Static Output Feedback Fuzzy Controller Of a Synchronous Machine

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Abstract— This paper study the design of a static output feedback fuzzy controller for a synchronous machine without damper. The non-linear mathematical model is described by a continuous-time Takagi-Sugeno (T-S) fuzzy model. Motivated by stability results developed for parallel distributed compensation (PDC) controller, an Output PDC (OPDC) controller that corresponds to a nonlinear static output feedback control law is proposed. Based on the Lyapunov stability criterion, the stability analysis is presented in terms of Linear Matrix Inequalitie (LMI) optimisation to seek the static output feedback gains that satisfy the Lyapunov stability inequalities. Simulation results for synchronous machine demonstrate the OPDC controller's effectiveness.

Keywords— Continuous systems, T-S model, OPDC controller, Quadratic stability, LMI formulation

I. Introduction

The issue of stability and the synthesis of controllers for nonlinear systems described by continuous-time Takagi-Sugeno models [14] has been considered actively. There has been also an increasing interest in the multiple model approach [17,13] which also uses the T-S systems to modeling. During the last years, many works have been carried out to investigate the stability analysis and the design of state feedback controller of T-S systems.

Nonlinear control syntheses based on the Takagi–Sugeno (T-S) fuzzy model have been successfully developed in the past decade [18–21]. Most of these designs [5–10, 18–25] are based on linear matrix inequality (LMI) techniques [12, 26], which concentrate on transferring various performance constraints and Lyapunov inequalities into LMI representations.

Afterwards, a LMI solver can be employed to compute the feedback gains of each fuzzy rule and the common positive definite matrix to satisfy all Lyapunov inequalities. Unfortunately, the static output feedback fuzzy control design becomes much more difficult and complex than the state feedback design because it is a nonlinear matrix inequalities (NLMIs) problem. Therefore, only a few investigators [1, 4, 7] have found a way to convert the NLMI problem into an LMI problem.

Unfortunately, the mathematical derivations of these approaches are too complex to apply in the real world. Also, these design results are conservative because some extra constraints have to be attached when converting the NLMIs into LMIs.

The advantage of the T-S fuzzy model lies in that the stability and performance characteristics of the system represented by a T-S model can be analyzed using Lyapunov function approach [13]. Tanaka and Sugeno in [13] showed that the stability of a T-S fuzzy model could be shown by finding a common symmetric positive definite matrix P for r sub-models, that satisfy a set of Lyapunov inequalities [10], [14].

Using a Quadratic Lyapunov function and PDC technique [6], [8] sufficient conditions for the stability and stabilisability have been established [5,6,10,13,15,16]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. The PDC control is a nonlinear state feedback.

Recently a number of control law have been derived from the PDC controller [4, 13,14,15,17]. For example, a static Output PDC (OPDC) [7], which is an output feedback controller PDC, used to stabilise T-S models. The LMI approach is used to develop a static output feedback controller for nonlinear systems described by continuous-time T-S models.

The purpose of this paper is to develop a simple and powerful method to solve the static output feedback gains for a synchronous machine in T-S fussy design, so each fuzzy rule such that all the Lyapunov stability inequalities can be satisfied Quadratic Lyaponov function. It is well known that the system performance depends on a PDC control that can be shifted by feedback gains. Since the feedback gains are given, the Lyapunov stability inequalities of the static output feedback syntheses can be dealt with using the LMI solver,

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who define the fitness function by using the computing results of the LMI solver

In this paper we study the stabilization of a synchronous machine, using fuzzy Takagi-Sugeno approach. By using Output Parallel Distributed Compensation (OPDC) as a nonlinear static output feedback control law, we give a formula for the fishing effort that allows stabilizing the system around a steady state. Only the stability conditions reformulated into solving an LMI problem was developed for the synchronous machine.

The rest of this paper is organized as follows. In section 2, we present an overview of dynamic Takagi-Sugeno systems and OPDC formulation in terms of LMI [21]. Section 3 deals with the description of the continuous model structured of Synchronous machine, which is transformed to a Takagi-Sugeno fuzzy model, and controlled by an OPDC law. The simulation results are shown. Finally, a conclusion is given.

II. T-S FUZZY MODEL AND ITS STABILITY

A. Model representation

The design procedure describing in this section begins with representing a given nonlinear plant by the so-called Takagi-Sugeno (T-S) fuzzy model. The fuzzy model proposed by Takagi and Sugeno [11] is described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy ''blending'' of the linear system models.

The i[th] rule of the T–S fuzzy models for a continuous fuzzy system is written as follows:

A T-S model [9], [10], is based on the interpolation between several LTI local models as follows in the continuous time case:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{A_i.x(t) + B_i.u(t)\} \\ y(t) = \sum_{i=1}^{r} h_i(z(t))\, C_i\, x(t) \end{cases} \quad (1)$$

where r is the number of submodels, $x \in \mathcal{R}^n$ is the state vector and $u(t) \in \mathcal{R}^m$ is the input vector, $y(t) \in \mathcal{R}^q$ is the output vector, $z(t) \in \mathcal{R}^p$ is the decision variables vector and $h_i(z(t))$ is the activation function.

$A_i \in \mathcal{R}^{n\times n}$, $B_i \in \mathcal{R}^{n\times m}$ and $C_i \in \mathcal{R}^{q\times n}$ are the state matrix, the input matrix and the output matrix respectively.

Different classes of models can be considered with respect to the choice of the decision variables and the type of the activation function.

In this paper, all the decision variables of the T-S model (1) are assumed measurable.

Each linear consequent equation represented by $(A_i.x(t) + B_i.u(t))$ is called "subsystem" or "submodel".

Model Rule i :

$$\textbf{\textit{if}}\ z_1(t)\ is\ M_i^1(z_p(t)) and\ ...\ and\ z_p(t)\ is\ M_i^p(z_p(t))$$

$$\textbf{\textit{Then}}\ = \begin{cases} \dot{x}(t) = A_i.x(t) + B_i.u(t) \\ y(t) = C_i\, x(t) \end{cases} \quad (2)$$

Where i=1, 2, …, r. r is the number of IF-THEN rules.

The normalized activation function $h_i(z(t))$ corresponding to the i[th] submodel is such that [2], [3], [14]:

$$\begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1 \\ h_i(z(t)) \geq 0 \quad \forall\, i\, \in \{1, \dots, r\} \end{cases} \quad (3)$$

B. Stability using Static Output Feedback

We have proposed an LMI-based design method using fuzzy state feedback control in [1]. However, in real-world control problems, the states may not be completely accessible. In such cases, one needs to resort to output feedback design methods.

Fuzzy static output feedback control is the most desirable since it can be implemented easily with low cost fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For the fuzzy models (1) we construct the following fuzzy controller via the OPDC:

Control Rule i :

$$\textbf{\textit{if}}\ z_1(t)\ is\ M_i^1(z_1(t))\ and\ ...\ and\ z_p(t)\ is\ M_i^p\left(z_p(t)\right)$$

$$\textbf{\textit{Then}}\ u(t) = K_i\, y(t), \qquad i = 1, \dots, r$$

Where K_i is the local output controllor feedback to determine the overall fuzzy control law is composed of several linear output feedbacks blended together using the nonlinear functions of the model.

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) K_i\, y(t) \quad (4)$$

In the sequel, we assume that Ci = C, i = 1, ... ,r, is full row rank.

By substituting (4) into (1), the closed-loop fuzzy system can be represented as:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mathrm{h_i}(\mathrm{z(t)}\,\mathrm{h_j}(\mathrm{z(t)}\big(A_i + B_i K_j C\big)\,\mathrm{x(t)} \tag{5}$$

The following theorem deals with sufficient conditions in LMIs form to ensure asymptotic stability of (5).

Theorem 1 [14]: The equilibrium of the continuous fuzzy system described by (5) is asymptotically stable in the large if there exists a P > 0 such that:

$$L_i^T P + P L_i \le 0, for\ i = 1,\ldots,r \tag{6}$$

$$R_{ij}^T P + P R_{ij} \le 0, for\ i,j = 1,..,r\ and\ i < j \tag{7}$$

Where:

$$L_i = (A_i + B_i K_i C) \tag{8}$$

$$R_{ij} = \frac{(A_i + B_i K_j C)^T + (A_i + B_i K_j C)}{2} \tag{9}$$

In the following, sufficient conditions in LMIs form are given to ensure asymptotic stability of (5).

Suppose that there exist matrices $X > 0$, *M and N,* such that:

$$\mathrm{X} = \mathrm{P}^{-1} > 0\ \ and\ \ CX = MC$$

$$\mathrm{A_i X} + (\mathrm{A_i X})^T + \mathrm{B_i N_i C} + (\mathrm{B_i N_i C})^T < 0 \quad pour\ i = 1,\ldots,r \tag{10}$$

$$(\mathrm{A_i} + \mathrm{A_j})\mathrm{X} + \big((\mathrm{A_j} + \mathrm{A_j})X\big)^T + (\mathrm{B_i N_j} + \mathrm{B_j N_i})\mathrm{C} - \big((\mathrm{B_i N_j} + \mathrm{B_j N_i})\mathrm{C}\big)^T < 0 \tag{11}$$

$$i < j\ for\ \ i,j = 1,\ldots,r\ \ \ and\ i \ne j$$

With $h_i(z(t)).h_j(z(t)) \ne 0$.

The LMIs (10,11) are obtained from (6,7) by using the changes of variables $\mathrm{X} = \mathrm{P}^{-1} > 0\ and\ CX = MC$. Since the matrix C is assumed full row rank, we deduce that there exist a non-singular matrix :

$$\mathrm{M} = \mathrm{CXC^T}(\mathrm{CC^T})^{-1} \tag{12}$$

And then

$$\mathrm{K_i} = \mathrm{N_i M^{-1}} \tag{13}$$

Or

$$K_i = \mathrm{N_i} C C^T (C X C^T)^{-1} \quad \forall\, i\ \in \{1,\ldots,r\}$$

III. Application: Stabilization of the Synchronous Machine

A. The synchronous Machine and its Mathematical Model

The mathematical model of the synchronous machine adopted in this work is obtained on consist to transform all stator quantities from phase a, b and c into equivalent d-q axis new variables.

The equations are derived by assuming that the initial orientation of the q-d synchronously rotating reference frame is such that the d-axis is aligned with stator terminal voltage phase. The details of their above equation and its parameters can be found in [16].

The mathematical model of the synchronous machine with damper was established as the follow:

The state equations:

$$\dot{\mathrm{x}}(\mathrm{t}) = f\big(x(t)\big) + g(x(t))u(t) \tag{13}$$

Where:

$$f(x(t)) = \begin{bmatrix} x_2(t) \\ -a_1 \sin\big(2x_1(t)\big) - a_2 x_3(t)\sin\big(x_1(t)\big) + a_3 x_5(t)\cos\big(x_1(t)\big) + a_4 u_1(t) \\ b_1\cos\big(x_1(t)\big) + b_2 x_3(t) \\ -c_1 \sin\big(x_1(t)\big) - c_2 x_4(t) + c_3 x_5(t) + c_4 u_2(t) \\ -d_1 \sin\big(x_1(t)\big) + d_2 x_4(t) + d_3 x_5(t) + d_4 u_2(t) \end{bmatrix}$$

$$g(x(t)) = \begin{bmatrix} 0 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & d_4 \end{bmatrix}^T$$

Where the state vector:

$$x(t) = \Big(\theta_d(t)\ \ \omega_d(t)\ E''_d(t)\ \ E'_q(t)\ \ E''_q(t)\Big)^T \in \mathcal{R}^5,$$

the input vector $u(t) = \big(\mathrm{E_{ex}(t)}, \mathrm{P_m(t)}\big)^T \in \mathcal{R}^2$

Augmented with an output vector:

$y(t) = \mathrm{C}\,x(t)$, with $C = (0\ \ 1\ \ 0\ \ 0\ \ 0)$, then $y(t) = (0 \ \ \omega_d \ \ 0 \ \ 0 \ \ 0)^T$.

The entire symbols for nonlinear model used in this paper are given in appendix A.

The synchronous machine model parameters are given in appendix B.

A Takagi-Sugeno fuzzy model of this system is given in the following section.

B. Construction of T-S Fuzzy Model

The TS fuzzy model represents exactly many nonlinear models on a limited interval of the state variables [14]. One of the main interests of this representation is that it allows systematic methods to design control laws [2]. Unfortunately, for this approach the stability criteria are only sufficient, such that numerous model rules are necessary to meet conservative specifications. Furthermore, the number of rules grow by 2^n where n is the number of non-linearities [14]. Still, it has been used successfully for the PDC control of synchronous machine [1, 2, 3, 4].

As commented earlier, the number of model rules goes as $e=2^n$ with n nonlinear terms [2] and [17].

Note that there are four non linearities in the non-linear dynamical model (1):

$$\sin(2x_1(t)) = 2\sin(x_1(t))\cos(x_1(t))$$

And $x_3(t)\sin(x_1(t))$.

Thus n = 3 indicating e= 2^3 = 8 rules are required. However, with some compromise the number of rules can be reduced to 2 while maintaining model [1], [17].

First, we can rewrite two of the nonlinear terms in $\sin(x_1(t))$ and $\cos(x_1(t))$ as:

$$\frac{\sin(x_1)}{x_1(t)} = \frac{x_{10}\ \sin(x_1(t)) - x_1(t)\sin(x_{10})}{x_1(t)\,(x_{10} - \sin(x_{10}))}.1 + \frac{x_{10}\ (x_1(t) - \sin(x_1(t)))}{x_1(t)\,(x_{10} - \sin(x_{10}))}.\frac{\sin(x_{10})}{x_{10}}$$

And

$$cos(x_1(t)) = \frac{cos(x_1(t)) + cos(x_{10})}{1 - cos(x_{10})}.1 + \frac{1 - cos(x_1(t))}{1 - cos(x_{10})}.cos(x_{10})$$

Whose membership functions are bounded in the range:

$x_1(t) = [-x_{10} \quad +x_{10}]\ for\ x_{10} = \theta_{d0} \in [0 \quad \frac{\pi}{2}]$ impliyng :

$$\left|\frac{x_{10}\ \sin(x_1(t)) - x_1(t)\sin(x_{10})}{x_1(t)\,(x_{10} - \sin(x_{10}))} - \frac{cos(x_1(t)) + cos(x_{10})}{1 - cos(x_{10})}\right| \leq 2{,}4\%$$

Therefore the transformation on $cos(x_1(t))$ can be eliminated with little compromise and the fuzzy model order reduced to 2^2 or 4 rules. Then, the final fuzzy model is described by only two rules.

Here the premise vector is independent of the input and often considered as a part of the state vector or as a linear combination of this one. And the premise vector is defined by

$$z(t) = [z_1(x_1(t)) \quad z_2(x_3(t))]$$

$$with\ z_1(x_1(t)) = \theta_d(t)\ and\ z_2(x_3(t)) = x_3(t)$$

For a premise terms, define, $z_i(t) = x_i(t), i = 1, 2.$

Next, calculate the minimum and maximum values of $z_i(t)$ under $\forall\ x(t) \in [-a, a],\ a > 0$.

They are obtained as follows:

$$\max z_i(t) = a, \qquad \min z_i(t) = -a$$

From the maximum and minimum values, $z_i(t)$ can be represented by:

$$z_i(t) = M_i^1(z_i(t)).a + M_i^2(z_i(t)).(-a)$$

Where $M_i^1(z_i(t)) + M_i^2(z_i(t)) = 1$

Therefore the membership functions can be calculated as:

$$M_i^1(z_i(t)) = \frac{z_i(t) + a}{2a}; \quad M_i^2(z_i(t)) = \frac{a - z_i(t)}{2a}$$

Finally, the complete fuzzy model is comprised of four rules, the premise variable is:

$$z_1(t) = \theta_d(t)\ and\ z_2(t) = x_3(t)$$

with the following membership functions respectively,

$$M_1^1(z_1(t)) = \frac{\sin z_1(t)}{z_1(t)}; \quad M_1^2(z_1(t)) = 1 - M_1^1(z_1(t))$$

$$M_2^1(z_2(t)) = z_2(t); \quad M_2^2(z_1(t)) = 1 - M_2^1(z_1(t))$$

The Takagi-Sugeno fuzzy model of the synchronous machine connected to infinite bus system can be rewritten by introducing submodels are described respectively by the four matrices Ai, Bi, Ci, i=1, .. , 4. as follows:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2\cdot a_1 + a_2\cdot a & 0 & 0 & 0 & a_3 \\ b_1 & 0 & b_2 & 0 & 0 \\ c_1 & 0 & 0 & c_2 & c_3 \\ d_1 & 0 & 0 & d_2 & d_3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 2\cdot a_1 - a_2\cdot a & 0 & 0 & 0 & a_3 \\ b_1 & 0 & b_2 & 0 & 0 \\ c_1 & 0 & 0 & c_2 & c_3 \\ d_1 & 0 & 0 & d_2 & d_3 \end{bmatrix}.$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ K_1 & 0 & 0 & 0 & a_3\cos(\theta_{d0}) \\ b_1\cos(\theta_{d0}) & 0 & b_2 & 0 & 0 \\ c_1(\sin(\theta_{d0})/\theta_{d0}) & 0 & 0 & c_2 & c_3 \\ d_1(\sin(\theta_{d0})/\theta_{d0}) & 0 & 0 & d_2 & d_3 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ K_2 & 0 & 0 & 0 & a_3\cos(\theta_{d0}) \\ b_1\cos(\theta_{d0}) & 0 & b_2 & 0 & 0 \\ c_1(\sin(\theta_{d0})/\theta_{d0}) & 0 & 0 & c_2 & c_3 \\ d_1(\sin(\theta_{d0})/\theta_{d0}) & 0 & 0 & d_2 & d_3 \end{bmatrix}$$

With;

$$K_1 = \frac{\sin(\theta_{10})}{\theta_{10}}\,[2.a_2.\cos(\theta_{d0}) + a_2.a],$$

$$K_2 = \frac{\sin(\theta_{10})}{\theta_{10}}\,[2.a_2.\cos(\theta_{d0}) - a_2.a]$$

$$B_1 = \begin{bmatrix} 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & d_4 \end{bmatrix}^T \ and\ C_1 = [0\ \ 1\ \ 0\ \ 0\ \ 0]^T$$

The T.S fuzzy model exactly represents the nonlinear systems. Notice that this fuzzy model has the commons B and C matrix.

In the OPDC design, each control rule is associated with the corresponding rule of a T–S fuzzy model. The designed fuzzy controller shares the same fuzzy sets as the fuzzy model and the same weights $w_i(z(t))$ in the premise parts. The following output feedback fuzzy controller is constructed via OPDC as follows:

$$u(t) = \sum_{i=1}^{2} h_i(z(t))K_i\, y(t)$$

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{2} w_i(z(t))},\ \ i = 1, \dots, r,$$

$$w_i(z(t)) = \prod_{j=1}^{r} M_i^j\left(z_j(t)\right), \qquad i = 1, \dots, r$$

Finally, the complete the closed loop model T-S fuzzy (5) is synthesized with the premise variable:

$$z_1(t) = \theta_d(t)\ \ and\ z_2(t) = x_3(t)$$

The synthesis of the controller consists of finding the feedback gains of the conclusion parts Ki which guaranteed the asymptotic stability of the output closed loop system.

IV. Simulations

To show the effectiveness of the proposed design, a simulation study is performed using simulations tets to a synchronous machine.

Many tests have been performed to prove the goodness of the proposed fuzzy control system. Some results, obtained by means of the SIMULINK program of MATLAB, are reported in what follows.

For these simulations, the model rules are chosen for $\theta_d(t) \in [-\theta_{d0}\quad \theta_{d0}]$ and $x_3 \in [-a\quad a]$.

Assume that the initial conditions $x(t)$:

$$x(0) = (-0.52\quad 0\quad 0.36\quad 0.80\quad 0.71)^T$$

We presents only the results for the value of $\theta_{d0} = \frac{\pi}{4}$ and a = 0.82. Every set of LMIs was solved via the Matlab LMI toolbox.

Using a software simulator, we have found matrixes X, N_i and M that satisfies (10) and (11), and also gains Ki for the output feedback controller (13) that stabilizes the system:

$$K_1 = [243.4088;\ 29.1002]$$

$$K_2 = [352.6021;\ 63.3890]$$

$$K_3 = [211.1034;\ 95.3898]$$

$$K_4 = [197.4842;\ 68.7478]$$

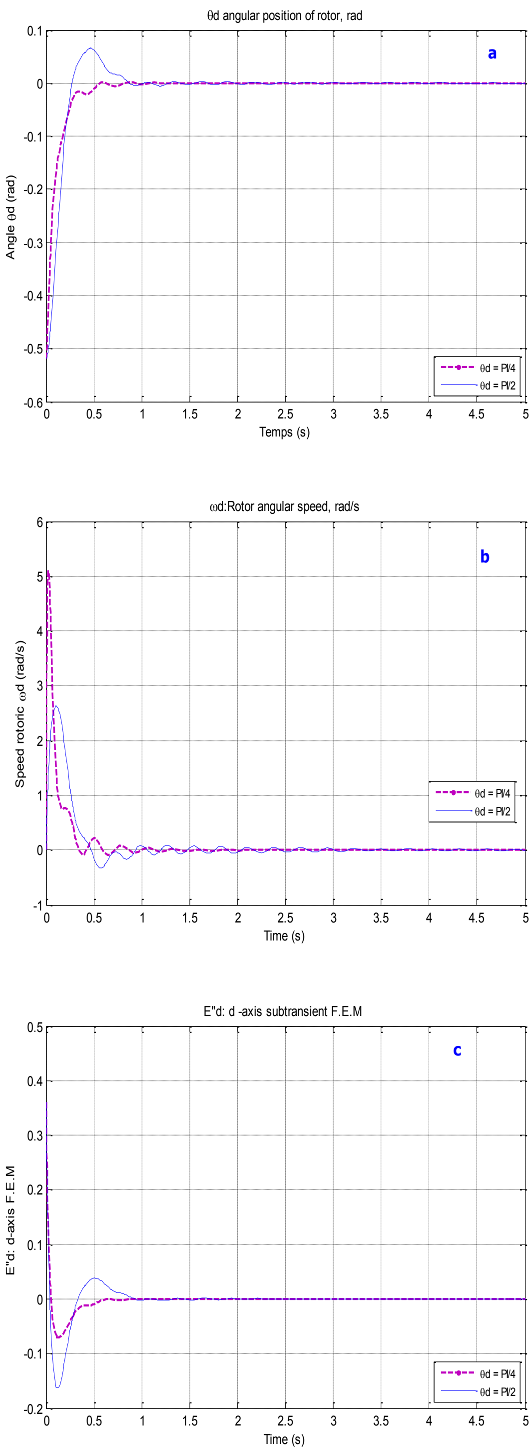
$$M_1 = [39.9401;\ \ 4.7750]$$

$$M_2 = [57.8573;\ 10.4013]$$

$$M_3 = [34.6392\ ;\ 15.6522]$$

$$M_4 = [32.4045;\ 11.2806]$$

The trajectories of the state vector and the system response were illustrated by the above simulation results in the case of Static output quadratic stability.Figure.1 displays overall simulation results of the state vector.



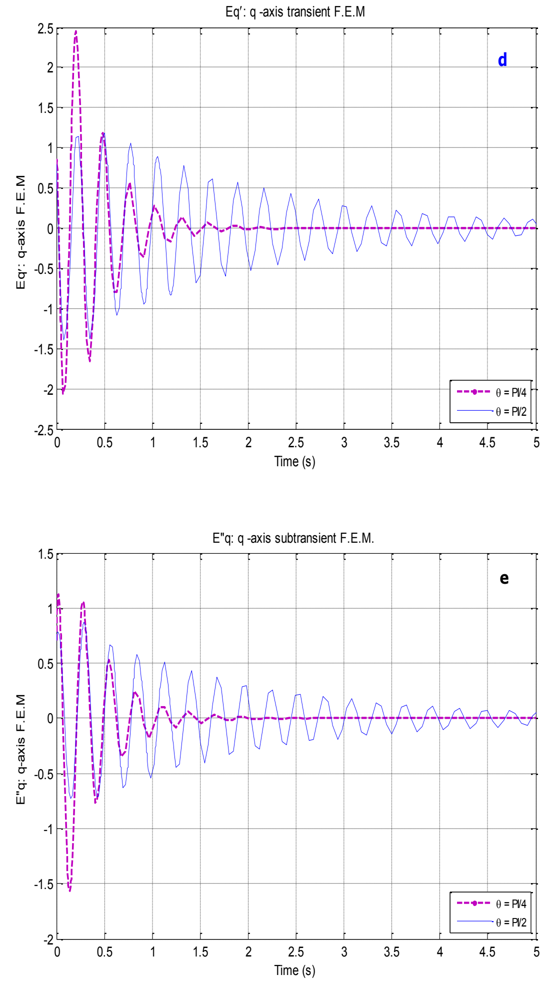


Figure.1 The trajectories of the state vector: a: θ_d angular position of rotor (rad), b: ω_d Rotor angular speed (rad/s), c: E''_d d -axis sub-transient F.E.M, d: E'_q q -axis transient F.E.M and e: E''_q q -axis sub-transient F.E.M.

The simulations results displays the trajectory of the state vector at different values of at startup and those simulations illustrate the effect of the different values of the angle θ_d.

In general, we conclude that in the same way of the parallel distributed compensation (PDC) concept stabilize the synchronous machine [1, 2, 3, 4], in this paper we can also show that the static output feedback fuzzy controller also features this property.

V. Conclusion

In this paper, we have described an output feedback control designed to stabilize the synchronous machine with damper infinite bus. The OPDC law was used to design a fuzzy controller from T-S fuzzy model.

The static output feedback fuzzy control problems are derived from the Lyapunov stability criterion, so the problem is formulated as the solution of LMIs set.

Another way to deal with the static output feedback control for Synchronous machine modeled by a T-S model is to transform the synthesis of the output feedback into a complementarity problem, which constitutes our future researches.

The numerical simulations and experimental results have illustrated the expected performance and indicate that the stability of the OPDC controlled system is very suitable in Synchronous Machine and it leads to an optimization problem.

APPENDIX

$$a_1 = \frac{-\omega_0}{2T_L}\left(\frac{1}{X''_d} - \frac{1}{X''_q}\right) ; a_2 = \frac{-\omega_0}{T_L X''_q} ;$$

$$a_3 = \frac{-\omega_0}{T_L X''_d} ; a_4 = \frac{\omega_0}{T_L} ;$$

$$b_1 = \frac{-1}{T''_{q0}}\left(1 - \frac{X_q}{X''_q}\right) ; b_2 = \frac{-X_q}{T''_{q0} X''_q} ;$$

$$c_1 = \frac{-1}{T'_{d0}}\left[\frac{X_d - X_a}{X'_d - X_a}\left(1 - \frac{X'_d}{X''_d}\right) - \left(1 - \frac{X_q}{X''_q}\right)\right]$$

$$c_2 = \frac{-1}{T'_{d0}}\left(\frac{X_d - X_a}{X'_d - X_a}\right) , c_3 = \frac{-1}{T'_{d0}}\left(\frac{X'_d}{X''_d}\frac{X_d - X_a}{X'_d - X_a} - \frac{X_d}{X''_d}\right) ;$$

$$c_4 = \frac{-1}{T'_{d0}}, d_1 = \frac{X''_d - X_a}{X'_d - X_a} c_1 + \frac{1}{T''_{d0}}\left(1 - \frac{X'_d}{X''_d}\right)$$

$$d_2 = \frac{X''_d - X_a}{X'_d - X_a} c_2 + \frac{1}{T''_{d0}} ; d_3 = \frac{X''_d - X_a}{X'_d - X_a} c_3 + \frac{1}{T''_{d0}}\frac{X'_d}{X''_d} ;$$

$$d_4 = \frac{X''_d - X_a}{X'_d - X_a} c_4 .$$

TABLE I
MACHINE SYNCHRONOUS DATA (CAPACITY POWER 200VA)

Symbol	***Quantity***	***Value (p.u)***
X_d	*d - axis magnetic reactance*	*1.10*
X'_d	*d -axis transient reactance*	*0.50*
X''_d	*d -axis subtransient reactance*	*0.35*
X_q	*q - axis magnetic reactance*	*1.10*
X''_d	*q -axis subtransient reactance*	*0.30*
X_a	*field leakage reactance*	*0.19*
T_L	*magnetic dipole moment*	*10.00*
T'_{d0}	*d -axis transient open circuit time constant*	*7.00*
T''_{d0}	*d -axis subtransient open circuit time constant*	*0.07*
T''_{q0}	*q -axis subtransient open circuit time constant*	*0.18*
ω_0	*synchronous rotor angular*	*100 π*

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