Obtaining an Optimum PID Controllers for Unstable Systems using Current Search

D. Puangdownreong and A. Sakulin

Abstract**—**A significant application of the current search (CS), a novel meta-heuristic intelligent search method in industrial control domain, is proposed in this article. The CS is inspired by an electric current flowing through electric networks. It is conducted to synthesize an optimum PID (proportional-integral-derivative) controller for stabilizing three unstable plants, i.e. inverted pendulum, ball-beam, and pitch control systems. The CS is used to search for the optimum controller's parameters denoted as proportional, integral, and derivative gains. As simulation results, optimum PID controllers can be achieved by the CS. The results obtained are very satisfactory.

Keywords**—**Control synthesis, current search, PID controller, unstable system.

I. Introduction

In 1922, Minorsky [1] proposed the proportional-integral-derivative (PID) controller. It has become to well-known controller widely used in industrial applications. The PID controller family consisting of P, PI, PD, and PID, has been played as the heart of control engineering practice. Over the past two decade, three hundred and eighty publications on the use of PI or PID controller for the compensation of process with time delays have been recorded, more than three times the number of publications in the previous five decades [2]. The ability of PI and PID controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications. For example, 98% of control loops in the pulp and paper industries are controlled by SISO PI controllers [3], and in process control applications, more than 95% of the controllers are of PID type [4].

To obtain appropriate controller's parameters, one can proceed with available analytical design methods or tuning rules. Mostly the analytical design methods assume known plant models [2],[5],[6],[7], while the tuning rules assume known process responses [8],[9]. Those analytical design methods and tuning rules, however, have some particular conditions concerning the plant models, such as dead time or transport lag, fast and slow poles, real and complex conjugated zeros and poles, as well as unstable poles, etc.

To date, artificial intelligent (AI) techniques have been accepted and used for the controller design in industrial control applications, for example, designing of an adaptive PID controller by genetic algorithm (GA) [10], self-tuning PID controller by GA [11], finite-precision PID controller by GA [12], and PID controller design by adaptive tabu search (ATS) [13]. Although the ATS are efficient to find the global minimum of the search space and widely used in control and engineering applications such as system and model identification [14], system performance optimization [15], assembly line balancing problem [16], surface optimization [17], and control synthesis [18], they consume too much search time in some particular cases. The current search (CS), one of the novel meta-heuristic optimization techniques, is an alternative. The CS is expected to be an alternative potential algorithm to obtain an optimum PID controller. In this article, the CS algorithm is briefly reviewed and then applied to design PID controllers for three unstable systems. This article consists of five sections. The problem formulation of PID control synthesis is described in Section 2. A brief of the CS algorithm is provided in Section 3. The CS-based design of the PID controllers is illustrated in Section 4, while conclusions are given in Section 5.

II. Problem Formulation

This section describes the problem formulation of PID controller design consisting of two parts, i.e. the PID control loop and the unstable plants used in this work.

A. The PID Control Loop

A conventional control loop is represented by the block diagram in Fig. 1. The PID controller receives the error signal, $E(s)$, and generates the control signal, $U(s)$, to regulate the output response, $C(s)$, referred to the input, $R(s)$, where $D(s)$ is disturbance signal, $G_p(s)$ and $G_c(s)$ are the plant and the controller transfer functions, respectively.

The transfer function of the PID controller is stated in (1), where K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain. So, the design problem is simplified to determine the parameters K_p, K_i, and K_d. The closed loop transfer function with PID controller is given in (2).

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \tag{1}$$

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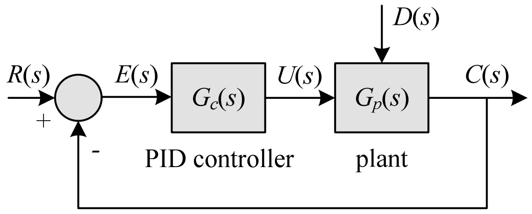

Fig. 1 PID control loop

$$\frac{C(s)}{R(s)} = \frac{\left(K_p + \frac{K_i}{s} + K_d s\right) G_p(s)}{1 + \left(K_p + \frac{K_i}{s} + K_d s\right) G_p(s)} \tag{2}$$

The use of the CS to synthesize the PID controller can be represented by the block diagram in Fig. 2. The cost function, J, the sum of absolute errors between $r(t)$ and $c(t)$ as stated in (3), is fed back to the CS tuning block. J is minimized to find the optimum PID controller's parameters.

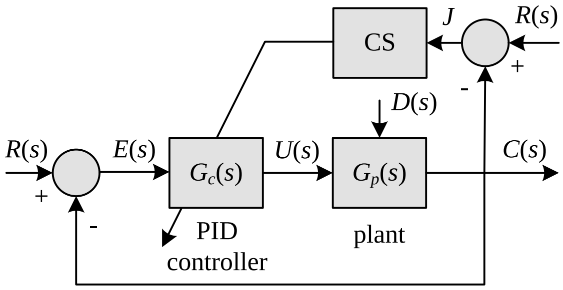

Fig. 2 CS-based PID control synthesis

$$J = \sum_{t=0}^{T} \left\| r(t) - c(t) \right\| \tag{3}$$

B. Unstable Plants

Three unstable plants used in this work, the inverted pendulum (IP) [19], the ball-beam (BB) [20], and the pitch control (PC) [21], are conducted by the survey of literature. They can be considered as the hard-to-be-controlled plants, because both are inherently unstable and difficult to be designed.

The IP is well-known unstable system. The schematic diagram of the IP is represented in Fig. 3. The model of the IP, $G_{p1}(s)$, is linearized as stated in (4), where M (cart mass) = 0.5 kg, m (pendulum mass) = 0.5 kg, l (pendulum length) = 0.3 m, b (viscous-friction coefficient of cart) = 0.1 N/m/s, I_p (moment of inertia of pendulum bar) = 0.006 kg-m^2, g (gravitational force) = 9.81 m/s^2, f is an external force as input, and θ is pendulum angle as output. The step response, the frequency response, and the root locus plots of the IP are depicted in Fig. 4 to Fig 6, respectively. Readers can find details of the IP system from [19].

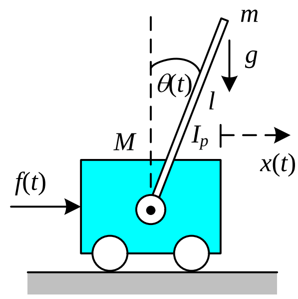

Fig. 3 inverted pendulum (IP) system

$$G_{p1}(s) = \frac{\Theta(s)}{F(s)}$$

$$= \frac{\frac{ml}{q} s}{s^3 + \frac{b(I_p + ml^2)}{q} s^2 - \frac{(M+m)mgl}{q} s - \frac{bmgl}{q}}$$

…, where $q = [(M+m)(I_p + ml^2) - (ml)^2]$

$$G_{p1}(s) = \frac{5.2632s}{s^3 + 0.1789s^2 - 51.6316s - 5.1649} \tag{4}$$

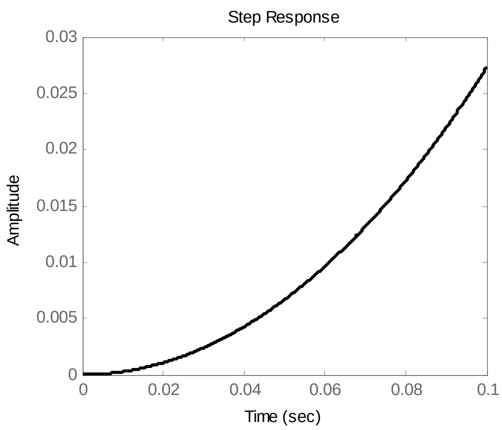

Fig. 4 step response of IP system

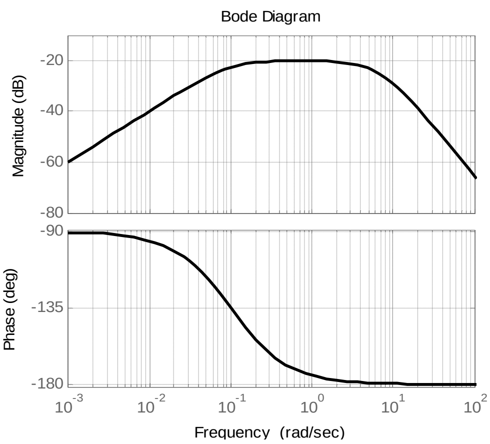

Fig. 5 frequency response of IP system

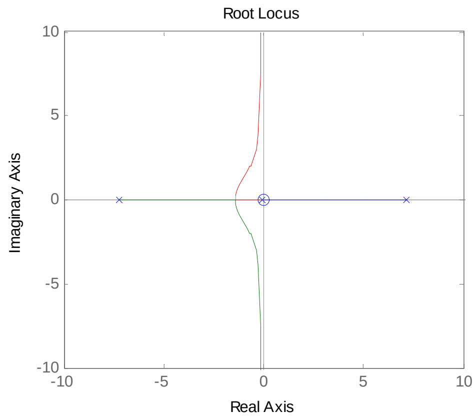

Fig. 6 root locus of IP system

The BB is also well-known unstable system. The schematic diagram of the BP is represented in Fig. 7. The model of the BB, $G_{p2}(s)$, is linearized as stated in (5), where R (radius of spherical ball) = 3.59 cm, r (displacement from ball's center to one of edges of beam) = 2.54 cm, g (gravitational force) = 9.81 m/s^2, ϕ is an angle of the beam as input, and x is the position of the ball as output. The step response, the frequency response, and the root locus plots of the BB are depicted in Fig. 8 to Fig 10, respectively. Readers can find details of the BB system from [20].

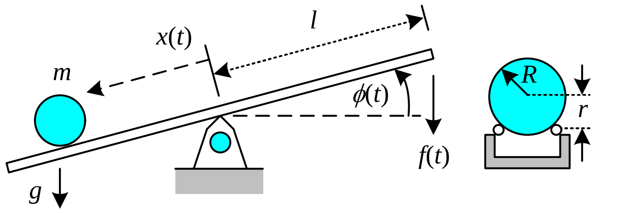

Fig. 7 ball-beam (BB) system

$$G_{p2}(s) = \frac{X(s)}{\Phi(s)} = \frac{g}{\left(1 + \frac{2}{5}\frac{R^2}{r^2}\right)s^2} = \frac{5.4528}{s^2} \qquad (5)$$

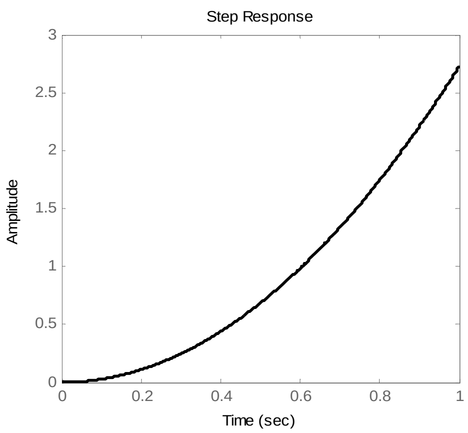

Fig. 8 step response of BB system

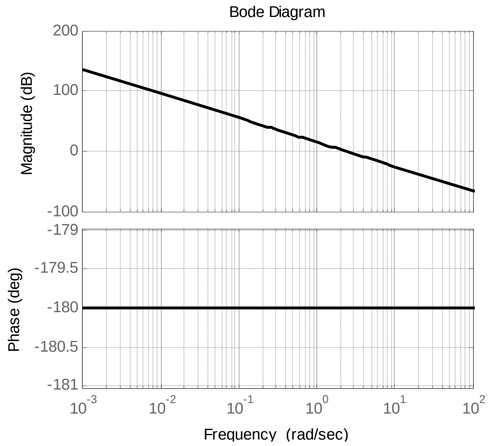

Fig. 9 frequency response of BB system

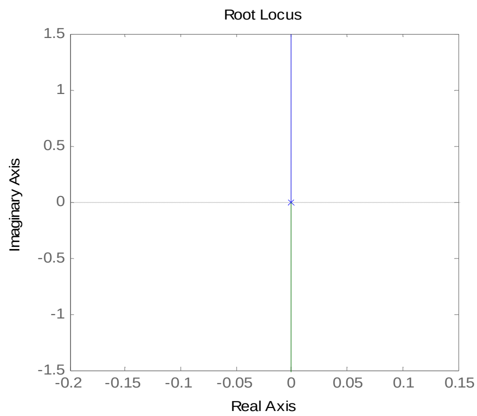

Fig. 10 root locus of BB system

The pitch control (PC) is also unstable system. It is used to design the appropriate autopilot for the aircraft Boeing 747-400 [21]. The schematic diagram of the PC is represented in Fig. 11. The model of the PC, $G_{p3}(s)$, is linearized as stated in (6), where θ is the pitch angle and δ_e is an elevator deflection angle. The step response, the frequency response, and the root locus plots of the PC are depicted in Fig. 12 to Fig 14, respectively. Readers can find details of the PC system from [21].

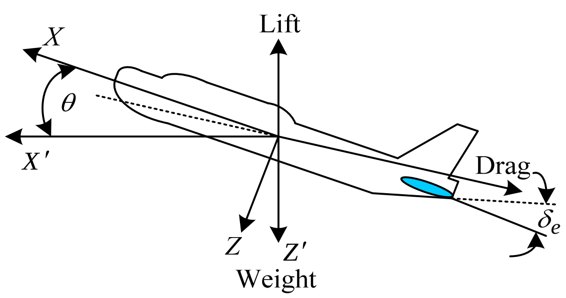

Fig. 11 pitch control (PC) system

$$G_{p3}(s) = \frac{\Theta(s)}{\Delta_e(s)} = \frac{1.6895s + 0.84445}{s^3 + 1.1752s^2 + 1.5887s} \qquad (6)$$

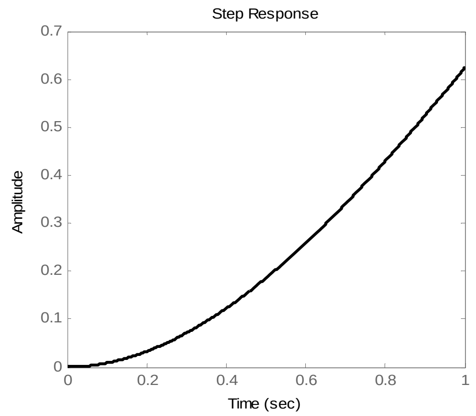

Fig. 12 step response of PC system

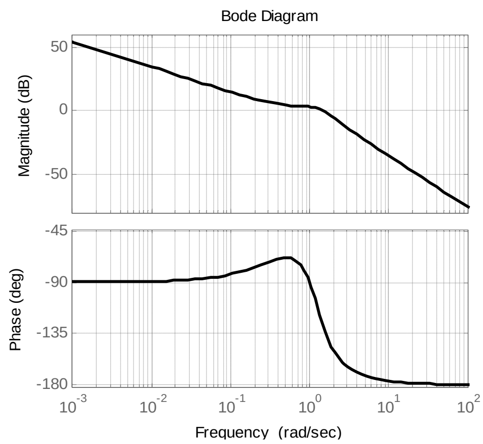

Fig. 13 frequency response of PC system

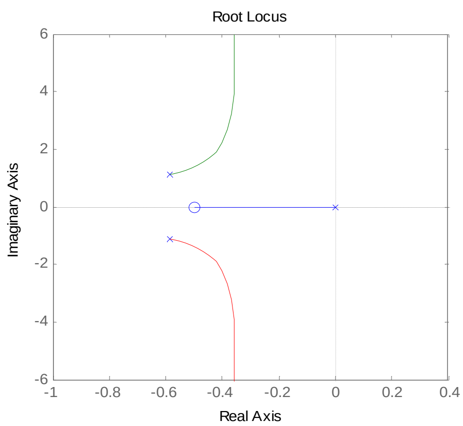

Fig. 14 root locus of PC system

III. Current Search Algorithm

The current search (CS) is proposed based on the principle of current divider in electric circuit theory [22]. The behavior of electric current is like a tide that always flow to lower places. All blanches of electric circuit represent the feasible solutions in search space. The less the resistance of blanch, the more the current flows. The local entrapment is occurred when the current hits the open circuit connection. The optimum solution found is the blanch possessing the optimum resistance. The diagram in Fig. 15 reveals the search process of the proposed CS algorithm. The CS algorithm is described step-by-step as follows.

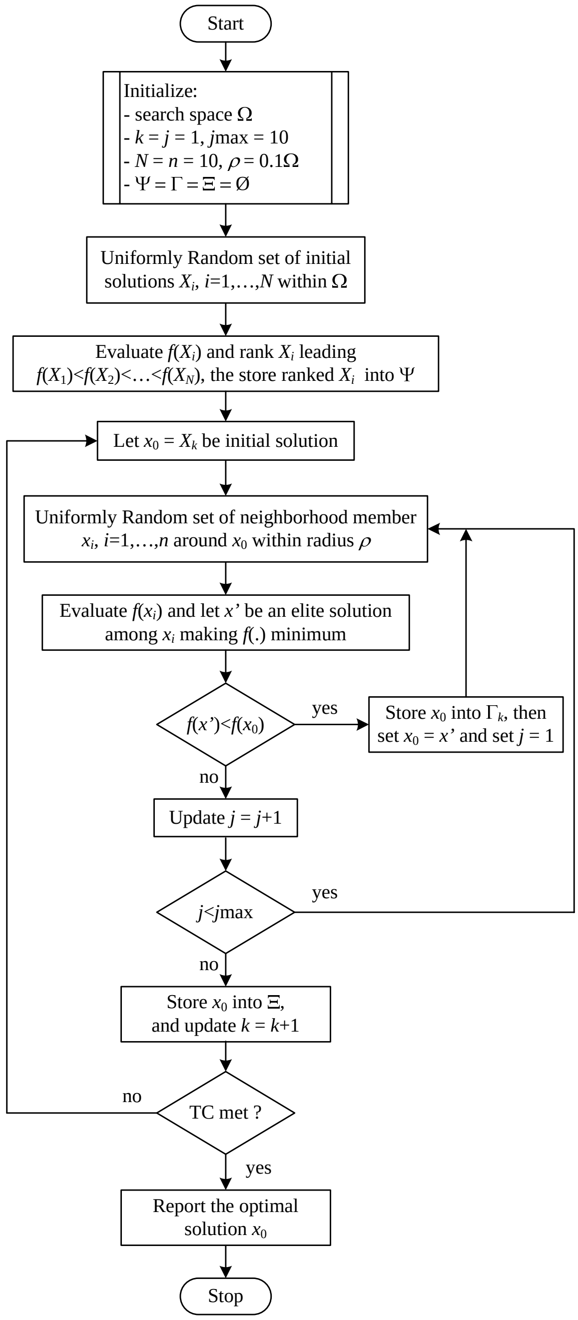

Fig. 15 diagram of the proposed CS algorithm

Step 1. Initialize the search space Ω, iteration counter $k = j = 1$, maximum allowance of solution cycling $j_{\max}$, number of initial solutions (feasible directions of currents in network) N, number of neighborhood members n, search radius ρ, and set $\Psi = \Gamma = \Xi = \varnothing$.

Step 2. Uniformly random initial solution X_i, $i = 1,\ldots,N$ within Ω.

Step 3. Evaluate the objective function $f(X_i)$ of $\forall X$. Rank $X_i, i = 1,\ldots,N$ that gives $f(X_1) < \cdots < f(X_N)$, then store ranked X_i into Ψ.

Step 4. Let $x_0 = X_k$ as selected initial solution.

Step 5. Uniformly random neighborhood x_i, $i = 1,\ldots,n$ around x_0 within radius ρ.

Step 6. Evaluate the objective function $f(x_i)$ of $\forall x$. A solution giving the minimum objective function is set as x'.

Step 7. If $f(x') < f(x_0)$, keep x_0 in set Γ_k and set $x_0 = x'$, set $j = 1$ and return to Step 5. Otherwise update $j = j + 1$.

Step 8. If $j < j_{\max}$, return to Step 5. Otherwise keep x_0 in set Ξ and update $k = k + 1$.

Step 9. Terminate the search process when termination criteria are satisfied. The optimum solution found is x_0. Otherwise return to Step 4.

The search space Ω is performed as the feasible boundary where the electric current can flow. The number of initial solutions N is set as feasible directions of the electric currents in network. The number of neighborhood members n is provided as the sub-directions of the electric currents in the selected direction, and the search radius ρ is given as the sub-search space where the electric current can flow in the selected direction. The maximum allowance of solution cycling $j_{\max}$ implies the local entrapment occurred in the selected direction.

IV. CS-based Control Synthesis

To synthesize optimum PID controller, the CS is conducted according to the block diagram represented in Fig. 2. The CS algorithm is coded by MATLAB running on Intel Core2 Duo 2.0 GHz 3 Gbytes DDR-RAM computer. The CS parameters must be assigned by the priory tests as summarized in Table 1, where N is number of initial solutions, n is number of neighborhood members, and ρ is search radius. Maximum search iteration = 100 is set as termination criteria. The tests were conducted 100 trial runs to obtain the optimum PID controller's parameters.

The boundaries of the PID parameters for IP system are set as follows: $K_p \in$ [4,000, 8,000], $K_i \in$ [500, 1,000], and $K_d \in$ [10, 100], for BB system are set as follows: $K_p \in$ [0, 100], $K_i \in$ [0, 50], and $K_d \in$ [0, 10], and for PC system are set as follows: $K_p \in$ [50, 150], $K_i \in$ [0, 10], and $K_d \in$ [0, 20].

The objective function J in (3) will be minimized according to an inequality constrain in (7), (8), and (9) for the $G_{p1}(s)$, $G_{p2}(s)$, and $G_{p3}(s)$, respectively, where t_r is rise time, M_p is percent maximum overshoot, t_s is settling time, and e_{ss} is steady state error.

$$\begin{aligned} &\min \quad J \\ &\textit{subject to} \quad t_r < 0.01\,\text{sec}, \\ &\qquad M_p < 10.00\%, \\ &\qquad t_s < 0.25\,\text{sec}, \\ &\qquad e_{ss} < 0.01 \end{aligned} \tag{7}$$

$$\begin{aligned} &\min \quad J \\ &\textit{subject to} \quad t_r < 0.05\,\text{sec}, \\ &\qquad M_p < 12.00\%, \\ &\qquad t_s < 0.10\,\text{sec}, \\ &\qquad e_{ss} < 0.01 \end{aligned} \tag{8}$$

$$\begin{aligned} &\min \quad J \\ &\textit{subject to} \quad t_r < 0.1\,\text{sec}, \\ &\qquad M_p < 10.00\%, \\ &\qquad t_s < 1.00\,\text{sec}, \\ &\qquad e_{ss} < 0.01 \end{aligned} \tag{9}$$

Table 1 CS parameter setting

Parameters	**IP**	**BB**	**PC**
N	10	10	10
n	30	30	30
ρ	1% of Ω	1% of Ω	1% of Ω

Table 2 obtained results by CS

Cost *J*	**IP**	**BB**	**PC**
Min	31.6927	18.8663	49.4223
Max	31.7006	29.2423	52.7812
Ave.	31.6969	23.9715	50.0154
Std.	0.0023	5.3276	0.0158
Time (sec)	**IP**	**BB**	**PC**
Min	280.8277	280.8108	152.1874
Max	283.2908	283.0496	165.1823
Ave.	281.7469	281.8419	161.9987
Std.	0.6976	0.6306	0.8604

Table 3 obtained optimum PID controllers

Parameters	**IP**	**BB**	**PC**
K_p	7,999.0964	99.9012	105.2512
K_i	517.7384	0.0994	4.8197
K_d	99.9994	9.9994	18.1701

Table 4 time-domain and frequency-domain responses

Responses	**IP**	**BB**	**PC**
t_r (sec)	0.0032	0.0290	0.0750
M_p (%)	9.4466	10.9325	9.2451
t_s (sec)	0.1000	0.0406	0.3544
e_{ss} (%)	0.12	00.00	0.00
GM (dB)	∞	∞	∞
PM (deg)	81.5	79.8	80.7
BW (rad/sec)	598.2	64.5	35.1
M_r (dB)	0.86	0.99	0.98
ω_r (rad/sec)	304.28	16.42	8.54

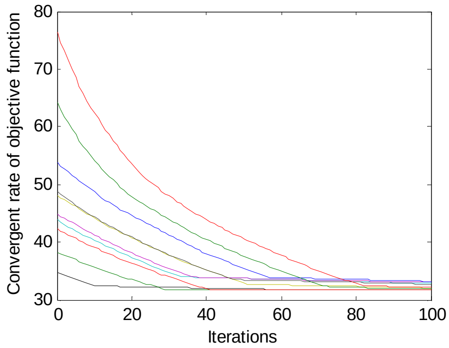

Fig. 16 convergent rate of IP system

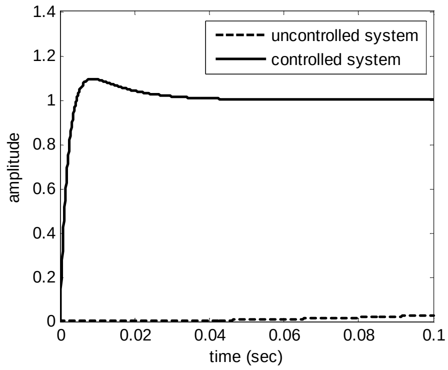

Fig. 17 time-domain (step) response of IP-controlled system

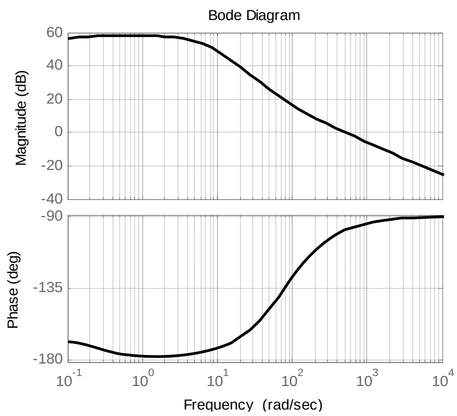

Fig. 18 open-loop frequency response of IP-controlled system

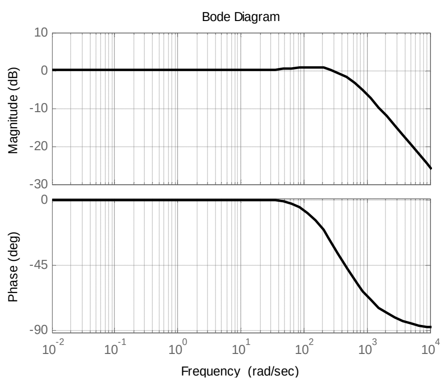

Fig. 19 closed-loop frequency response of IP-controlled system

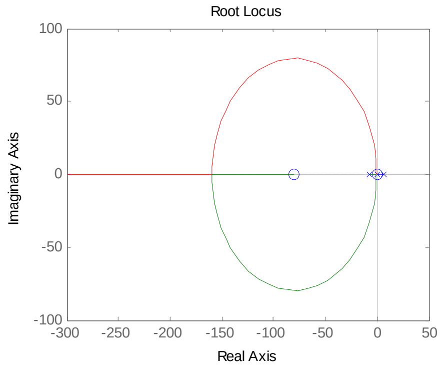

Fig. 20 root locus of IP-controlled system

For IP system, after the search process stopped, the convergent rate of the objective cost values are depicted in Fig. 16. The cost functions found and the search time consumed are summarized in Table 2. The optimum PID controller is given in Table 3. Very satisfactory time-domain (step) and frequency-domain responses both open and closed loop system of IP-controlled system are shown in Fig. 17, Fig. 18, and Fig. 19, respectively, and summarized in Table 4, where *GM* is gain margin, *PM* is phase margin, *BW* is bandwidth, M_r is maximum resonant peak, and ω_r is resonant frequency. The root locus of IP-controlled system is depicted in Fig. 20.

For BB and PC systems, after the search process stopped, the cost functions found and the search time consumed are summarized in Table 2. The optimum PID controller is given in Table 3. The convergence rate curves of the objective cost values are omitted because they have a similar form to those of the IP shown in Fig. 16. Very satisfactory time-domain (step), frequency-domain responses both open and closed loop systems, and root locus plots of BB- and IP- controlled systems are shown in Fig. 21 – Fig. 28, respectively, and summarized in Table 4. As results obtained, optimum PID controllers can be successfully achieved by the CS. The results obtained are very satisfactory.

V. Conclusion

An application of the current search (CS), one of the most efficient and powerful AI search techniques, to synthesize optimum PID control system has been proposed in this article. The CS algorithm has been developed by concept of an electric current flowing through electric networks. The CS is flexible and suitable for a variety of optimization problems in which control application is one of the domains. The PID controllers of three unstable plants, i.e. inverted pendulum (IP), ball-beam (BB), and pitch control (PC) systems are successfully optimized by the CS. As simulation results, very satisfactory responses can be achieved. However, the proposed CS is still problem-dependent, selecting an appropriate parameter is essential for successful applications. For future trends of research, the CS is still needed to solve real-world problems in control engineering context both control synthesis and system identification problems.

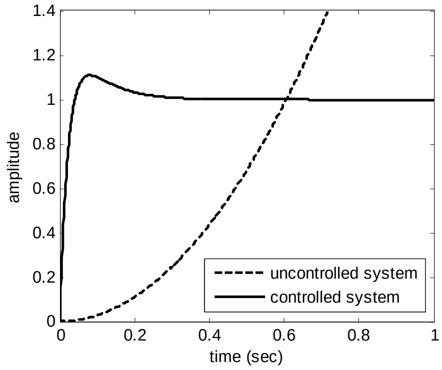

Fig. 21 time-domain (step) response of BB-controlled system

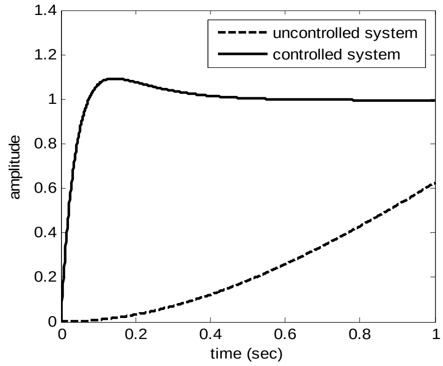

Fig. 25 time-domain (step) response of PC-controlled system

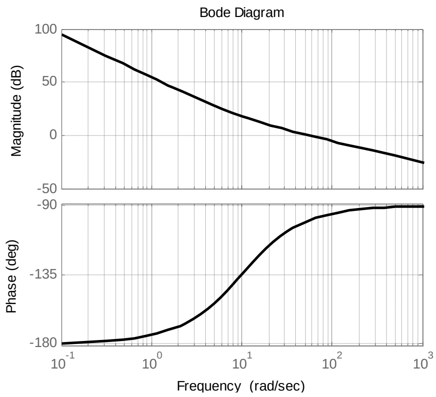

Fig. 22 open-loop frequency response of BB-controlled system

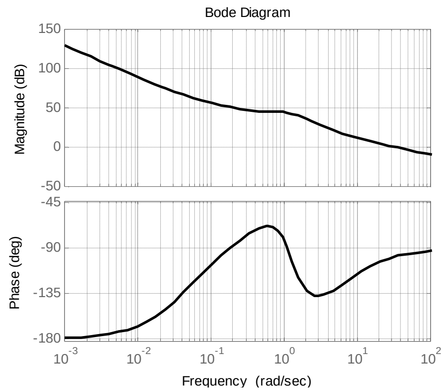

Fig. 26 open-loop frequency response of PC-controlled system

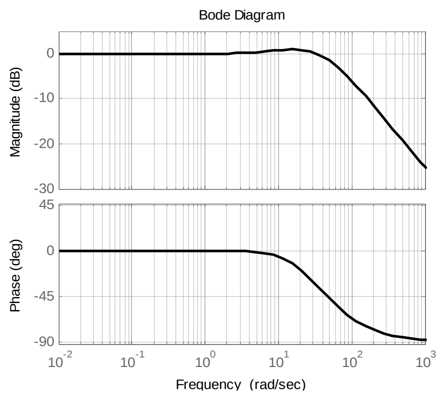

Fig. 23 closed-loop frequency response of BB-controlled system

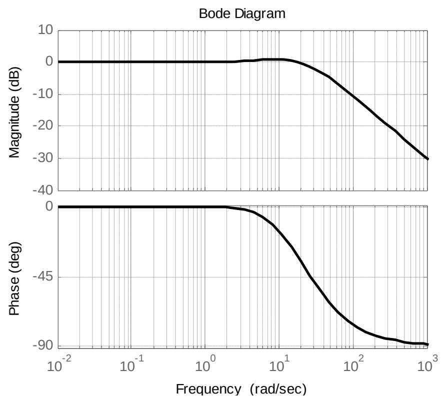

Fig. 27 closed-loop frequency response of PC-controlled system

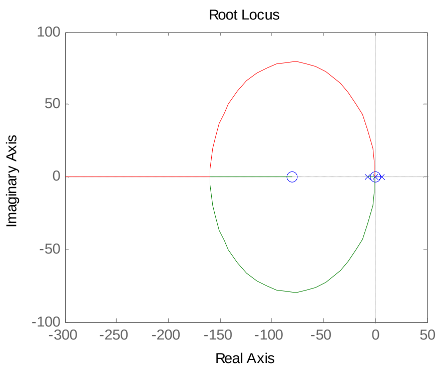

Fig. 24 root locus of BB-controlled system

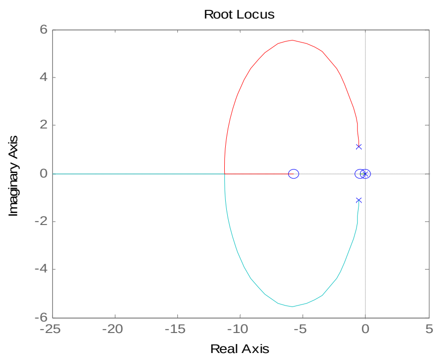

Fig. 28 root locus of PC-controlled system

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