A Model for Vehicle Fuel Consumption Estimation at Urban Operating Conditions

Michael Ben Chaim, Efraim Shmerling

Abstract**—**A mathematical model for Estimation of vehicle fuel consumption on a 100 km interval at standard operating conditions is presented. Vehicle fuel consumption is presented via mathematical modeling for urban operating conditions. Vehicle fuel consumption is calculated separately for four different operating modes: average speed, accelerations, decelerations and idle mode. This approach was chosen due to the fact that a significant part of fuel consumption at urban conditions occurs at idle mode. The efficiency of the engine is expressed as a function of the speed mode of the engine and of the degree of power utilization of the engine. The adequacy and accuracy of the model is verified using experimental calculations. Moreover, it is shown that the effect of various design parameters on vehicle fuel consumption can be studied utilizing the proposed model.

Keywords**—**fuel economy, vehicle fuel consumption, urban driving cycle

I. INTRODUCTION

THE transportation sector accounts for a growing amount of total end-use energy consumption. Therefore, conserving transportation energy and reducing transportation petroleum consumption is an important short-term and long-term goal. Reducing energy consumption can be achieved by creating new, more fuel efficient vehicle models, as well as by operating existing vehicles more efficiently. Both of these require the utilization of some method of fuel consumption estimation.

At present the most widely used methods of fuel consumption estimation are simulations and road tests. However, these can be costly and are not always obtainable. Utilizing mathematical models is simpler and more readily available. Several mathematical models for estimating fuel efficiency are described in literature; however, the adequacy and accuracy of these models have to be improved in order to make them feasible for practical use.

In [1]-[3], energy expenditure is determined on the basis of modeling. However, fuel consumption estimation in the models presented there is based on specific fuel consumption under optimal conditions, and engine and vehicle mode changes are not incorporated in the models. It is assumed that the vehicle is constantly in acceleration mode and that the effective efficiency is constant. Formulas presented in these papers, as well as in others similar to them, enable one to evaluate fuel consumption only for specific modes of the car and the engine.

In [3] it is proposed to calculate fuel consumption based on specific hourly fuel consumption and energy expenditure.

In [2], it is proposed to determine energy expenditure based on computer simulation. A special program called ADVISOR is used in order to do this. It gives good results for the qualitative analysis of fuel economy.

In [4], the vehicle fuel consumption and emission rates of environment-adaptive driving with or without inter-vehicle communications are estimated using an autonomous running traffic flow simulator. In this study, a microscopic fuel consumption and emission model is used. Simulation results show that environment-adaptive driving can reduce both the average fuel consumption and vehicle emission. It also shows that inter-vehicle communications can improve these impacts under high vehicle densities and long traffic light cycle times. It is proposed to determine energy expenditure on the basis of statistical modeling.

In [5], the estimation method of fuel consumption from vehicle information through OBD-II is proposed. It was proposed that RPM, TPS had a relationship with fuel consumption. In order to verify the effectiveness of the proposed method, a 5 km road-test was performed. The results showed that the proposed method can estimate precisely the fuel consumption from vehicle multi-data. It was observed that the proposed models using instantaneous engine RPM, TPS and (RPM, TPS) can predict the fuel consumption quite well.

The authors of [6] propose an innovative analytically based method of calculating corrected fuel consumption of parallel and series hybrid electric vehicles at balanced energy content of the electric storage devices. The proposed analytical method is generally applicable and features highly accurate corrected fuel consumption results. It enables calculation of the corrected fuel consumption based on a single fuel consumption test run in a single analytic post-processing step. An additional fuel consumption test run might be needed to obtain highly accurate results if the ratio of the energy content deviation of the electric storage devices to the energy used for vehicle propulsion over the test cycle is high. The proposed method enables consideration of non-linear energy flow changes and non-linear hybrid electric vehicles component efficiency changes caused by the energy management strategy or by the component characteristics. The method therefore features highly accurate results out of the minimum number of fuel consumption test runs and thus optimizes workload for

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M. Ben Haim, Department of Mechanical Engineering and Mechatronics, Ariel University Center of Samaria, Israel, Ariel University Center of Samaria, Science Park ,Ariel 44837, Israel. (e-mail: michailbc@ariel.ac.il).

E. Shmerling, Department of Computer Science and Mathematics, Ariel University Center of Samaria, Science Park ,Ariel 44837, Israel. (e-mail: efraimsh@ariel.ac.il; efraimsh@yahoo.com).

development or optimization of hybrid electric vehicles. The input data of the method are characteristic energy flows and efficiencies that are derived from the energy flows on selected energy paths of hybrid electric vehicles.

The paper [7] deals with the fuel consumed by cars for each element of an urban trip by drawing on the results of a survey conducted in Leeds with two instrumented vehicles. Those characteristics of urban car trips which are most likely to influence fuel consumption are identified, and the data obtained from the survey is used to quantify the importance of each element of an urban trip. The fuel consumed during a stop/start maneuver is analyzed, and this is followed by a comparison of the fuel consumption rate estimated when the Leeds results are applied on the standard European driving cycle and the corresponding manufacturer's published data. Finally, the results obtained are used to calibrate two fuel consumption expressions to be used with the output of traffic simulation and assignment models.

Currently, the major set of regulations governing vehicle operating modes for estimating fuel consumption of vehicles are the rules of the UN Economic Commission for Europe [8]. The abovementioned models and formulas for calculating fuel consumption, which do not take into consideration the changes in the mode of motion, are unfit for evaluating fuel consumption in accordance with the accepted regulations. The development of a mathematical model which can be used for this purpose is the contribution of this article.

In the second section of this paper a model for calculating fuel consumption in accordance with UN ECE regulations for urban operating conditions is described. Vehicle fuel consumption is determined separately for four different operating modes: average speed, accelerations, decelerations and at idle mode. The efficiency of the engine is a function of the degree of power utilization and the engine speed mode. We used two correction coefficients in calculating the efficiency of the engine: the coefficient of power utilization and the coefficient of the engine speed.

This approach has allowed us to obtain a universal formula for determining vehicle fuel consumption, which is suitable for practical use.

In the third section of our paper we present the verification of the adequacy and accuracy of the obtained formula. To assess its adequacy and accuracy, we carried out calculations of fuel consumption using our formula and compared the results to experimental data provided by the manufacturers. In order to carry out calculations via the formula, we first identified parameters common for all automobiles. The vehicle-specific parameters that we used were the type of engine, automobile mass, maximum power and the engine speed at maximum power. We compared the results of our calculations to data on modern automobiles manufactured in 2011 by leading automotive firms.

In the fourth section the results of the abovementioned comparison are analyzed. It is concluded that since the discrepancy between the results of our calculations and the experimental data was between 4-5%, the obtained formula provides a sufficiently good approximation. The discrepancy can be partly explained by the fact that since the parameters for specific types of vehicles do not appear in the literature, we used averaged values for diesel and petrol engines.

II. Equation for Estimating Fuel Consumption

The formula for calculating fuel consumption incorporates UN ECE regulations. As in [1, 2], we calculate fuel consumption based on energy expenditure. Energy expenditure is divided into four parts, (Table I, Fig. 1) [8]:

Table I. Parameters for the urban cycles [10]

Characteristics	Unit	ECE 15
Distance	km	4×1.013=4.052
Duration	s	4×195=780
Average Speed	km/h	18.7 (with idling)
Maximum Speed	km/h	50

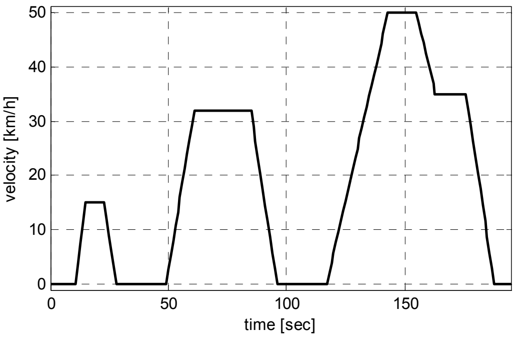

Fig. 1. Scheme of urban cycle

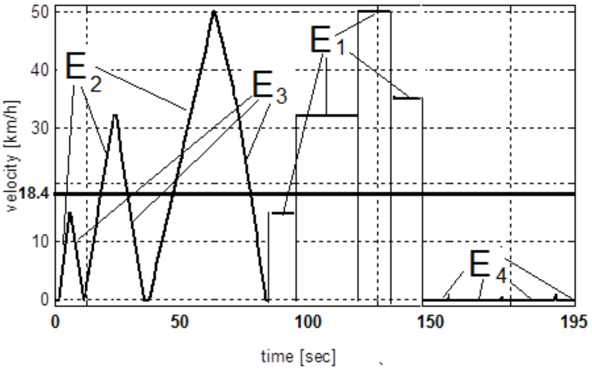

Fig. 2. Scheme for calculating E_1, E_2, E_3 and E_4

Table II. Parameters of vehicle motion during the ECE 15 Cycle

n/n	time	Acceleration (Deceleration)	Average speed	Distance
	s	m/s^2	km/h	m
1 E_4	15	-	-	-
2 E_2	5	0.78	7.0	10
3 E_1	10	-	14.0	50
4 E_3	5	-0.75	7.0	15
5 E_4	20	-	-	-
6 E_2	20	0.60	25.0	60
7 E_1	10	-	50.0	180
8 E_3	20	-0.60	25.0	50
9 E_4	15	-	-	-
10 E_2	20	0.60	25.0	175
11 E_1	10	-	35.0	170
12 E_3	10	-0.60	42.5	40
13 E_1	10	-	35.0	130
14 E_3	20	-0.60	18.0	133
15 E_4	5	-	-	-

The equation for estimating fuel consumption must take this into account:

1. The energy required for overcoming the forces of resistance at constant average speed,
2. The energy required for movement with accelerations or the energy required for overcoming the resistance of the inertial force,
3. The energy required for movement with decelerations or braking,
4. The energy expended at idle mode.

In the formula the energy expenditure is expressed as a sum:

$$E_s = E_1 + E_2 + E_3 + E_4.$$

The equation for fuel consumption takes the form:

$$Q_{S(e)} = \frac{E_s}{H_L} = \frac{E_1 + E_2 + E_3 + E_4}{H_L},$$

where

E_1 is the energy required for overcoming the forces of resistance at constant average speed,

E_2 is the energy required for movement with accelerations, or the energy required for overcoming the resistance of the inertial force,

E_3 is the energy required for movement with decelerations,

E_4 is the energy expended at idle mode,

H_L is the calorific value of one liter of fuel.

The first component E_1 has the form:

$$E_1 = \frac{1}{\eta_T \eta_{P,n}} \left(m_a \cdot g \cdot c_r + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V_a^2 \right) \cdot S$$

where

η_T is the efficiency of the transmission,

V_a is the average speed of the vehicle, m/sec,

m_a is car mass, kg,

c_r is the rolling resistance coefficient,

C_D is the coefficient of aerodynamic resistance of the car,

A_f is the characteristic area of the car, m^2.

The values of parameters c_r and A_f are determined by empirical equations [9]:

$$c_r = 0.0136 + 0.40 \cdot 10^{-7} V_a^2,$$

$$A_f = 1.6 + 0.00056(m_a - 765).$$

S is the car mileage, which equals 100000 m, i.e. 100 km,

$\eta_{P,n}$ is the efficiency of the engine, which depends on the peak efficiency of the engine, degree of power utilization and the engine speed mode in the following way:

$$\eta_{P,n} = \eta_e \mu_P \mu_n,$$

where

η_e is the engine's peak efficiency,

μ_P is the coefficient through which the influence of the degree of power utilization (the part-load) on the peak efficiency of the engine is expressed,

μ_n is the coefficient through which the influence of engine speed mode on the peak efficiency of the engine is expressed.

Unlike formula (1), in which the efficiency (or g_e, the specific fuel consumption) of the engine is assumed to be constant, in our formula it is variable and a function of the peak efficiency of the engine at optimal mode η_e , the degree of power utilization and engine speed mode. Variable efficiency of diesel engines (with the peak efficiency $\eta_e = 0.40,\ g_e = 206,$ g/kWh, [10]), or gasoline engines (with the peak efficiency $\eta_e = 0.30,\ g_e = 275,$ g/kWh, [10]), determines the actual fuel consumption in regular driving conditions and is smaller (on average 0.30 for diesel and 0.25 for gasoline engines) (Fig. 3), [10].

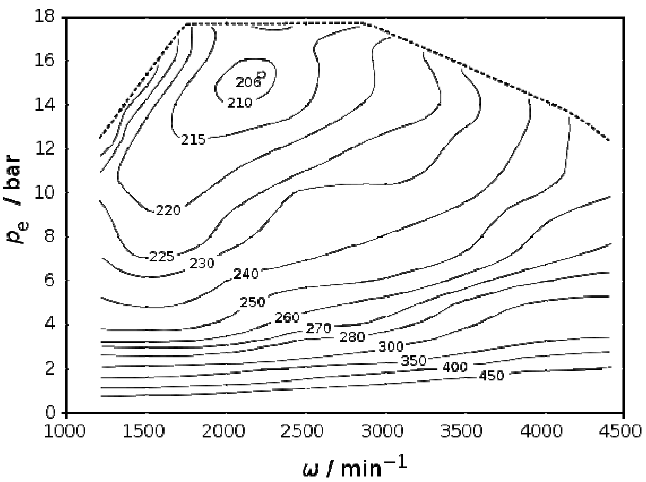

Fig. 3. Variable fuel consumption of diesel engines

Diesel engine's peak efficiency [10] η_e = 1/(0.0119531 g_e) ~0.40

Gasoline engine's peak efficiency [10] η_e = 1/(0.0122225 g_e) ~0.30

To include the effect of the degree of power utilization and of the engine speed mode on the efficiency of the engine, we have introduced the correction coefficients μ_P and μ_n, respectively.

In order to obtain functions μ_P and μ_n, the dependences $\mu_P = f(P_i / P_e)$ and $\mu_n = f(n_i / n_N)$ were analyzed for a number of modern gasoline and diesel engines, information about which is available in the literature. As a result of the data analysis [11] – [15], the following graphic dependences (Fig. 4, 5, 6) were obtained.

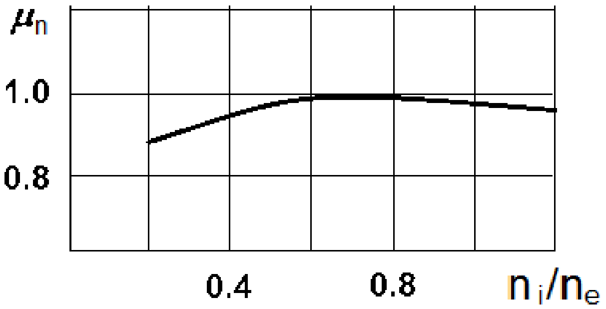

Fig. 4. Coefficient of the engine speed mode (same for diesel and gasoline engines)

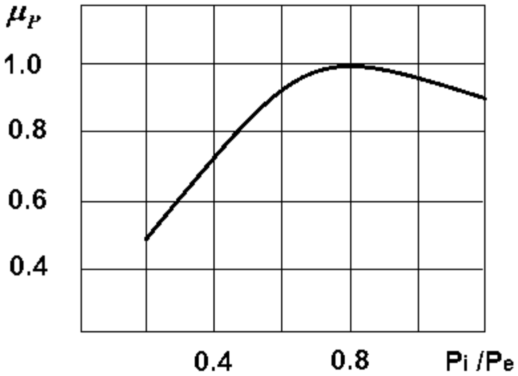

Fig. 5. Coefficient of power utilization for gasoline engines

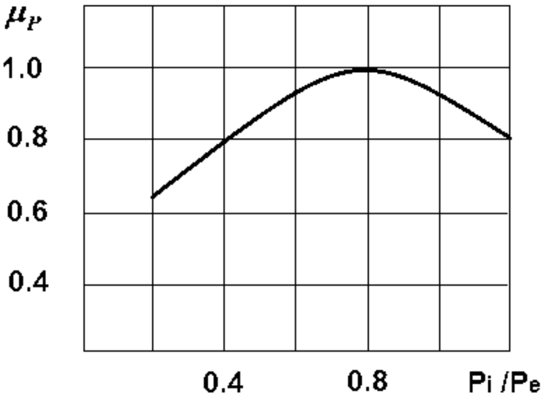

Fig. 6. Coefficient of power utilization for diesel engines

These graphical relationships were approximated and the following results were obtained:

a formula for calculating μ_P for diesel engines:

$$\mu_P = 0.2688 \ln\left(\frac{P_i}{P_e}\right) + 1.0546,$$

a formula for calculating μ_P for gasoline engines:

$$\mu_P = 0.3232\ln\left(\frac{P_i}{P_e}\right) + 1.0118,$$

and a formula for calculating μ_n for diesel and gasoline engines:

$$\mu_n = 0.1055\ln\left(\frac{n_i}{n_P}\right) + 1.0446,$$

where

P_i is the engine power required for the given mode (*P*) of motion,

P_e is the engine power by the performance characteristics of the engines, corresponding to vehicle speed V_{a_i},

n_P is the engine speed at maximum power of engine, min^{-1},

n_i is the engine speed at average speed of vehicle, V_{a_i}, min^{-1}.

Here P_e and n_i are calculated via the formulas given in [3, 16].

According to the definition

$$\frac{P_i}{P_e} = \frac{\left(m_a g c_r + 0.5 c_D A_f V_a^2 + m_a a \gamma_m\right) V_a}{P_e}.$$

Here the numerator is the engine power required for the given mode of motion, and the denominator is the engine power by the performance characteristics of the engine for the corresponding vehicle speed (Fig. 3.). It is a function of engine speed and maximum engine power and is determined by the empirical formula [3, 16].

$$P_e = P_{\max}\left[a\left(\frac{n}{n_P}\right) + b\left(\frac{n}{n_P}\right)^2 - c\left(\frac{n}{n_P}\right)^3\right]$$

where

$P_{\max}$ is the engine's maximum power, kW,

a, *b*, *c* are the polynomial coefficients, different for different types of engines (see Table III).

Table III. Polynomial coefficients [3]:

Gasoline engine	Diesel engine	Coefficients
1.0	1.0	a
1.0	0.5	b
1.0	0.5	c

n_P is the engine speed at maximum power of engine, min^{-1},

n_i is the engine speed at average speed of vehicle, V_a, min^{-1}.

The formula for determining it has the following form

$$n = \frac{9.55 V_a \xi_{ax} \xi_n}{r_d},$$

where

r_d is the rolling radius of the tire, *m*,

ξ_{ax} is the finale drive gear ratio,

ξ_n is gear ratio in the gearbox,

ρ is air density, $N \cdot s^2 / m^4$,

g is the acceleration of gravity, m / s^2.

The energy required for motion with accelerations is calculated via the following formula:

$$E_2 = \sum_{j=1}^{\sigma} \frac{S_j}{\eta_{Pnj}}\left[\left(m_a g f_{rl} + \frac{\rho}{2} \cdot C_D A_f V_{a_j}^2\right) + q m_a a_j \gamma_{mj}\right]$$

where

γ_{m_j} is the mass factor of the vehicle,

a_j is the acceleration of the vehicle, m / s^2.

S_j is the acceleration distance of the vehicle, *m*.

σ is the number of acceleration intervals,

q is the number of accelerations in each acceleration interval.

The energy E_3 is the energy expended during deceleration or braking by engine. It is known that for diesel engines, gasoline engines with injection of fuel, and carbureted engines with economizer of forced idle mode, it is equal to zero [14] – [18], and because most modern car engines belong to the above listed groups, we assume that $E_3 = 0$.

The energy E_4 is the energy expenditure at idle mode or fuel consumption at time of idling mode of the engine. The following empiric formula is used for its calculation in the publications [19]:

$$Q_{i.m.} = 0.55 \cdot 10^{-6} V_h n_{i.m.} t$$

where

V_h is engine displacement, *l*,

$n_{i.m.}$ is engine speed at time of idling mode,

t is time of idling mode, *h*.

For its calculation we propose the following formula:

$$E_4 = \frac{n_{i.m.}}{60} \frac{1}{2} \frac{1}{\lambda} \frac{\rho_a}{AFR_{teor}} V_h \varphi \frac{1}{\rho_f} H_L t, \quad j,$$

where

ρ_a is the air-fuel mixture density at time of idling mode of the engine, kg/l,

H_L is the calorific value of one kilogram of fuel, J/kg,

φ is the filling ratio by air–fuel mix at time of idling mode (*φ<1)*.

AFR_{teor} is the air–fuel ratio. It is the ratio between the mass of air and the mass of fuel in the fuel–air mix [16]:

$$AFR_{teor} = \frac{m_{air(teor)}}{m_{fuel(teor)}} = \frac{14.7\ kg}{1\ kg} = 14.7$$

where

$m_{air(teor)}$ is the mass of air theoretically required for complete combustion of 1 kilogram of fuel, which is equal to 14.7 kilograms,

$m_{fuel(teor)}$ is the mass of fuel which is equal to one kilogram in a theoretically normal mixture.

At time of idling mode for gasoline engines $m_{air} < 14.7\ kg$ for combustion of 1 kilogram of fuel, and for diesel engines $m_{air} > 14.7\ kg$

To estimate the composition of the mixture at different operating modes of the engine, the following lambda coefficient is used:

$$\lambda = \frac{AFR_i}{AFR_{teor}},$$

where

AFR_i is the air–fuel ratio for the given engine mode. Thus, for idling mode for gasoline engines $AFR_i < 14.7$, while for diesel engines $AFR_i > 14.7$, and therefore, we obtain that for gasoline engines (λ<1.0) and for diesel engines (λ>1.0).

The total energy required for driving 100 kilometers is:

$$E_S = \frac{1}{\eta_T}\Bigg(\sum_{i=1}^{k} \frac{S_i}{\eta_{(P,n)_i}}\left(m_a \cdot g \cdot f_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V_{a_i}^2\right) + \sum_{j=1}^{\sigma} \frac{S_j}{\eta_{(P,n)j}}\left[\left(m_a g f_{rl} + \frac{\rho}{2} \cdot C_D A_f V_{a_j}^2\right) + q m_a a_j \gamma_{mj}\right]\Bigg) + \frac{n_{i.m.}}{60}\frac{1}{2}\frac{1}{\lambda}\frac{\rho_a}{AFR_{teor}} V_h \varphi H_L t$$

When we substitute E_S, the equation for fuel consumption defined by energy expenditure takes the following form:

$$Q_{S(e)} = \frac{1}{\eta_T H_L}\Bigg(\sum_{i=1}^{k} \frac{S_i}{\eta_{(P,n)_i}}\left(m_a \cdot g \cdot f_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V_{a_i}^2\right) + \sum_{j=1}^{\sigma} \frac{S_j}{\eta_{(P,n)j}}\left[\left(m_a g f_{rl} + \frac{\rho}{2} \cdot C_D A_f V_{a_j}^2\right) + q m_a a_j \gamma_{mj}\right]\Bigg) + \frac{n_{i.m.}}{60}\frac{1}{2}\frac{1}{\lambda}\frac{\rho_a}{AFR_{teor}} V_h \varphi \frac{1}{\rho_f} t.$$

III. VERIFICATION OF THE FORMULA BASED ON EXPERIMENTAL CALCULATIONS

To assess the validity of the formula which we obtained and the feasibility of conclusions based on it, we carried out calculations of fuel consumption via the formula and compared the results to experimental data available from manufacturers [17].

In order to carry out calculations via the formula, we first identified vehicle motion parameters at average speed and during accelerations according to the ECE 15 cycles, which are given in Tables IV and V:

Table IV. Parameters of vehicle motion at average speed according to the ECE 15 cycle

Vehicle motion parameters at average speed	Average speed V_i km/h			
	14.0	32.0	50.0	35.0
Distance, m, S_i	45.0	210.0	175.0	125.0
Duration of motion, t, s	12.5	22.5	12.5	12.5

Table V. Parameters of vehicle motion during accelerations according to the ECE 15 cycle

Vehicle motion parameters during acceleration	Acceleration from speed to speed, km/h		
	0 - 14.0	0 - 32.0	0 - 50.0
Acceleration time, t, s	5.0	12.0	23.0
Average Acceleration, m/s2, a_i	0.78	0.75	0.60
Acceleration Distance, m, S_i	10.00	50.00	160.0
Average speed during acceleration, km/h, V_i	7.0	16.0	25.0

In addition we also identified parameters common for all automobiles, which are given in Tables VI and VII:

Table VI. Common parameters for all automobiles

Type of Engine	P_e, kW	$n_P, \min^{-1}$
Diesel	100	4500
Gasoline	100	6000
Type of Engine	ξ_{n_i}	$c_d, N \cdot s^2 / m$
Diesel	1.0-3.5	0.30
Gasoline	1.0-3.5	0.30

Table VII. Common parameters for all automobiles

Type of Engine	η_e	η_T	ξ_{ax}
Diesel	0.40	0.95	3.5
Gasoline	0.30	0.95	3.5
Type of Engine	γ_{m_i}	A_f, m^2	c_r
Diesel	1.06-1.12	1.8	0.015
Gasoline	1.06-1.12	1.8	0.015

The calculation results are summarized in Table VIII.

Table VIII. Fuel consumption rates of different vehicles based on experimental data vs. the results obtained using the formula

Vehicle	Fuel consumption based on experimental data	Fuel consumption by formula
Volkswagen Polo Sedan	7.7	7.4
Toyota Yaris	7.8	8.2
Toyota Sienna AWD	14.7	15.5
Toyota Camry AWD3.5	10.6	10.7
Hyundai Genesis Coupé 2.0 T	10.2	10.8

Table IX. Parameters of different vehicles from Table VIII

	Technical Specifications		
Vehicle	Mass, kg	Max. Power, kW	rpm
Volkswagen Polo Sedan	1106	62.6	5000
Toyota Yaris	1005	73.1	6000
Toyota Sienna AWD	2080	197.6	6000
Toyota Camry AWD3.5	1570	196.9	6000
Hyundai Genesis Coupé 2.0 T	1570	157.3	6000

IV. CONCLUDING REMARK

Comparison of the data presented in the Table 4 and similar data available in literature [17], suggests that the obtained formula reflects general trends in the influence of design and performances parameters on fuel consumption.

The discrepancy between the results of our calculations and the experimental data was between 4-5%, which indicates that the obtained formula provides a sufficiently good approximation. The abovementioned discrepancy can be partly explained by the fact that since the parameters η_e and η_T for specific types of vehicles do not appear in the literature, we used averaged values for diesel and petrol engines.

Based on the above comparison we concluded that the obtained formula is sufficiently accurate and fit for evaluating fuel consumption.

REFERENCES

[1] L. Guzzella and A. Sciarretta, Vehicle propulsion systems, Springer Verlag, Berlin, 2007.

[2] L. Guzzella, A. Sciarretta, Vehicle propulsion systems, Springer Vlg. Berlin, Heidleberg, 2005, ISBN-10 3-540-25195-2.

[3] V. Gaevsky and A. Ivanov, Theory of Ground Vehicles, MADI, Moscow, 2007.

[4] S. Widodo, T. Hasegawa, S. Tsugawa, Vehicle fuel consumption and emission estimation in environment-adaptive driving with or without inter-vehicle communications: Intelligent Vehicles Symposium, 2000, IV 2000, Proceedings of the IEEE, pp.382 – 386, 2000.

[5] M. G. Lee, Y. K. Park, K. K. Jung and J. J. Yoo, Estimation of Fuel Consumption using In-Vehicle Parameters, International Journal Science and Technology, Vol. 4, No. 4, pp 37-47, 2011.

[6] T.Katrasnik. Analytical method to evaluate fuel consumption of hybrid electric vehicles at balanced energy content of the electric storage devices. Applied Energy. V. 87, I. 11, Pages 3330–3339. 2010.

[7] R. Akcelik. Efficiency and drag in the power-based model of fuel consumption. Transportation Research Part B: Methodological, V. 23, I. 5, 1989, pp. 376–385.

[8] United Nations Economic Commission for Europe (UNECE), Vehicle Regulations, from accessed on 2011-08-01.

[9] Wong, J., Theory of Ground Vehicles, John Wiley and Sons, 2001.

[10] http://www.fueleconomy.gov/feg/printGuides.shtml

[11] Bosch, Gasoline Engine Management, 2004.

[12] R. Stone, Introduction to Internal Combustion Engines, Macmillan, 1985.

[13] J. Jimenez-Palacios, Understanding and Quantifying Motor Vehicle Emissions and Vehicle Specific Power and TILDAS Remote Sensing, Ph.D. thesis, Massachusetts Institute of Technology, February 1999.

[14] J.B. Heywood, Internal Combustion Engine Fundamental, McGraw-Hill, Inc., New York, 1988.

[15] Colin R. Ferguson, A. Thomson Kirkpatrick, Internal Combustion Engine, John Wiley & Sons, 2001.

[16] V. N. Kravets, Theory of Vehicles, Handbook, University of Nizhniy Novgorod: NNSU, p. 368. (in Russian), 2007.

[17] http://www.metrompg.com/posts/speed-vs-mpg.htm