithm of the maximal order*

This algorithm chooses the biggest order vector element characterized by the biggest coefficient $\gamma_n^{k-1}$ in the state $S^{k-1}$. To produce element $a$, $1 \le a \le N$ the condition in the form (27) must be met, where $\gamma_n^{k-1} = z_n^{k-1}$.

$$(q^k = a) \Leftrightarrow \left[ \gamma_a^{k-1} = \max_{1 \le n \le N} \gamma_n^{k-1} \right] \tag{27}$$

This approach is justified by avoiding excessive bringing the production line to a standstill in order to change an element to be manufactured. If only minimal orders were chosen in state $S_n^{k-1}$, in consequence the number of orders might be reduced. Such control is favourable because the $n$th production line is blocked and must be stopped only in order to replace the tools in certain stands (on condition that the replacement process disturbs the flow of the material).

### B. *The algorithm of the minimal order*

This algorithm chooses the smallest order vector element characterized by the smallest coefficient $\gamma_n^{k-1}$ in the state $S^{k-1}$. To produce the element $a$, the condition (28) must be met, where $\gamma_n^{k-1} = z_n^{k-1}$.

$$(q^k = a) \Leftrightarrow \left[ \gamma_a^{k-1} = \min_{1 \le n \le N} \gamma_n^{k-1} \right] \tag{28}$$

This approach is justified by the need to eliminate the elements of the order vector $Z$ which could be sent to the customer just after the $n$th product leaves the production line on condition that the customer sets such a requirement.

### C. *The algorithm of the relative order*

This algorithm chooses the order characterized by the maximal relative order coefficient $\gamma_n^{k-1}$ in the state $S^{k-1}$. To produce the element $a$, the condition (29) must be met, where $\gamma_n^{k-1} = z_n^{k-1} / z_n^0$.

$$(q^k = a) \Leftrightarrow \left[ \gamma_a^{k-1} = \max_{1 \le n \le N} \gamma_n^{k-1} \right] \tag{29}$$

It is assumed that the orders are realized one after another that is to say each order element $z_n$ in the state $S_n^{k-1}$ is reduced partly. Such control may be advantageous when some parts of the order are needed earlier.

## IV. Sub-line heuristic algorithms

In order to control the choice of the line we need to implement heuristics which determine the subsystem for producing the order on the basis of the flow capacity of the routes (subsystems) defined in the vector (30), where $p(n)_i^{k-1}$ is the flow capacity of the $i$th subsystem.

$$P_i^{k-1} = \left[p(n)_i^{k-1}\right] \tag{30}$$

The algorithms of the maximal and minimal capacity of the subsystem are put forward.

### A. The algorithm of the maximal flow capacity of the sub-system

This algorithm chooses the route characterized by the maximal flow capacity of the subsystem i.e. the maximal coefficient $\rho_i^{k-1}$. To choose the route $b,\ 1 \le b \le I$, the condition in the form (31) must be met, where $\rho_i^{k-1} = p(n)_i^{k-1}$.

$$(q_i^k = b) \Leftrightarrow \left[\rho_b^{k-1} = \max_{1\le i\le I} \rho_i^{k-1}\right] \tag{31}$$

It is assumed that the $n$th order is manufactured in the minimal number of subsystems which may not lead to splitting the order.

### B. The algorithm of the minimal flow capacity of the sub-system

This algorithm chooses the route characterized by the minimal flow capacity of the subsystem i.e. the minimal coefficient $\rho_i^{k-1}$. To choose the route $b,\ 1 \le b \le I$, the condition in the form (32) must be met, where $\rho_i^{k-1} = p(n)_i^{k-1}$.

$$(q_i^k = b) \Leftrightarrow \left[\rho_b^{k-1} = \min_{1\le i\le I} \rho_i^{k-1}\right] \tag{32}$$

It is assumed that the $n$th order is manufactured in the bigger number of subsystems which may not lead to faster manufacturing the order.

## V. The regeneration procedure

Let us introduce the matrix $\Delta = \left[\delta_{i,j}\right]$ which elements represent the number of allowable regeneration processes of the tool for the stand located in the $j$th column. If the formula (33) is valid, then tools from the stands in the $j$th column cannot be regenerated and must be replaced by a new one. If the formula (34) is valid and at the same time the number of already carried out regeneration procedures has been exceeded, the tool cannot be regenerated anymore and must be replaced with a new one. We assume that the replaced tools are available at once.

$$r_j = 0 \;\Rightarrow\; \underset{1\le i\le I}{\forall} \delta_{i,j} = 0 \tag{33}$$

$$\underset{1\le i\le I}{\forall} \delta_{i,j} \ne 0 \tag{34}$$

Each active regeneration plant $r_j$ uses the FIFO procedure consisting in regenerating the worn out elements which are in the queue as the first ones. Then, in due course, the element after completing the regeneration procedure is returned to the adequate production stand which has been in the standstill mode the longest period of time. Machines in the production stands use tools which either can be regenerated or must be replaced by a new one. Each tool subjected to regeneration is indexed. The matrix of the number of allowable regeneration procedures of a tool is given in the vector $R^{reg} = \left[\varsigma_j^{reg}\right]$, where $\varsigma_j^{reg} = \varphi$, $\varphi = 0,1,\ldots,\Phi$. If $\varsigma_j^{reg} = 0$, then tools from stands in the $j$th column are not subjected to the regeneration procedure and must be replaced in order to resume the manufacturing process.

## VI. Manufacturing criteria

The criteria presented hereby are meant to either maximize the production output or minimize the lost flow capacity of the production stands or minimize the tool replacement time. Let us propose production criteria for the logistic system along with the necessary bounds.

### A. The production maximization criterion

The production maximization criterion in the form (35) is reduced to the tool replacement bound specified in the form (36) and the flow capacity bound (37), where $x_n^k$ is the number of units of the $n$th element realized at the $k$th stage and $c$ is the maximal allowable tool replacement time.

$$Q_1 = \sum_{k=1}^{K} q_1^k = \sum_{k=1}^{K}\sum_{n=1}^{N} x_n^k \to max \tag{35}$$

$$\sum_{i=1}^{I} y_{i,j}^k \tau_j^{repl} \le c \tag{36}$$

$$y_{i,j}^k \sum_{i=1}^{I} p(n)_{i,j}^k \le g_{n,j} \tag{37}$$

### B. The lost flow capacity criterion

It is assumed that the total order realization time can be minimized by reducing the lost flow capacity of tools (the so-called residual pass) as other processes (e.g. the time of tool replacement is defined) cannot be shortened. Such a sum can be minimized by means of the method given in [29]. Therefore, the maximal possible use of tools is one of the most important goals and leads to implementing the lost flow capacity criterion in the form (38) which is reduced to the tool replacement bound specified in the form (39) and the order bound (40).

$$Q_2 = \sum_{k=1}^{K} q^k = \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J} y_{i,j}^k \sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{J} p(n)_{i,j}^k \to \min \tag{38}$$

$$\sum_{j=1}^{J} y_{i,j}^k \tau_j^{repl} \le c \tag{39}$$

$$\sum_{n=1}^{N} x_n^k \le z_n \tag{40}$$

### C. The minimal tool replacement time criterion

The minimal tool replacement time criterion in the form (41) is reduced to the flow capacity bound specified in the form (42) and the order bound (43).

$$Q_3 = \sum_{k=1}^{K}\sum_{i=1}^{I} y_{i,j}^k \tau_j^{repl} \to \min \tag{41}$$

$$y_{i,j}^k \sum_{i=1}^{I} p(n)_{i,j}^k \le g_{n,j} \tag{42}$$

$$\sum_{n=1}^{N} x_n^k \le z_n \tag{43}$$

## VII. Manufacturing Strategies

There are two proposed strategies on the basis of which the order realization can be carried out. The first one consists in realization the whole order $z_n^{k-1}$ and only then can another order $z_\nu^{k-1}$ enter the production system. However, there are a lot of disadvantages expected (e.g. the lost production availability which may result in big financial losses if the time factor is analyzed). In this case, the model approach discussed in the paper can be used for calculations. The state of the system is then thought of as $S(n)^{k-1}$.

The second strategy is based on the assumption that elements of the order vector are realized one after another i.e. the element $z_\nu^{k-1}$ enters the production system even then when the order $z_n^{k-1}$ is still realized in the logistic system. The state is then calculated in the stated even moments of time e.g. every second. However, the state is calculated for each element separately: $S(n)^{k-1}$ for the $n$-th order vector element, $S(\nu)^{k-1}$ for the $\nu$th order vector element where $\nu \ne n$, $S(\nu')^{k-1}$ for the $\nu'$th order vector element where $\nu \ne \nu' \ne n$, etc.

## VIII. Optimization matters

Let us now analyze the discussed production system from the point of view of the optimization approach. Here, we can distinguish the production system, its regeneration plants, a charge flux characterized by the charge vector and a product flux characterized by the product vector.

$N$ types of products can be manufactured from $M$ types of charge, at the same time: $M < N$.

Let us introduce the charge vector in its initial state in the form (44), where $w_m^0$ is the number of units of the $m$th charge type in the initial state.

$$W^0 = \left[w_m^0\right], \; m = 1,\dots,M \tag{44}$$

Let us introduce the order vector in the initial state in the form (45), where $z_n^0$ is the number of units of the $n$th product in its initial state.

$$Z^0 = \left[z_n^0\right], \; n = 1,\dots,N \tag{45}$$

Static optimization is indispensably connected with planning. The manufacturing process can be optimized in order to minimize charge costs, minimize lost flow capacity of stands, maximize production or minimize the standstill of stands.

Now, the multi-stage planning model is introduced. The multi-stage process model is presented in the Fig. 1.

Stage $\downarrow$

$$1 \left\{ \begin{array}{ll} t_0 \mapsto & \Delta W^0, \Delta Z^0, S^0 \\ t_1 \to & \Delta W^1, \Delta Z^1, S^1, Z^1 \end{array} \right\} \to X^1, Y^1$$

$\vdots$

$$k \left\{ \begin{array}{ll} t_{k-1} \to & \Delta W^{k-1}, \Delta Z^{k-1}, S^{k-1} \\ t_k \to & \Delta W^k, \Delta Z^k, S^k, Z^k \end{array} \right\} \to X^k, Y^k$$

$\vdots$

$$K \left\{ \begin{array}{ll} t_{K-1} \to & \Delta W^{K-1}, \Delta Z^{K-1}, S^{K-1} \\ t_K \to & \Delta W^K, \Delta Z^K, S^K, Z^K \end{array} \right\} \to X^K, Y^K$$

Fig. 1 The scheme of the multi-stage process model

It is assumed that the production process consists of $K$ stages. The stage $k = 1$ begins at moment $t_0$, the final stage $K$ finishes at the moment $t_K$. At the moment $t_0$, the initial state $S^0$ is given as well as the charge vector $W^0$ and the order vector $Z^0$. During the stage $k = 1$, the quantity $X^1$ of a product is manufactured and $Y^1$ decisions are made in order to replace the worn out assemblies.

During each stage *k*, where $k = 1,...,K$ the quantity $X^k$ of a product is manufactured and $Y^k$ decisions are taken to replace assemblies. The charge vector increases by $\Delta W^1$ after finishing the stage 1 and $\Delta W^k$ after finishing the stage *k*. Similarly, the order vector increases, after finishing the stage $k = 1$ by $\Delta Z^1$, after finishing the stage *k* by $\Delta Z^k$.

## IX. Conclusions

The paper hereby focuses on modelling a sample manufacturing system. However, its assumptions are based on real logistic systems described in the preceding works [30]. Nevertheless, each logistic structure requires completely different assumptions to be put forward and another heuristic-criterion approach. The main goal remains to simplify assumptions in order to enable a sample simulator to be created. It would also be advisable to verify another regeneration approach i.e. the LIFO procedure. Moreover, implementing the regeneration heuristic consisting in choosing the tool with the highest number $\varsigma_j^{reg}$ may result in minimizing the total order realization time. The number of production lines forming the discussed logistic system should be minimized as their bigger number leads to generating unnecessary costs due to the fact that there must be an extra number of operating factors employed. In conclusion, the need for carrying out a computer simulation must be met in order to project the logistic production system which will be able to realize the order in the shortest possible time at the lowest possible costs. To verify the correctness of this kind of modelling, there must also be a multi-criterion model with adequate bounds created. Another idea accelerating the order realization process consists in beginning realization of the next element of the order vector without having to wait for completing manufacturing the remaining units of the *n*th element which means an immediate employing the stand with no current manufacturing duty. However, it requires implementing sub-control for each *i*th production line.

## References


[1] J. D. Hall and R. O. Bowden. “A unified strategy for simulation optimization research and development”, *In Proceedings of 7th Industrial Engineering Research Conference*. Alberta, Canada, 1998.

[2] H. T. Papadopoulos and C. Heavey. “Queueing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines,” *European Journal of Operational Research*, Volume 92, Issue 1, July 1996, pp. 1-27, ISSN 0377-2217.

[3] T. Murino, E. Romano and P. Zoppoli. “Maintenance policies and buffer sizing: an optimization model,” *WSEAS Transaction on business and economics*, Issue 1, Volume 6, pp. 21-30, January 2009.

[4] M. Savsar, and A. S. Youssef, “An Integrated Simulation-Neural Network Meta Model Application in Designing Production Flow Lines”, *WSEAS Transactions on Electronics*, Issue 2, Volume 1, pp. 366-371, April 2004.

[5] C. Ciufudean and C. Filote. “Discrete event formalism for throughput estimation of production lines,” *In Proceedings of the 9th WSEAS international conference on Simulation, modelling and optimization (SMO'09),* Stevens Point, Wisconsin, USA, WSEAS Press, 2009, pp 152-155.

[6] K. Onan and B. Sennaroglu. “A comparative study of production control systems through simulation” *In Proceedings of the 12th WSEAS International Conference on Applied Mathematics (MATH'07),* Cairo, Egypt, WSEAS Press, 2007, pp. 377-382, ISBN: 777-222-4444-66-1.

[7] G. Guizzi, T. Murino and E. Romano. “A discrete event simulation to model passenger flow in the airport terminal”, *In Proceedings of the 11th WSEAS international conference on Mathematical methods and computational techniques in electrical engineering (MMACTEE'09).* Athens, Greece, WSEAS Press, 2009, pp. 427-434.

[8] B. Chramcov, P. Beran, L. Daníček and R. Jašek. “A simulation approach to achieving more efficient production systems,” *International Journal of Mathematics and Computers in Simulation [online]*., Volume 5, Issue 4, 2011, [cit. 2011-06-30]. Available from: < http://www.naun.org/journals/mcs/ >.ISSN 1998-0159.

[9] I. C. Dima, J. Gabrara, V. Modrák, P. Piotr and C. Popescu. “Using the expert systems in the operational management of production”, *In Proceedings of the 11th WSEAS Int. Conf. on Mathematics and Computers in Business and Economics*. Iasi, WSEAS Press, 2010, pp 307-312.

[10] R. Bucki, Z. Frąckiewicz and F. Marecki. “Modelling and Simulation of Complexes of Operations in Logistic Systems”, *In Proceedings of the 29th International Conference on Mathematical Methods in Economics*. Janská Dolina, Slovakia, 2011, pp. 88-93.

[11] V. Modrák. “On the conceptual development of virtual corporations and logistics”, *In Proceedings of the International Symposium on Logistics and Industrial Informatics - LINDI 2007*, University of Applied Sciences Wildau, 2007, pp. 121-125.

[12] R. Schmidt. “Application of simulation for order sequencing in flow-shop production systems,” *In Proceedings of the 21st annual symposium on Simulation (ANSS '88)*, Los Alamitos, CA, USA, IEEE Computer Society Press, 1988, pp. 71-89.

[13] J. G. Shanthikumar, D. Yao, W. Zijm and M. Henk. “Stochastic modeling and optimization of manufacturing systems and supply chains,” *International Series in Operations Research & Management Science,* Vol. 63, 424 p. 2003. ISBN 978-1-4020-7508-7.

[14] M. Shaked and J. G. Shanthikumar. *Stochastic Orders (Springer Series in Statistics),* New York: Springer, 2007, 475 p. ISBN 978-0-387-32915-4

[15] K. Z. Lee and M. A. El-Sharkawi. *Modern Heuristic Optimization Techniques: Theory and Applications to Power Systems*, J. Wiley & Sons, 2007. ISBN 9780471457114.

[16] A. Abraham and C. Grosan. “Engineering Evolutionary Intelligent Systems: Methodologies, Architectures and Reviews,” *Studies in Computational Intelligence (SCI)*, Berlin: Springer-Verlag, 82, pp. 1–22 2008.

[17] L. Castro. *Fundamentals of Natural Computing: Basic Concepts, Algorithms, and Application,* Chapman & Hall/CRC Computer & Information Science Series, 2006, 696 p. ISBN 9781584886433.

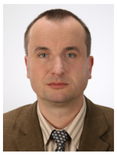
[18] D. Fogel. *Evolutionary Computation: Toward a New Philosophy of Machine Intelligence*, Wiley IEEE Press Series on Computational Intelligence, 2005, 296 p. ISBN 978-0471669517.

[19] Z. Michalewicz and D. B. Fogel. *How to Solve It: Modern Heuristics*, Springer, 2nd ed. Revised and Extended, 2004, 554 p. ISBN 978-3-540-22494-5.

[20] J. Jirsa, “Environments for modelling and simulation of production processes,” *In Proc. Conference Tvorba softwaru 2004*. Ostrava : VŠB-TU Ostrava, Faculty of economics, 2004. p. 65-70. (in Czech)

[21] A. E. Rizzoli, *Simulation Tools* [online]. December 2009 [cit. 2010-09-15]. A Collection of Modelling and Simulation Resources on the Internet. Available from WWW: <http://www.idsia.ch/~andrea/sim/simtools.html>.

[22] M. C. Fu, F. W. Glower and J. April. "Simulation optimization: A review, new developments, and applications", *In Proceedings of the 2005 Winter Simulation Conference*. Orlando, 2005, pp 83-95.
[23] E. U. Olugu and Y. W. Kuan, "Simulation study on lens manufacturing process flow," *In: Proc. of the 2nd Asia In.Conference on Modelling and Simulation, AICMS 2008*. Kuala Lumpur, 2008. pp. 806-811.
[24] M. D. Sarder, "Production design by simulation software *Witness* - A case study," *In: Proceedings of the WRI World Congress on Computer Science and Information Engineering, CSIE 2009*. Los Angeles, 2009. Vol 7:780-786.
[25] K. Belkadi and A. Tanguy, "Modelling and simulation of the opthalmology service for RMUHO," *In Proceedings of the 12th WSEAS international conference on Automatic control, modelling & simulation (ACMOS'10)*, Stevens Point, Wisconsin, USA, WSEAS Press, 2010, pp. 278-286. ISSN: 1790-5117, ISBN: 978-954-92600-1-4.
[26] E. Briano, C. Caballini, R. Mosca, and R. Revetria. "Using *WITNESS* simulation software as a validation tool for an industrial plant layout," *In Proceedings of the 9th WSEAS international conference on System science and simulation in engineering (ICOSSSE'10),* Stevens Point, Wisconsin, USA, WSEAS Press, 2010, pp. 201-206. ISSN: 1792-507X, ISBN: 978-960-474-230-1.
[27] A. M. Law and M. G. McComas. "Simulation-based optimization", *In Proceedings of the 2002 Winter Simulation Conference*. San Diego, 2002, pp 41-44.
[28] A. Mehta and I. Rawles. "Business solution using Witness", *In Proceedin of the 1999 Winter Simulation Conference*. Phoenix, 1999, pp. 230-233.
[29] V. Amirkhanov, E. Pavlušová, M. Pavluš, T. P. Puzynina, I. V. Puzynin and I. Sarhadov, "Numerical solution of an inverse diffusion problem for the moisture transfere coefficient in a porous material," *Materials and Structures*, Vol. 41, 2008, pp. 335-344.
[30] R. Bucki and B. Chramcov, "Control of the serial production system," *In Recent Researches in Automatic Control: Proceedings of 13th WSEAS Int. Conference on Automatic Control, Modelling & Simulation*. Lanzarote: WSEAS Press, 2011. pp. 352-356. ISBN 978-1-61804-004-6.
[31] P. Fiala et al. *Operační výzkum - Nové trendy*. Praha: Professional Publishing, 2010, ISBN: 978-80-7431-036-2 (in Czech).

**Robert Bucki** graduated from Agricultural University in Prague in 1986 where he studied at the Mechanization Faculty. He received his Ph.D. in Computer Sciences at The State Scientific and Research Institute of Information Infrastructure in Lviv in 2004. He is the assistant professor at The College of Informatics and Management in Bielsko-Biała. Scientific interests: mathematical modelling of logistic processes, computer simulation and modelling in computational linguistics.

**Bronislav Chramcov** was born in Uherské Hradiště, Czech Republic, in 1975. He studied automatization and control technology at the Faculty of Technology in Zlin of the University of Technology in Brno and he took his degree in 1998. In 2006 he graduated his doctoral degree from the Faculty of Applied Informatics of Thomas Bata University in Zlin. His doctoral thesis was focused on the utilization of time series prediction for control of technological process. He is working now as a lecturer at the Faculty of Applied Informatics of Thomas Bata University in Zlin. His research activities are focused on control algorithms for district heating systems, time series forecast in energy or discrete event systems simulation.

**Roman Jasek** is working as a Associate Professor, head of Department of Informatics and Artificial Intelligence in Faculty Applied Informatics in Tomas Bata University in Zlin. He graduated from computer science in Palacky University in Olomouc, his Ph.D. recieved in Charles University in Prague. His research interests are: Artificial Intelligence, Computer Science and Data Security.