Experiments on energy management in active suspension of vehicles

Katerina Hyniova, Miloslav Krajl and Ladislava Smitkova-Janku

Abstract—Nowadays the theoretical research concerning the active suspension of mechanical vibrations and improving ride comfort and handling properties of vehicles is concentrating on various suspension innovations.

The main goal of the paper is to describe H∞ controlled active suspension design with respect to the management of the energy flow distribution. In the time of growing interest in the overall minimization of energy consumption, the presented paper could be taken as a contribution to these efforts. Especially in the application field of automotive vehicles, the energy consumption optimization plays an important role in the design process. Suspension system influences both the comfort and safety of the passengers. In the paper, energy recuperation and management in automotive suspension systems with linear electric motors that are controlled by a designed H∞ controller to generate a variable mechanical force for a car damper is presented. Vehicle shock absorbers in which forces are generated in response to feedback signals by active elements obviously offer increased design flexibility compared to the conventional suspensions with passive elements (springs and dampers). The main advantage of the proposed solution that uses a linear AC motor is the possibility to generate desired forces acting between the unsprung (wheel) and sprung (one-quarter of the car body mass) masses of the car, providing good insulation of the car sprung mass from the road surface roughness and load disturbances. As shown in the paper, under certain circumstances linear motors as actuators enable to transform mechanical energy of the vertical car vibrations to electrical energy, accumulate it, and use it when needed. Energy flow control enables to reduce or even eliminate the demands on the external power source. In particular, the paper is focused on experiments with active shock absorber that has been taken on the designed test bed and the way we developed an appropriate input signal for the test bed that as real road disturbance acts upon the vibration absorber and the obtained results are evaluated at the end. Another important point the active suspension design should satisfy is energy supply control that is made via standard controller modification, and which allows changing amount of energy required by the system. Functionality of the designed controller modification was verified taking various experiments on the experiment stand as mentioned in the paper.

Keywords—Vehicle, suspension, linear motor, controller, energy

I. INTRODUCTION

At the Czech Technical University in Prague various alternative strategies and innovations to classical passive suspension systems improving ride comfort of the passengers, providing steering stability, maximizing safety and improving handling properties of vehicles has been researched. In order to improve handling and comfort performance instead of a conventional static spring and damper system, semi-active and active suspension systems has been developed. Certainly there are numerous variations and different configurations of vibration suspension. In known experimental active systems hydraulic or pneumatic actuators usually provide the force input. As an alternative approach to active suspension system design, the research group has studied electromechanical actuators. Such actuators would provide a direct interface between electronic control and the suspension system.

In most active suspension systems, the biggest disadvantage consists in energy demands. Regarding linear electric motors, this drawback can be minimized or even eliminated because under certain circumstances there is a possibility to recuperate energy, accumulate it and use it later for the shock absorber when necessary. This way, it is possible to reduce the posted claims on the external power source as much as possible. In the next paragraphs also the proposed strategy how to control the energy distribution will be described. In order to regenerate electric power from the vibrations excited by road unevenness a new energy-regenerative active suspension for vehicles has been designed. The active system has been modeled and simulated to show the performance improvement and the performance experiments of the actuator prototype-testing stand have been carried out.

All suspension systems are designed to meet variable specific requirements. In suspension systems, mainly two most important points are supposed to be improved - disturbance absorbing (videlicet passenger comfort) and attenuation of the disturbance transfer to the road (videlicet car handling). The first requirement could be understood as an attenuation of the sprung mass acceleration or as a peak minimization of the sprung mass vertical displacement. The second one is characterized as an attenuation of the force acting on the road or - in the simple car model - as an attenuation of the unsprung mass acceleration. The goal is to satisfy both the above given contradictory requirements. Satisfactory results can be achieved when an active suspension systems generating variable mechanical force acting between the sprung and unsprung masses is used. Such an actuator can be a linear electric motor. In comparison with traditional actuators that use revolving

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The authors are with the Department of Digital Design, Faculty of Information Technology, Czech Technical University in Prague, Prague, Czech Republic (e-mail: hyniova@fit.cvut.cz, phone: (++420)722575989) miloslav.krajl@gmail.com, janku@ikt.cz)

electro-motors and a lead screw or toothed belt, the direct drive linear motor enables contactless transfer of electrical power according to the laws of magnetic induction. The gained electromagnetic force is applied directly without the intervention of a mechanical transmission then. Linear electric motors are easily controllable and for features like low friction, high accuracy, high acceleration and velocity, high values of generated force, high reliability and long lifetime their usage as shock absorbers seems to be ideal.

II. ONE-QUARTER-CAR SUSPENSION MODEL AND SHOCK ABSORBER TEST STAND

A traditional one-quarter-car model has been used to design a suspension controller design and to simulate the system behavior. The basic configuration of the model is shown in Fig.1. The model involves unsprung and sprung masses, conventional passive suspension (a spring and a damper), stiffness of the tire, and linear electric motor as actuator placed in parallel to the traditional passive suspension.

In Fig. 1:

F_a control input (active suspension force) [N]
m_w unsprung mass (wheel) [kg]
m_b sprung mass supported by each wheel and taken as equal to a quarter of the total body mass [kg]
k_2 stiffness of the tire [N/m]
$z_r(t)$ road displacement (road disturbance) [m]
$z_b(t)$ displacement of the sprung mass [m]
$z_w(t)$ displacement of the unsprung mass [m]
k_1 stiffness of the passive suspension [N/m]
c_1 damping quotient of the passive suspension [Ns/m]

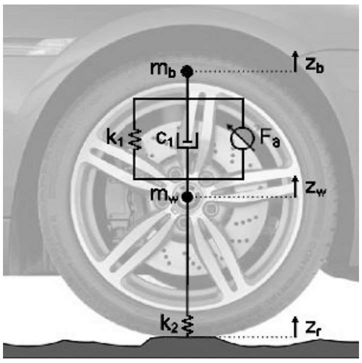

Fig.1one-quarter-car model

The same configuration has been used for real experiments. Mechanical configuration of the test stand is obvious from Fig.2. Under the tire there is placed another linear electric motor that uses an input experimental signal described in next paragraphs to generate road displacement (road deviations) under the running wheel.

As will be mentioned later the controller has been developed via Matlab implemented into dSpace a connected to the test stand system.

Fig.2 experiment stand

III. LINEAR ELECTRIC MOTOR

A. Linear Motor Description

From the point of view of the aspects discussed above, the authentic application of a linear electrical AC motor seems to be very perspective. The beauty of linear motors is that they directly translate electrical energy into usable linear mechanical force and motion and vice versa. Compared to conventional rotational electro-motors, the stator and the shaft (translator) of direct-drive linear motors are linear-shaped (see Fig.3). One can imagine such a motor taking infinite stator diameter. The direct drive AC linear motor exhibits the property of contact-less transfer of electrical power according to the laws of magnetic induction. The electromagnetic force is applied directly without the intervention of a mechanical transmission. Low friction and no backlash resulting in high accuracy, high acceleration and velocity, high force, high reliability and long lifetime enable not only effective usage of modern control systems but also represent the important attributes needed to control vibration suspension efficiently. Linear motor translator movements take place with high velocities (up to approximately 200m/min), large accelerations (up to g-multiples), and forces (up to kN). As mentioned above, the electromagnetic force can be applied directly to the payload without the intervention of a mechanical transmission, what results in high rigidity of the whole system, its higher reliability and longer lifetime. The main advantage of the proposed solution using a linear AC motor is the possibility to generate desired forces acting between the unsprung and sprung masses of the car, providing good insulation of the car sprung mass from the road surface disturbances. In addition, under certain circumstances using linear motors as actuators enables to recuperate energy i.e. to transform mechanical energy of the car vertical vibrations generated by the road disturbances to electrical energy, accumulate it, and use it when needed.

For the automotive suspension system, the application of the synchronous three-phase linear motor TBX 3810 produced by opley Controls Cooperation (technical parameters: peak force 2027N, peak current 21.8A, continuous stall force 293.2N, electrical time constant 1.26ms, continuous working voltage 320Vac, maximum phase temperature 100^{o}C) has been designed by the research team.

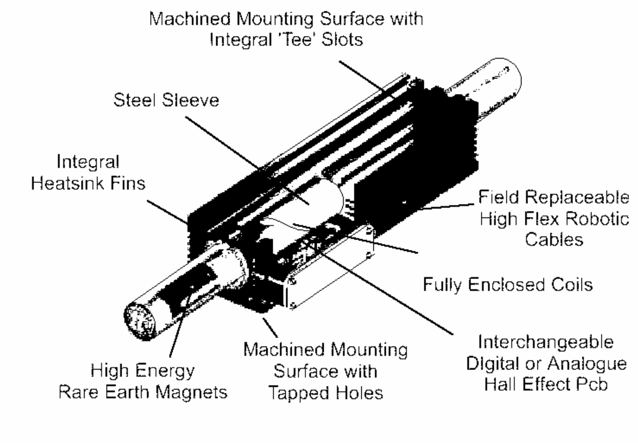

Fig. 3 linear motor - basic design (adapted manufacturer spreadsheet)

B Linear motor implementation

It is necessary to answer one important question - if it is more advantageous to include the model of the linear electric motor in the model for active suspension synthesis or if it should be used only for simulations.

Comparing advantages and disadvantages of the model inclusion, it can be said that the closed-loop provides more information so that better control results can be achieved. Unfortunately, there are also some significant disadvantages in such a solution. The first one consists in the rank of the system (and consequently the rank of the controller which increased up to 5) and the second one is that the D matrix in the state space description of the motor model does not have full rank and that is why implementation functions are limited or too complicated. On the base of this comparison the linear motor has not been included in the model for active suspension synthesis.

There is another important question whether the linear motor model could be omitted and a linear character of the desired force could be supposed. The answer is "yes". Both the mechanical and the electrical constants are very small – just about 1ms. Moreover it will be shown that the robustness of the H∞ control design has been verified using numerous simulation results and experiments.

To verify control algorithms a linear motor model including the power amplifier has been created in Matlab/Simulink. The model enables to demonstrate the conversion of electrical energy to mechanical energy.

In the model, it is assumed that: the magnetic field of the secondary part with permanent magnets is sinusoidal, the phases of the primary part coils are star-connected, and the vector control method is used to control the phase current. Here, PWM voltage signal is substituted by its mean value to shorten (about 10 times) the simulation period (inaccuracies caused by such a substitution can be neglected). The principal inner representation of the model is shown in Fig.4. The model input vector is given by the instantaneous position [m] (necessary to compute the commutation current of the coils), instantaneous velocity [m/s] (the induced voltage of the coils depends on the position and velocity) and desired force [N].

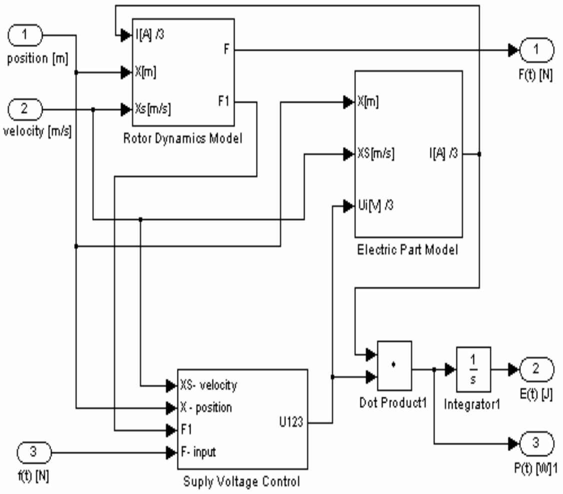

Fig. 4 inner model schema

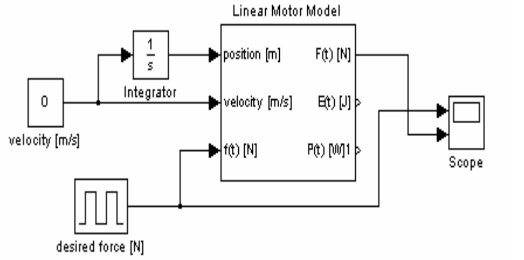

Fig. 5 linear motor input/output model for dynamics verification

The designed model function has been verified comparing dynamics of the model (see Fig.5) and the real motor. The simulation parameters correspond to the catalogue parameters of TBX3810 linear motor produced by Copley Controls Cooperation.

C Energy balance

As mentioned above, linear electric motors are able to recuperate energy. When the generated force is of the same direction as the suspension velocity, the energy has to be supplied into the system. Otherwise, it can be recuperated and accumulated for the future usage.

In fact, there are some nonlinearities in the recuperation process and that is why the energy management is a bit difficult. Just for imagination, the 3-D plot (shown in Fig. 6.) represents the force-velocity profile of the recuperated energy. It shows how much recuperated (and only recuperated) energy can be obtained under the given forces and velocities. In the plot, when the recuperated energy is equal to zero or bigger it is necessary to supply the energy into the system.

This characteristic surface gives an important information regarding one of the requirements on the control system as the optimization objectives are equal to maximization of the recuperated energy (with necessary trade-offs).

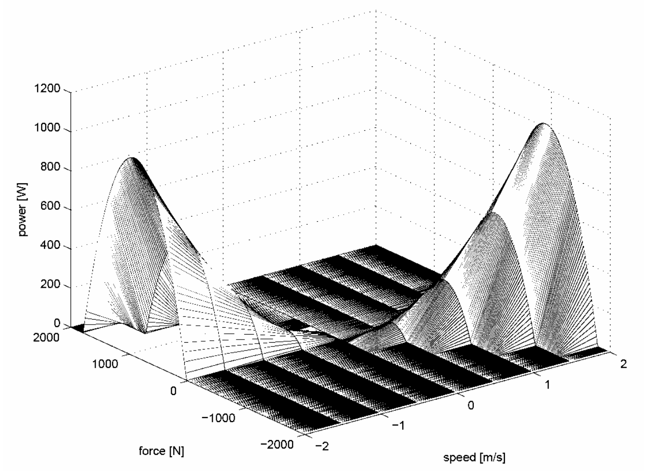

Fig. 6 recuperated energy

IV. EXPERIMENTAL SIGNAL USED TO SIMULATE ROAD DISPLACMENT

In order to bring about simulation and practical experiments at the test stand it is necessary to find a proper experimental signal that represents the road profile and excites the active suspension. Although the simplified suspension model seems to be linear the truth is that there are many nonlinear parts in the system. Now the question is what signal to generate for experimental testing to reach a true model of the uneven road under the wheel. We can define two types of input signals regarding objectives:

- to prove results of simulations and pre-calculations
- to test real behavior on the road

Let's start with the first objective – to verify simulation results. The best signal might be considered probably white noise because full frequency spectrum could be analyzed then. But it should be noted that the system is nonlinear and even white noise is not satisfactory. Moreover it is not possible to generate easily white noise by the test stand.

For these reasons a bump has been chosen as a signal, that often occurs on the road profiles and that can be generated by the test bed easily. This signal allows observing both directions – bump-up and bump-down. Since it is not possible to generate infinite slope by the test stand the following signal approximation has been used (Eq.1)(widely known approximation). Different magnitudes of the signal have been tested because the system is nonlinear. Magnitudes have been chosen according to mechanical dimensions of the suspension system:

$$\dot{z}_r(t) = 0.5\,\pi\,\sin\big(20\,\pi(t-0,1)\big) \qquad (1)$$

Thus this signal is used to verify the quality of the simulation model, it does not confirm the usability of he controller on a real road surface.

More significant results can be obtained from the input signal, which is similar to the road profile. A deterministic random signal is used to approximate it. The input signal for simulation is described by the following equation (2):

$$z_r = \sum_{i=1}^{n}\sqrt{\frac{\dot{\omega}_i}{\pi\cdot v_x}}\Big\{\ Re\Big(\frac{b_o}{-\omega_i^2+a_1 j\omega_i+a_o}\Big)\cdot\cos(\omega_i t+\alpha)+$$
$$+\,Im\Big(\frac{b_o}{-\omega_i^2+a_1 j\omega_i+a_o}\Big)\cdot\sin(\omega_i t+\alpha)\ \Big\} \qquad (2)$$

$$b_o = 0.121\cdot v_x$$
$$a_o = 2.249\cdot v_x$$
$$a_1 = 30.36\cdot v_x$$

where v_x represents the car velocity.

Thus resulted signal is obtained as a superposition of the sinusoids with deterministic "random" angles (α_i in Eq.2). In this case 128 random angles have been calculated. The used pseudo-random signal is plotted in Fig.7.

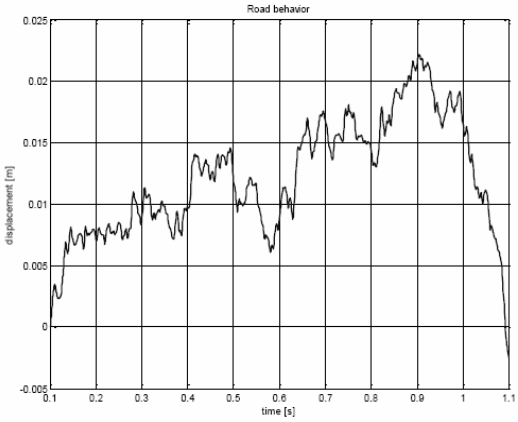

Fig.7 random signal approximation

V. QUANTIFICATION

Some quantitative ratios have to be defined to evaluate the results achieved by the closed loop system and to compare the active and passive systems.

A. Car stability

First requirement in the active suspension system is to improve car stability and "road friendliness" that can be characterized as the attenuation of the tire pressure, or more precisely the attenuation of the unsprung mass force acting on the road. To get a measurable parameter, the following RMS function has been introduced:

$$J_{stab} = \sqrt{\int_0^T (z_w - z_r)^2\,dt} \qquad (3)$$

where z_w represents wheel displacement and z_r road displacement.

B. Passenger comfort

Second important requirement in the active suspension system is to improve passenger comfort. This requirement can be formulated as the sprung mass acceleration attenuation when the RMS function is defined as:

$$J_{comf} = \sqrt{\int_0^T G_w * \ddot{z}_b^{\,2}\,dt} \qquad (4)$$

where $\ddot{z}_b$ represents body acceleration, G_w is a weighting function for human sensitivity to vibrations and * denotes convolution.

VI. ON THE CONTROL LOOP

A robust controller is necessary to design for the suspension system, because the system parameters often vary in a wide range. Especially the body mass varies for every single drive. For this reason H∞ control theory [8] has been chosen for controller design [2] as the ideal controller design. Matlab Toolbox procedures have been used for H∞ controller computation.

Nevertheless the standard H∞ controller cannot handle energy consumption. Modification of the controller had to be done during experiments. Additional input, which controls energy demands was supposed to be connected to the controller. Then a master controller can use this input to keep energy balance (Design of the master controller is not involved in the paper.). The structure of the modified controller is shown in Fig.8.

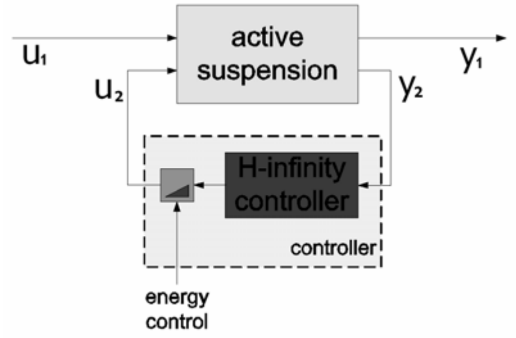

Fig.8 modified controller structure

Then energy management is supposed to be controlled by an external signal (Fig.8) depending on the car and road parameters, i.e. on the energy accumulator capacity and the road profile, respectively.

First possible way to control energy supplied into the suspension system consist in the analysis of the driving conditions and cyclic re-computing of the control signal in real time. For high sampling frequency (over 1 kHz) and because of the performance of the controller can not be guaranteed for all operating conditions this approach has been rejected.

The second possibility is to control the energy consumption by controller deterioration. Then the designed controller is reliably robust and the active suspension system is relatively stable. Simply said, the controller deterioration sort of devalues the suspension performance, but enables to gain some energy to be stored. Let us assume two kinds of driving conditions:

- the terrain /road surface the car is going on is very rough and uneven and there is enough energy stored in the accumulator system - then the controller works in the standard mode, the linear motor consumes energy from the accumulator (supercapacitors) and the suspension performance is preserved.
- the terrain /surface under the running wheels is relatively smooth and there is not enough energy stored in the accumulator system (supercapacitors) as it was consumed because of the situation described above. The external signal provides the information to the controller to deteriorate its performance and to reduce the energy consumption. The deterioration is stated by the desired force attenuation so that the suspension gets devaluated.

If the force is completely attenuated the suspension system works only via the passive suspension part while the linear electric motor works as a generator generating electrical energy to be stored in the accumulator system. Of course, the suspension performance is devaluated now (to the passive suspension level in the worst case).

That is the basic description how the control loop with the modified controller works. Some results (percentage indicators) of the real experiments taken at the test stand that endorse presumed above will be discussed in paragraph VIII.

VII ENERGY MANAGEMENT ANALYSIS

In paragraph V, the energy management has been discussed as an extension of the H∞ controller abilities. Now the influence on the performance and robustness will be presented. The H∞ controller is deteriorated by the desired force attenuation using the input coefficient that is given by a superior (master) controller.

At first robustness tests have to be done to find the range of the input coefficient in the energy management block. To test robustness the direct numerical method has been chosen because the rank of the closed system is relatively small (4 for the plant + 6 for weights = 10 for the system, 10 for the system + 10 for the controller = 20 total for the closed loop rank). Hence the poles have been tested for stability for a given input coefficient range.

The stability test in graphical form is shown in Fig. 9 and Fig. 10. In the figures, closed loop poles are plotted for the input coefficient range of (-0.5 ÷ 1.7). Zoomed surroundings of the stability region from Fig. 9 is shown in Fig.10. The original H_∞ pole placement is presented by *, pole placement for stable region by # and unstable region by ■, respectively.

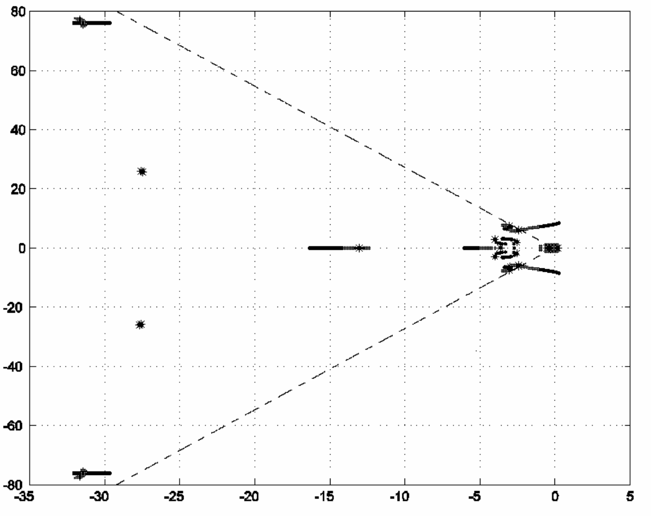

Fig. 9. pole plot in energy control

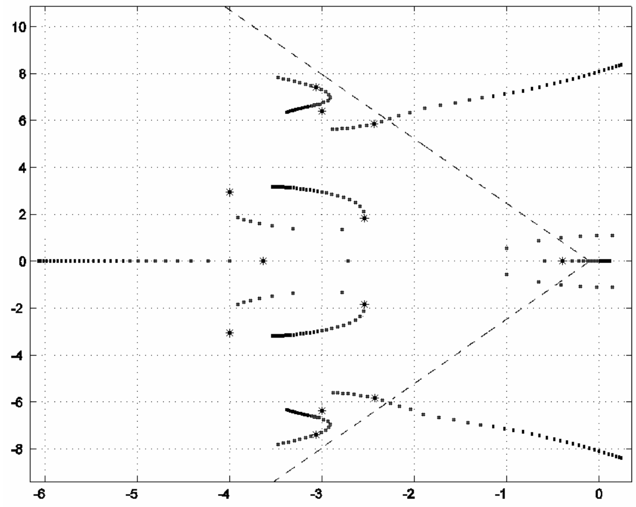

Fig.10 pole plot in energy control (zoom)

On the base of the test mentioned above, we have stated the maximum and minimum stable input coefficients. To achieve stability the coefficient must not exceed the range of (0.000 ÷ 1.613).

The coefficient range should be determined to achieve also certain robustness. That is why we have chosen the pole region of relative damping 1.4 and maximum real part -0.1 as a condition. In Fig. 9, the selected region is represented by the dashed line. According to the previous section it does not have any sense to set the input coefficient greater than one. The resulting input coefficient range that satisfies the defined conditions is as follows:

- minimum: 0.512
- maximum: 1.000

At the end, the influence of the input coefficient on the active suspension performance has been tested.

The quantitative measures we have compared using passive suspension performance. The random road disturbance we have used as a first test input and the driving over a bump as a second input. The comparison for minimum and maximum input coefficients and their influence on the active suspension system performance is summarized in Tab.1. The percentage values are computed as relative improvements of the active system compare to the passive suspension.

Table 1 Influence of the input coefficient on the system performance

coefficient	0.512	1
H_∞ norm	0.455	0.359
comfort	20.13%	29.89%
stability	8.92%	12.83%
energy	-71.1J	127.6J

VIII.. ENERGY ACCUMULATION MANAGEMENT

With respect to the usage of the accumulated energy in the automotive suspension system, two storage devices - a supercapacitor and a board power net with a lead-acid battery – are considered. The reason for using two storage devices is to combine their advantageous properties [2]. The battery has a large capacity and small energy leakage over time. However, the losses during charging and discharging and battery wear are large, especially when using high powers. The open cell voltage of the battery is linear with the energy level but with a large offset. Because of this offset, the battery can be operated between 20% and 100% state of charge (SOC) while maintaining an acceptable board net voltage. The supercapacitor has a smaller capacity, but the charge and discharge losses are also much smaller. This makes the supercapacitor advantageous to use for high peak powers. A supercapacitor has considerable energy leakage, so it is not suitable for long term storage. The open cell voltage of the supercapacitor is linear with the energy level. If the supercapacitor is connected directly to the board net, it can only be operated within a small SOC window while maintaining an acceptable board net voltage. By connecting it to the board net using a DC-DC converter, it can be used in its full range.

There are several ways of energy accumulation, but some of them do not allow holding the supercapacitor voltage at the upper limit of the range. One of the possibilities is shown in Fig.11.

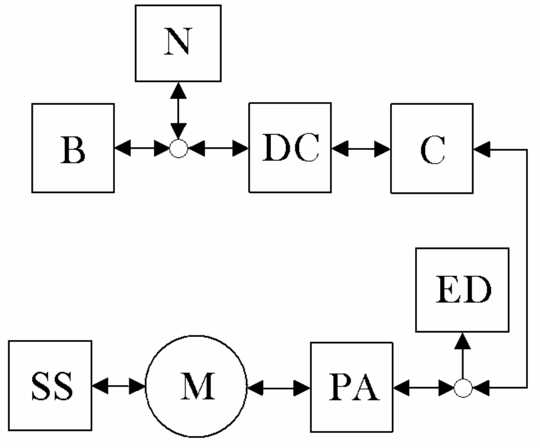

Fig.11 energy management

The power amplifier (PA) requires a direct current power supply for the switch bridge. To achieve a good function of the power amplifier, the voltage level of the supercapacitor (C) cannot exceed a permitted range. Because the maximum of the linear motor force is limited within the given permitted range, it is useful to hold the supercapacitor voltage at the upper limit. The supercapacitor voltage does not have to be necessarily time-invariant and can vary within the range of $U_{min} \div U_{max}$. The supercapacitor is connected directly to the power amplifier. The energy dissipator (ED) dissipates energy in case the supercapacitor voltage exceeds the permitted level U_{max}. The main advantages of such a solution are: its simplicity, the fact that the energy taken from the motor is stored with the efficiency of 100% (loss resistance of the supercapacitor is neglected), and no problems concerning the DC/DC converter (DC) like efficiency, disturbances, cooling etc. As the main disadvantage is taken the fact that the stored energy is limited by:

$$E_c = \frac{1}{2} C (U_{max}^2 - U_{min}^2) \tag{5}$$

Consider the average efficiency of the DC converter as equal to η = 0,85. The recuperated energy can be reused with the efficiency of $\eta^2 = 0.72$. It results in a disadvantage when 28% of the recuperated energy is lost during one cycle of the energy accumulation and its following re-usage. While the power management between the supercapacitor and the board power net (N) with a battery (B) holding the supercapacitor voltage within the range of $U_{min} \div U_{max}$, the power management between the supercapacitor and the AC linear motor (M) is given by the control algorithms applied in the suspension system (SS).

To verify power management strategy a power management model has been created in Matlab/Simulink. The model enables to demonstrate the conversion of electrical energy to mechanical energy and vice versa. The simulation parameters correspond to a one-quarter car (sedan) with TBX3810 linear motor produced by Copley Controls Cooperation.

IX. RESULTS

Deterministic random signal stated in (2) has been used for real experiments taken on the shock absorber test stand. Two important things should be followed during experiments – suspension comfort improvement and energy consumption (see Fig. 7). In Fig 7, the road profile input, energy consumption, and corresponding body displacement for standard controller setting are displayed.
As an indicator of the comfort improvement body (sprung) mass displacement can be taken

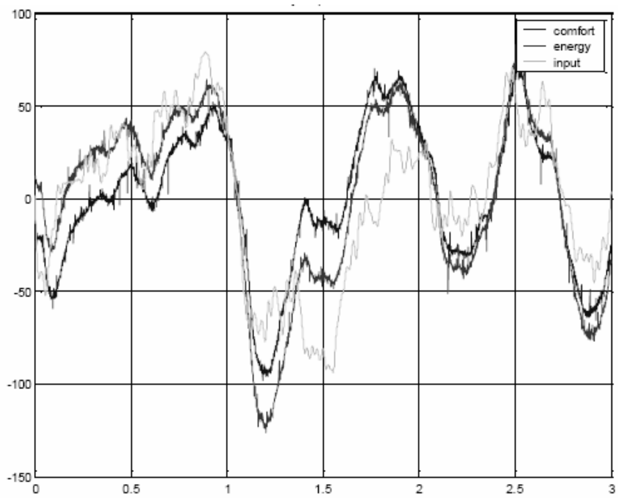

Fig.12 sprung mass displacement

Fig.8 shows two curves - energy demand for standard energy consumption setting (called “comfort setting”) and energy demands for lower consumption (called “energy setting”). Both curves were measured excited with the same input - the deterministic random signal (road profile). Actually, negative values of energy in the figure represent the recuperated energy.

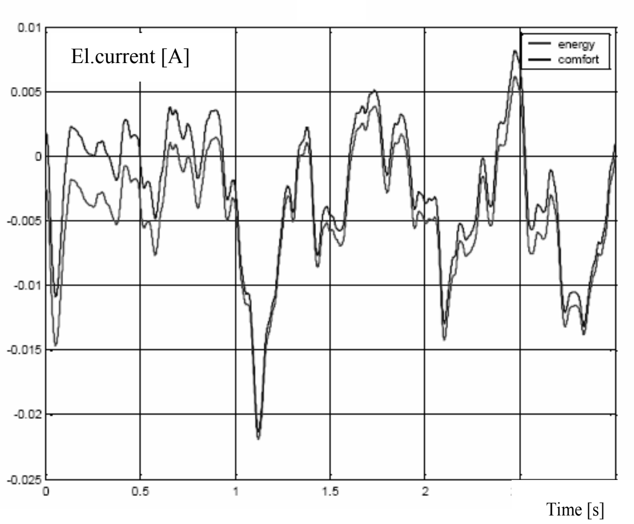

Fig.13 energy demands (demands on electrical current)

Let’s show the results as mean values of “comfort” and “energy” indicators stated in [1]. Tab.1 involves mean values for the body displacement as absolute values of the defined body displacement indicator and also as a percentage of its improvement.

Table 1 Displacement mean values

	Mean value	Percentage
Body displacement – comfort	38.6	100%
Body displacement – energy	47.4	123%

The first line shows that for standard controller setting the comfort indicator (38.6) is taken as 100%. In the second line, when the controller was deteriorated, the comfort is devaluated up to 47.4, i.e. to 123% .

Tab.2 involves mean values of energy indicator. Lower value corresponds to the lower energy demand. Similarly the first line shows that for standard controller setting the comfort indicator (2.689) is taken as 100%. In the second line, when the controller was deteriorated, the energy indicator is devaluated up to 1.598, i.e. to 59% then.
Briefly, controller deterioration causes comfort devaluation to 123% while energy consumption decreases to 59 %.

Table 2 Power mean values

	Mean value	Percentage
Comfort setting	2.689	100%
Energy setting	1.598	59%

X . CONCLUSION

In this paper the H∞ controller for active suspension with linear electric motor has been used for experiments done on the experiment stand. An experiment signal for real road profile simulation has been developed and then it has been used for experiments. The method for the direct real-time energy control with respect to reduction of the energy consumption has been used. Experiments verified results of the simulations and showed that it is possible to change energy demands according to the road profile and status of the energy storage in the car (battery or supercapacitor). The method can be extended to general plants with considerable energy demands, where the decreasing actuator signal in a given range can preserve the system stability. Thus this controller with linear motor as an actuator can be used in any suspension system.

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