

Improved Demand Forecasting Using Local Models Based on Delay Time Embedding

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Abstract—Due to growing dynamics and complexity of today's markets, customer demands are often highly volatile. In order to achieve a well-founded forecast of customer demands, a company has to consider several dynamic influences. Classical simple statistical prediction methods are mostly easy to apply but are not able to react on dynamic behavior. More complex statistical methods achieve better forecasts but also do not include dynamic means. Prediction methods of nonlinear dynamics consider qualitative in addition to quantitative information within time series of past customer orders in order to achieve better forecasts into the future. In particular, local models use the information fostered by delay time embedding of nonlinear time series analysis. In this paper, a research approach is presented that has the goal of outlining suitable prediction methods for future customer demands of a forecasting company in a production and delivery network.

Keywords— demand forecast, forecasting methods, nonlinear dynamics, time series analysis.

I. INTRODUCTION

TODAY'S markets are characterized by a strong competition among globally dispersed companies. Along with the continuing trends of outsourcing and the companies' concentration on their core competences these markets are highly complex. In addition, mutual dependencies within the related production and logistics processes as well as changing conditions in the economic, political, and ecologic environment foster the development of dynamics on all time scales. These conditions entail volatile markets complicating the accurate forecast of future customer demands [1]. However, a well-founded prediction of upcoming customer demands is highly important for a company's long- and mid-term planning, especially considering procurement strategies and production resources. Here, low-quality data on future customer demands in combination with limited flexibility in reaction can implicate undesirable consequences. In the case of overestimation, low efficiencies can result. Vice versa, an

underestimation of the customer demand can lead to capacity overloads, followed by delayed deliveries and even out-of-stock-situations which can cause a loss of customers in an extreme case. Therefore, forecasting methods are required which achieve high-quality predictions on future customer demands considering all available information within the data at hand.

The incoming data of customer demands can be regarded as time series. In order to grasp the dynamic evolution one can distinguish between external and internal dynamic influences that impact on the time series and accordingly on a customer's demand. These influences are illustrated in Fig. 1. External dynamic influences are given by general market trends, e.g. seasonal fluctuations, changes of currency rates, political issues or personal relationships. These factors and their dependencies are boosted by the continuing globalization and its impact on complexity and volatility. From a forecasting company's point of view, the external dynamic influences are superposed by internal dynamic influences which arise from its direct customers. Here, the internal settings of a customer's production system and its control, e.g. considering the system's organization, order policies etc., are of eminent importance for the impressed dynamics on the time series of its demand. Generally, each production system can be interpreted as a dynamic system [3], [4], [5]. Information on the system's structure, capacities, operational rules, and queuing policies determine the system's dynamics. This insight is instructive for the prediction of its demand as it accounts for deterministic and qualitative properties within the related time series which can be considered in addition to quantitative information.

In the following, we present the research approach of the project "Forecasting in Production Considering Prediction Models of Nonlinear Dynamics" which is funded by German Research Foundation. The main goal of the research project is to improve forecasting considering customer demands in production systems. It bases on the assumption that the analysis of dynamic properties within the time series representing customer demands leads to a better comprehension of their behavior and allows more precise predictions of future demands. The project considers quantitative as well as qualitative properties. In the next section we give a short overview on forecasting methods including their ability to deal with volatile customer demands and qualitative information. Furthermore, we detail the project's approach to consider qualitative data for an improved

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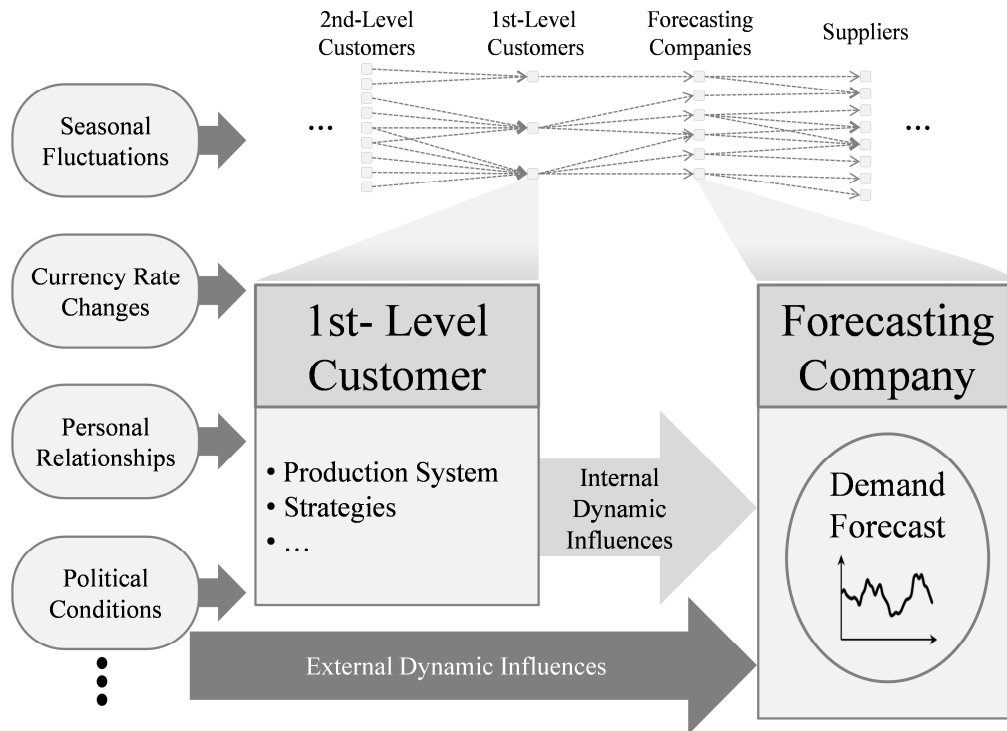


Fig. 1 Dynamic Influences on Demand Forecasts Within a Production and Delivery Network (in Accordance to [2])

selection and application of forecasting methods. Within this, time series analysis as a tool for characterization is linked with prediction methods, especially emphasizing methods of nonlinear dynamics. In particular, prediction by local models of nonlinear dynamics is considered. These models use qualitative data from the reconstruction of dynamic properties by delay time embedding in order to improve forecasting results.

II. FORECASTING METHODS

Over the years diverse forecasting methods have been developed. In the course of the twentieth century, these methods have been adapted and furthered with the aid of new arising powerful computers. There are three main classes in which the forecasting methods can be grouped: statistical methods, methods of soft computing, and methods of nonlinear dynamics, Fig. 2.

In general, classical statistical methods, e.g. moving average, exponential smoothing or linear regression, split an observed time series based on past values into components of trend, season, and random noise. After that, the resulting model is applied to extrapolate the time series into the future [7]. Most of the methods are quite simple and applicable for non-experts. Associated with the introduction of computers they were advantageous because of entailing low computing efforts. However, an application of the mentioned simple statistical methods is only reasonable to receive vague estimations of future demands. The increased power of today's computers allows the adaptation of more sophisticated

prediction methods. As an example, the Box-Jenkins-method provides improved forecasting results while requiring intensified computation capacity [8].

Other computational intensive procedures are contemporary methods of soft computing. These are characterized by a strongly increased model complexity and are currently rather of academic interest than applicable for a forecasting company. Examples of these models are expert systems [9], artificial neural networks [10], methods of fuzzy logic [11] or genetic programming [12], [13]. Furthermore, the indicated methods allow their combination to hybrid systems, e.g. expert systems and artificial neural networks were used in order to gain predictions of outward stock movement [14]. Equally, a combined Neuro-Fuzzy system was applied to the daily sales rate prediction of newspapers [15]. However, although the application of soft computing methods provides the capability to achieve high-quality prediction results when applied correctly, they are still in the period of development and feature various potentials for improvement. In addition, these models are rather complex which complicates their application and interpretation by non-experts, e.g. the application of neural networks requires the choice, training, and optimization of networks and their underlying functions [10].

Dealing with a time series which represents a customer's demand influenced by his production system, the question arises whether the applied prediction method considers qualitative structures of the time series obtained by time series analysis. Here, methods of nonlinear dynamics offer the potential to take deterministic as well as quantitative and

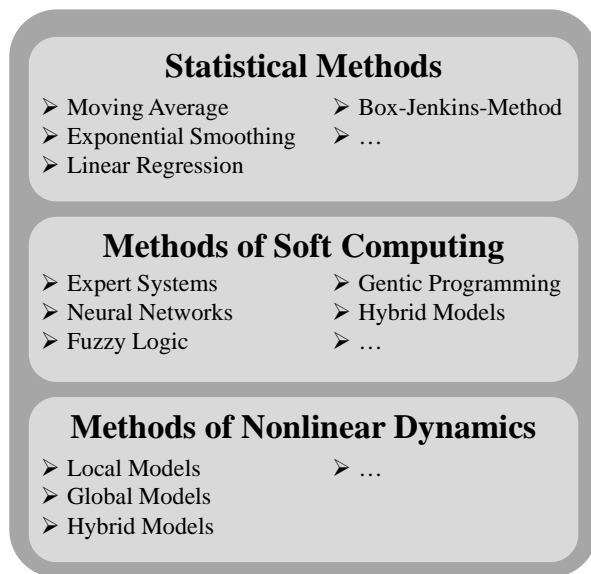


Fig 2 Forecasting Methods (in Accordance to [6])

stochastic properties of observed time series into account. These methods enable to include dynamic effects in production and logistics systems which have been studied in various publications [3], [5], [16], [17], [18]. Suitable methods for this challenge are found among local, global, and hybrid models. Local models create individual prediction functions for each single point within the time series [19]-[21]. A special prediction algorithm that uses local models is described in section IV. Global models strive to describe the whole time series in terms of a polynomial as a basis for predictions [22]. Hybrid models are a combination of both, local and global models [23]. Besides, approaches were developed to combine methods of nonlinear dynamics with methods of soft computing [24]. Indeed, these approaches of combined application are in development and have potentials to be specified and extended.

In contrast to the before mentioned forecasting methods, the methods of nonlinear dynamics are applicable to real systems even without a previous idealization. Moreover, they have the potential to predict the future more precisely without the necessity of splitting the time series into its components of trend, season, and random noise. However, equally to the methods of soft computing, the application of methods of nonlinear dynamics requires expert knowledge especially considering the choice of parameters associated with the methods. On that account, an understanding of the time series' properties facilitates adequate adjustments of the model parameters. These properties can be inspected by methods of linear and nonlinear time series analysis. Thus, the next section describes suitable methods for analysis and characterization of time series.

III. ANALYSIS AND CHARACTERIZATION OF TIME SERIES

Generally, the choice of suitable forecasting methods is

supported by qualitative information on a time series' evolution. Methods of linear and nonlinear time series analysis are applicable to analyze and characterize a time series in terms of its structure and patterns. Linear methods are capable of giving a preliminary estimate of the time series' properties. In terms of stationarity the mean and the variance can be used to derive first indications. However, being indexes of the statistical overall-properties, an application of these methods does not provide sufficient information about the time series' structure. In order to deal with the related characteristics, Fourier analysis as a measure of power spectrum or the autocorrelation function which observes the self-similarity of the time series are adaptable. These methods are appropriate to characterize time series arisen out of linear processes. In order to deal with nonlinear effects, e.g. originating from interdependent production and logistics processes, methods of nonlinear time series analysis have to be adapted as well.

The state of a system is determined uniquely by a set of variables which span the phase space. The time series' evolution over time is described by a sequence of points within phase space, a so-called trajectory. The analysis of a trajectory's movement in phase space allows investigating the time series' properties and characterizing their dynamic structure [25], [26], [27]. Over the years, several methods to analyze and characterize the properties and dynamics of nonlinear time series have been developed. A method to provide vectors that span a reconstructed phase space is the model of delay time embedding that is described in the next section [28]. Here, the box counting dimension and the method of false neighbors are applicable to estimate the dimension of the reconstructed phase space and the delay time [29], [30]. Postulating similar future evolutions from two contiguous points in phase space, the method of false neighbors allows determining whether these points are true neighbors or if their neighborhood originates in a projection of a higher-dimensional space. Counting these false neighbors the system's dimension can be estimated. In addition, the calculation of Lyapunov-exponents is a way to investigate the divergence or convergence of contiguous trajectories. As a result, sensitivity in terms of starting conditions on future evolutions can be analyzed. In case the trajectories deviate from each other even if the initial values are very close, the behavior is called chaotic. Therefore, the calculation of Lyapunov-exponents implies the ability to separate areas in which the trajectory acts chaotic from those showing regular dynamic movement [27].

A procedure of approximating a complex and multi-dimensional trajectory by a sequence of discrete points is the so-called Poincaré mapping. By applying this method, a complex dynamic system can be reduced to a simplified and a more comprehensive appearance [25].

Furthermore, the calculation and illustration of recurrence plots represents a trajectory's recurrences within a high-dimensional phase space on a two-dimensional squared matrix. This can be instructive for future developments of a time series and recurrence quantification analysis [31], [32], [33]. Finally, entropy as a term of physics or information

theory can be adapted to measure a system's level of dynamic disorder and indicate its predictability [34]. In order to deal with stochastic and deterministic effects within a time series, the signal-noise-ratio characterizes to which extend a given signal is overlaid by a noise-signal, determining the ratio of stochastic to deterministic influences.

Our approach is to analyze an incoming time series representing a customer's demand in terms of its dynamic characteristics and link these with suitable forecasting methods. The next section details a usage of local prediction models after a dynamic reconstruction by delay time embedding of nonlinear time series analysis.

IV. LOCAL MODELS OF NONLINEAR DYNAMICS

In the wide range of different forecasting methods for time series, models of nonlinear dynamics promise high forecasting accuracy because of considering qualitative in addition to quantitative data information. For instance, local models of nonlinear dynamics use the delay time embedding method of nonlinear time series analysis to build a reconstructed phase space in which the attractor is equivalent to the attractor in the unknown original phase space. Based on this qualitative information about the time series structure, the time series is extrapolated into the future by using a nearest neighbor prediction algorithm. In the case when a time series on hand represents customer demands in production and delivery networks, the described procedure is applied in order to identify deterministic structures within past customer orders and subsequently to achieve better forecasts of future customer demands. In this section, firstly, we give a short overview of dynamic systems and its properties. Afterwards, we describe the method of delay time embedding and subsequently a prediction algorithm basing on this method.

A. Dynamic Systems and Properties

In the case of discrete time, a dynamic system can be described by a system of difference equations

$$\mathbf{x}_{i+1} = \mathbf{F}(\mathbf{x}_i) \quad \text{with } \mathbf{x}_i = \begin{bmatrix} x_{i,1} \\ \vdots \\ x_{i,m} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} F_1 \\ \vdots \\ F_m \end{bmatrix}, i \in \mathbb{Z}. \quad (1)$$

In the continuous case, a dynamic system can be described by an autonomous system of ordinary differential equations of first order

$$\frac{d}{dt} \mathbf{x}(t) = \mathbf{F}(\mathbf{x}(t)) \quad \text{with } \mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}, \mathbf{F} = \begin{bmatrix} F_1 \\ \vdots \\ F_m \end{bmatrix}, t \in \mathbb{R}. \quad (2)$$

The vector \mathbf{x} (or respectively \mathbf{x}_i in the discrete case) denotes the state of the system at the time t (at time event i) and \mathbf{F} denotes a slope vector.

In the following, we will refer to the continuous case. A

reduction to differential equations of first order in the continuous case is no restriction since every ordinary system of higher order can be transformed into a system of first order by introducing new variables. Moreover, every non-autonomous ordinary system can be converted into an autonomous by introducing a new variable $x_{m+1} = t$ and the trivial condition $\dot{x}_{m+1} = 1$. The components of the state vector \mathbf{x} span an m -dimensional space that is called the phase space $M \subset \mathbb{R}^m$. A point \mathbf{x} in phase space represents a state of the dynamic system. The dynamic evolution of a state over time is described by the flow, a mapping

$$\Phi: \mathbb{R} \times M \rightarrow M \quad (3)$$

with the properties

$$\Phi(0, \mathbf{x}) = \mathbf{x} \quad \text{for all } \mathbf{x} \in M \quad (4)$$

$$\Phi(\tau, \Phi(t, \mathbf{x})) = \Phi(t + \tau, \mathbf{x}) \quad \text{for all } t, \tau \in \mathbb{R}, \mathbf{x} \in M. \quad (5)$$

In the following, we will write $\Phi_t(\mathbf{x})$ instead of $\Phi(t, \mathbf{x})$. The velocity field of this evolution is characterized by \mathbf{F} . A mapping

$$\begin{aligned} \varphi_{\mathbf{x}}: \mathbb{R} &\rightarrow M \\ t &\mapsto \Phi_t(\mathbf{x}) \end{aligned} \quad (6)$$

that illustrates the system state's evolution over time in phase space is called a trajectory [25]. The dynamic system is defined as totally deterministic. This means that a trajectory is uniquely defined by a state vector \mathbf{x} and thus, any two trajectories in phase space cannot intersect. Through every point \mathbf{x} in phase space M there exists a unique trajectory with \mathbf{x} as the initial condition [25], [35].

Dynamic systems can be classified into the dissipative systems and the non-dissipative systems. Dissipation means that a phase space volume containing initial conditions is contracted under the dynamics [25]. Within non-dissipative systems no frictional-losses occur. In order to model states and evolutions according to production systems, a use of dissipative dynamic systems is more reasonable than a use of non-dissipative [36], [37]. For that reason, only dynamic systems with dissipation are considered within this article. In such systems, after a certain time, a set of initial conditions will be attracted to a subset $A \subset M$ of the phase space which is called the attractor of the system. By a definition of Lanford [38], an attractor A has the following properties:

- (Invariance under the flow) For $\mathbf{x} \in A$ it follows that $\Phi_t(\mathbf{x}) \in A$.
- (Attractiveness) It exists an open environment U with $A \subset U$ so that $\Phi_t(U) \subset U$ for $t > 0$ and

$$A = \bigcap_{t>0} \Phi_t(U). \quad (7)$$

- (Indecomposability) A cannot be divided into two nontrivial closed invariant pieces.

The set of initial conditions for which the trajectories converge against the attractor is called the basin of attraction [25]. In general, the space that contains the attractor A has a smaller dimension than the whole phase space M . This smaller space contains the relevant dynamic properties of the system. Therefore, attractors are of particular interest for an investigation on dissipative dynamic systems.

If all dependencies within the dynamic system are known, trajectories can be illustrated in the phase space M which is spanned by the known components of the state vector \mathbf{x} . The analysis of a trajectory's movement in phase space allows investigating its properties and characterizing its dynamic structure.

Commonly, not all dependencies are known. In many cases, only a scalar time series of measurements on the system is available. Here, more complicated methods have to be used in order to reconstruct the system's dynamic properties. It is possible to embed the original unknown phase space M into another room E , the embedding room, without losing the dynamic properties. An embedding is a one-to-one mapping that maps a manifold M onto another manifold E with $\dim(E) \geq \dim(M)$. In the following, the terms smoothness and diffeomorphism are needed within an embedding theorem. A smooth mapping is an at least two times continuously differentiable mapping. A diffeomorphism is a smooth mapping with smooth inverse mapping. The following theorem is a result by Takens [39]:

(Takens' Embedding Theorem)

Let M be a compact manifold of dimension m , let $\Phi_t : M \rightarrow M$ be the flow on M , let $g : M \rightarrow \mathbb{R}$ be a scalar measurement function of some quantity on the system, and let \mathbf{x} be the vector of the system's state. If Φ_t is a diffeomorphism and g is a smooth mapping, then the mapping $\mathbf{h} : M \rightarrow E \subseteq \mathbb{R}^{2m+1}$ with

$$\mathbf{h}(\mathbf{x}) = [g(\mathbf{x}), g(\Phi_1(\mathbf{x})), g(\Phi_2(\mathbf{x})), \dots, g(\Phi_{2m}(\mathbf{x}))] \quad (8)$$

is an embedding of M into $E \subseteq \mathbb{R}^{2m+1}$.

By embedding a phase space M into an embedding room E the dynamic properties of M can be reconstructed. In this process, the topological and differential characteristics of the attractor A in M are invariant which means that they are also reconstructed in E . According to Takens' theorem, an embedding dimension of $n \geq 2m+1$ for $m \in \mathbb{N}$ is sufficient for this reconstruction. In addition, Sauer, Yorke, and Casdagli [35] proved an extension of this theorem. They showed that it also holds if n is bigger than two times the box-counting dimension, a generalized fractal dimension that needs not to be an integer value.

Applying one of the mentioned embedding theorems, a system's state in the original phase space can be reconstructed

in the embedding space by the mapping \mathbf{h} . A well-known method to find a suitable measurement function g to define the embedding \mathbf{h} is the delay time embedding method.

B. Delay Time Embedding

Given a time series of past customer orders, a forecasting company attempts to extrapolate in order to predict future customer demands. Local prediction models of nonlinear dynamics firstly need information about qualitative structures within the time series on hand. Thus, the dynamic properties of a complex dynamic system that considers time evolutions within the considered production and delivery network have to be reconstructed. Therefore, the theoretical state vector \mathbf{x} whose components are unknown has to be replaced by a concrete vector based on the available time series data. Let the components of the given time series of length N

$$\mathbf{y} = \{y_0, y_1, \dots, y_{N-1}\} \quad (9)$$

$$y_k = x_j(t_k), \quad t_k = t_0 + k\tau_s, \quad y_k \in \mathbb{R}$$

be successive equidistant measurements (starting at t_0) of the component x_j of the state vector $\mathbf{x} = [x_1, \dots, x_m]^T$. τ_s is called the sampling time. In order to apply Takens' embedding theorem, the scalar measurement function g of some quantity on the system in (8) can be chosen as

$$g : M \rightarrow \mathbb{R} \quad (10)$$

$$g(\Phi_k(\mathbf{x})) = y_k = x_j(t_k).$$

Thus, the given time series can also be written as

$$\mathbf{y} = \{g(\mathbf{x}), g(\Phi_1(\mathbf{x})), \dots, g(\Phi_{N-1}(\mathbf{x}))\} \quad (11)$$

$$= \{x_j(t_0), x_j(t_1), \dots, x_j(t_{N-1})\}$$

$$= \{x_j(t_0), x_j(t_0 + \tau_s), \dots, x_j(t_0 + (N-1)\tau_s)\}.$$

Applying Taken's theorem, a state \mathbf{x} in the original phase space M can be embedded into an embedding room $E \subseteq \mathbb{R}^{2m+1}$ by the mapping \mathbf{h} in (8). Thus, define

$$\mathbf{h}_k : M \rightarrow E \subseteq \mathbb{R}^n \quad (12)$$

$$\mathbf{h}_k(\mathbf{x}) := \mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L}$$

where for $\tau_L \in \mathbb{R}_{>0}$, $\tau_L = c\tau_s$, $c \in \mathbb{N}$, the vector $\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L}$ is defined as

$$\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L} = [g(\Phi_k(\mathbf{x})), g(\Phi_{k+\tau_L}(\mathbf{x})), \dots, g(\Phi_{k+(n-1)\tau_L}(\mathbf{x}))]^T \quad (13)$$

$$= [x_j(t_k), x_j(t_k + \tau_L), \dots, x_j(t_k + (n-1)\tau_L)]^T$$

$$= [x_j(t_k), x_j(t_k + c\tau_s), \dots, x_j(t_k + (n-1)c\tau_s)]^T.$$

This vector is called a delay coordinate vector of length n starting at k . τ_L is called the delay time or lag time and is a multiple of the sampling time τ_s . The delay coordinate vector can also be displayed as

$$\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L} = [y_k, y_{k+\tau_L}, \dots, y_{k+(n-1)\tau_L}]^T. \quad (14)$$

The original time series \mathbf{y} consists of N measurements of the component x_j of the state vector \mathbf{x} with successive distance τ_s between each two measurements. The vector $\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L}$ is a segment of the original time series containing n of the N components of \mathbf{y} with a delay time $\tau_L = c\tau_s$, $c \in \mathbb{N}$, between each two components. The length n of the vector $\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L}$ is called the embedding dimension.

By the vector $\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L}$ one original state vector \mathbf{x} can be reconstructed. The set of all these reconstructed vectors

$$H = \{\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L} : k = 1, \dots, N\} \quad (15)$$

describes a discrete trajectory ψ_h in the embedding room E that is a unique and invertible mapping of the original trajectory φ_x in the original phase space M . For the applicability of one of the embedding theorems, it is required that Φ_t is a diffeomorphism and g is a smooth mapping. Hence, ψ_h has the same topological and differential properties as φ_x . In addition, all geometric and stability properties of the original attractor A can be calculated out of the reconstruction.

Two important parameters to choose for the embedding are the embedding dimension n and the delay time τ_L . These parameters have to be adjusted suitably in order to keep all essential information on the one hand and to leave out unnecessary information on the other hand.

The embedding dimension n has to be chosen as the smallest dimension for that all dynamic properties of the original attractor are kept. According to a theorem of Whitney [40] and according to Taken's embedding theorem [39] an embedding dimension of $n \geq 2m+1$ is sufficient for embedding where m is the dimension of the original phase space. A weaker condition is given by Sauer, Yorke, and Casdagli [35] who take the box-counting dimension as upper bound for n . In general, also smaller values of n can be sufficient for embedding. In order to apply these suggested upper bounds for n , the dimension m of the original phase space has to be known. In practice, this dimension is unknown. Hence, an algorithm to calculate a reasonable embedding dimension is needed. This can be achieved by the method of false nearest neighbors which was introduced by

Kenel, Brown, and Abarbanel [30]. A point \mathbf{h}_{NN} in the embedding space is called the nearest neighbor of the point \mathbf{h} if it has the shortest distance to this point. The term of distance is subjected to the dimension n of the embedding room. If two points are nearest neighbors in a room of dimension n_1 but the distance between them grows noticeable for a dimension $n_2 > n_1$ then they are called false nearest neighbors. The reason for this increasing distance is a too small embedding dimension n_1 in which the reconstructed attractor cannot unfold its full dynamic properties. Now, an embedding room which includes none of such false nearest neighbors has a sufficient embedding dimension. The algorithm calculates the number of false nearest neighbors for an arbitrary chosen start value n of the embedding dimension and increases n successively by one as long as all false nearest neighbors have vanished. In this way, an appropriate value of the embedding dimension n is calculated.

The second parameter to choose for a good embedding is the delay time τ_L . If this parameter is too small then successive elements of the delay vectors can be almost equal. Probably a reconstructed attractor cannot unfold its whole dynamic properties and no structure is visible. If it is too big then successive elements of the delay vectors are almost independent [25]. This leads to a connection of areas in the reconstructed attractor that are far apart from each other in the original phase space. For instance, reasonable values of the delay time can be determined by using the first root of the autocorrelation function [41] or the first minimum of the mutual information [42]. By taking the first root of the autocorrelation function as value for τ_L it is assured that successive components of the delay vectors are linear independent. The mutual information function was used by Fraser and Swinney [42]. It is a measure for the information a measurement at time t contains about a measurement at time $t + \tau_L$.

Altogether, the delay time embedding method can be used to reconstruct the dynamic properties for a state \mathbf{x} in an unknown phase space M by a vector $\mathbf{h}_{k,g,\mathbf{x}}^{n,\tau_L}$ in an embedding space E if a time series of equidistant measurements of one component x_j of \mathbf{x} is available. The course of action within this reconstruction algorithm is illustrated in Fig. 3.

C. Prediction

After the reconstruction of the dynamic properties of a dynamic system by methods of nonlinear time series analysis, this information can be used in order to achieve better forecasts into the future of a time series. Here, several methods can be applied. We now deal with a prediction algorithm that was introduced by Sauer [43]. This algorithm is based on local models and has the ability to use the available information from delay time embedding. For reasons of convenience, we define equivalent versions of the given time series

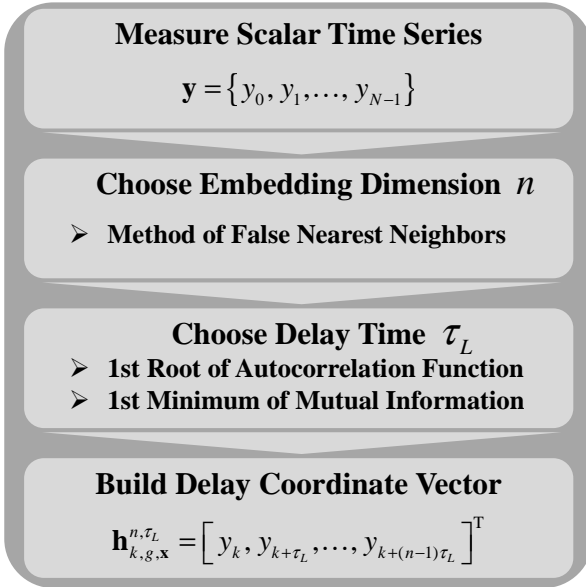


Fig 3 Course of Action in Delay Time Embedding

$$\mathbf{y} := \{g(\Phi_{-(N-1)}(\mathbf{x})), \dots, g(\Phi_{-1}(\mathbf{x})), g(\mathbf{x})\} \\ = \{x_j(t - (N-1)\tau_s), \dots, x_j(t - \tau_s), x_j(t)\} \quad (16)$$

and a delay coordinate vector

$$\mathbf{h} := [g(\Phi_{-(n-1)\tau_L}(\mathbf{x})), \dots, g(\Phi_{-\tau_L}(\mathbf{x})), g(\mathbf{x})]^T \\ = [x_j(t - (n-1)\tau_L), \dots, x_j(t - \tau_L), x_j(t)]^T. \quad (17)$$

For an unknown present state \mathbf{x} of the dynamic system at time t , a measured quantity $g(\mathbf{x})$ of one component x_j of \mathbf{x} is available. By the assumption of a deterministic system, the state \mathbf{x} contains all information to extrapolate the state r time units into the future by calculating $\Phi_r(\mathbf{x})$. The reconstructed state \mathbf{h} is used instead of the unknown original state \mathbf{x} . Now, a prediction function

$$P_r(\mathbf{h}) := g(\Phi_r(\mathbf{h})) \quad (18)$$

has to be found.

Denote the available time series in (16) as training set. Let \mathbf{h} represent the state of the system at time t . In order to evaluate the function $P_r(\mathbf{h})$ that extrapolates the time series r time units into the future, the training set is inspected for the z nearest neighbors $\mathbf{h}_{\text{NN},1}, \dots, \mathbf{h}_{\text{NN},z}$ of \mathbf{h} in \mathbb{R}^n . Here, the q -th nearest neighbor of \mathbf{h} is defined as the point $\mathbf{h}_{\text{NN},q}$ in the reconstructed phase space that has the q -th shortest distance to \mathbf{h} in terms of a defined distance like for example the n -dimensional squared Euclidian distance. For the

prediction, only one nearest neighbor is chosen from each nearby trajectory. Since every nearest neighbor $\mathbf{h}_{\text{NN},q}$ is an element of the known training set, the value $P_r(\mathbf{h}_{\text{NN},q})$ is also known. Thus, a prediction $P_r(\mathbf{h})$ can be achieved by using the knowledge of the $P_r(\mathbf{h}_{\text{NN},q})$, $q=1, \dots, z$. On that account, firstly, the center of mass \mathbf{c} of the nearest neighbors $\mathbf{h}_{\text{NN},1}, \dots, \mathbf{h}_{\text{NN},z}$ is calculated. For a fixed dimension $l \leq n$, the linear subspace \mathbb{R}^l of \mathbb{R}^n has to be found that passes through the point \mathbf{c} and minimizes the squared distances to the neighbors. This is achieved by computing the singular value decomposition. For that reason, calculate

$$\begin{pmatrix} \mathbf{h}_{\text{NN},1} - \mathbf{c} \\ \vdots \\ \mathbf{h}_{\text{NN},z} - \mathbf{c} \end{pmatrix} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (19)$$

where $\mathbf{\Sigma}$ is a diagonal matrix with non-increasing non-negative entries and \mathbf{U} and \mathbf{V} are orthogonal matrices [43]. The first l columns of \mathbf{V} span the desired space \mathbb{R}^l that minimizes the squared distances to the neighbors. Define a projection $\Pi: \mathbb{R}^n \rightarrow \mathbb{R}^l$. Now, the prediction can be achieved by linear or nonlinear models. We describe an approach by Sauer [43] with a local linear model.

The affine model $L: \mathbb{R}^l \rightarrow \mathbb{R}$ which best fits the points

$$(\Pi(\mathbf{h}_{\text{NN},1} - \mathbf{c}), P_r(\mathbf{h}_{\text{NN},1})), \dots, (\Pi(\mathbf{h}_{\text{NN},z} - \mathbf{c}), P_r(\mathbf{h}_{\text{NN},z})) \quad (20)$$

is formed by projecting the points $\mathbf{h}_{\text{NN},1} - \mathbf{c}, \dots, \mathbf{h}_{\text{NN},z} - \mathbf{c}$ onto \mathbb{R}^l . The model has the form

$$L(\mathbf{h}) = \mathbf{a} \cdot \mathbf{h} + d \quad (21)$$

with an l -dimensional vector \mathbf{a} and a constant scalar d . In order to extrapolate a given time series with present state \mathbf{h} r time units into the future, the following prediction function results:

$$P_r(\mathbf{h}) = L(\Pi(\mathbf{h} - \mathbf{c})). \quad (22)$$

Concluding, this prediction algorithm uses information of neighboring trajectories in order to predict the future evolution of an unknown trajectory. This approach is illustrated in Fig. 4 where the arrow constitutes the present state of the system and the dashed curve depicts a prediction into the future.

Helpful extensions of the described prediction algorithm can be filtration and interpolation. In a filtration step, the delay coordinate vector is transformed into a filtered delay coordinate vector

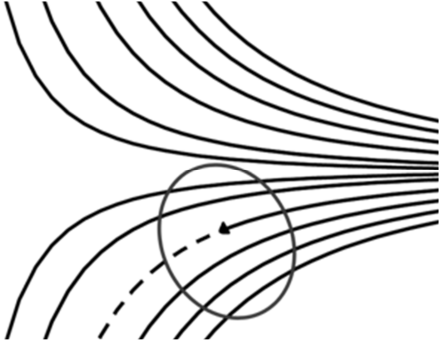


Fig 4 Estimation of a Future Trajectory Segment by Using Information of Nearest Neighbors

$$\mathbf{h} := \mathbf{W} \left[g(\Phi_{-(w-1)\tau_L}(\mathbf{x})), \dots, g(\Phi_{-\tau_L}(\mathbf{x})), g(\mathbf{x}) \right]^T. \quad (23)$$

\mathbf{W} is an $n \times w$ -matrix of rank n with $w \geq n$. So, \mathbf{h} remains an n -dimensional vector. \mathbf{W} is defined to be a composition of three linear operations: $\mathbf{W} = \mathbf{W}_3 \mathbf{W}_2 \mathbf{W}_1$. Here, \mathbf{W}_1 is a discrete Fourier transform of order w , \mathbf{W}_2 sets all but the lowest $\frac{w}{2}$ frequency contributions to zero, and \mathbf{W}_3 is an inverse Fourier transform of order n using the remaining $\frac{w}{2}$ frequencies. The three applied matrix operations yield to a low-pass filtration of the length w window of the time series and, as a result, to a low-pass embedding. The use of a filtered delay coordinate vector instead of an unfiltered leads to a reduction of noise within the data. In the absence of noise, a filtration still has the effect of decimating the data set [43].

In addition, an interpolation can also improve prediction accuracy if the available data set is sparse and there are too few neighbors in an appropriate neighborhood of the present state. In this case, without an interpolation, too distant neighbors are used to fit the model and low prediction accuracy results. This deficit can be overcome by an interpolation of the time series. Here, different types of interpolation can be used. For example, a section of the given time series is fit with a Fourier polynomial and the polynomial is sampled s equidistant times each sampling period of the original time series. By an interpolation of the given time series it can be assured that the direction from a present state to the nearest point on a neighboring trajectory is perpendicular [43].

Thus far, we have described the different steps of the prediction algorithm. Now, we detail the process of its application. Given a scalar time series

$$\mathbf{y} = \{y_0, y_1, \dots, y_{N-1}\} \quad (24)$$

of measurements of one component x_j of the state vector \mathbf{x} , future values of the underlying dynamic system can be estimated by using the prediction algorithm of Sauer [43]. For the prediction, five parameters have to be chosen ahead of

time. These are the number s of interpolation steps per sample period, the length w of the input window for filtered embedding, the low-pass embedding dimension n , the model dimension l , and the number of neighbors z to calculate in order to fit a one-dimensional linear model. Sauer [43] outlined that the best results can be obtained by choosing $w \geq n \geq l$ with l significantly smaller than n . Furthermore, the number z of nearest neighbors has to be selected weighing up the effects of variance and bias. Because of noise within the data, a too small number of neighbors can lead to a high variance of the model. On the other hand, too high values of z reduce the variance but can lead to bias because of considering faraway neighbors [43].

The interpolation step leads to a time series of length sN . Then, a filtered delay coordinate vector of dimension n can be obtained by filtering each original time series window of length w . Let \mathbf{h}_k be a filtered delay coordinate vector that represents the state at time t_k . Now, a future time series value y_i for $i > N-1$ can be estimated by $P_{i-k}(\mathbf{h}_k)$. In order to achieve better predictions, the predictions from the previous w time steps are averaged. This results in the following prediction algorithm for an unknown future value y_i of the given time series for $i > N-1$:

$$\hat{y}_i = \frac{1}{w} \sum_{k=i-w}^{i-1} P_{i-k}(\mathbf{h}_k). \quad (25)$$

This algorithm can be applied to predict future customer demands by using information of time series representing past customer orders. In this section, we described a prediction algorithm that was introduced by Sauer [43] and the required theory of dynamic systems and delay time embedding. This prediction algorithm will be applied to forecast future customer demands within a production and delivery network. This is part of a research project whose course of action is presented in the next section.

V. COURSE OF ACTION

The result of the research project "Forecasting in Production Considering Prediction Models of Nonlinear Dynamics" will be a data base containing various time series classified into groups of similar characteristics and suitable forecasting methods for every group. In order to establish this data base, the project follows a specified work scheme that is illustrated in Fig. 5. Initially, a discrete-event simulation model was developed and implemented that is detailed in Fig. 6. This model is applied to generate typical and representative time series of customer orders in production and delivery networks. Within the model, the influence of varying production and control concepts on the structure of the time series is observed. Therefore, it contains several generic elements of production systems with arbitrary numbers of suppliers and customers emphasizing variable job-shop systems. The variation of diverse system parameters like

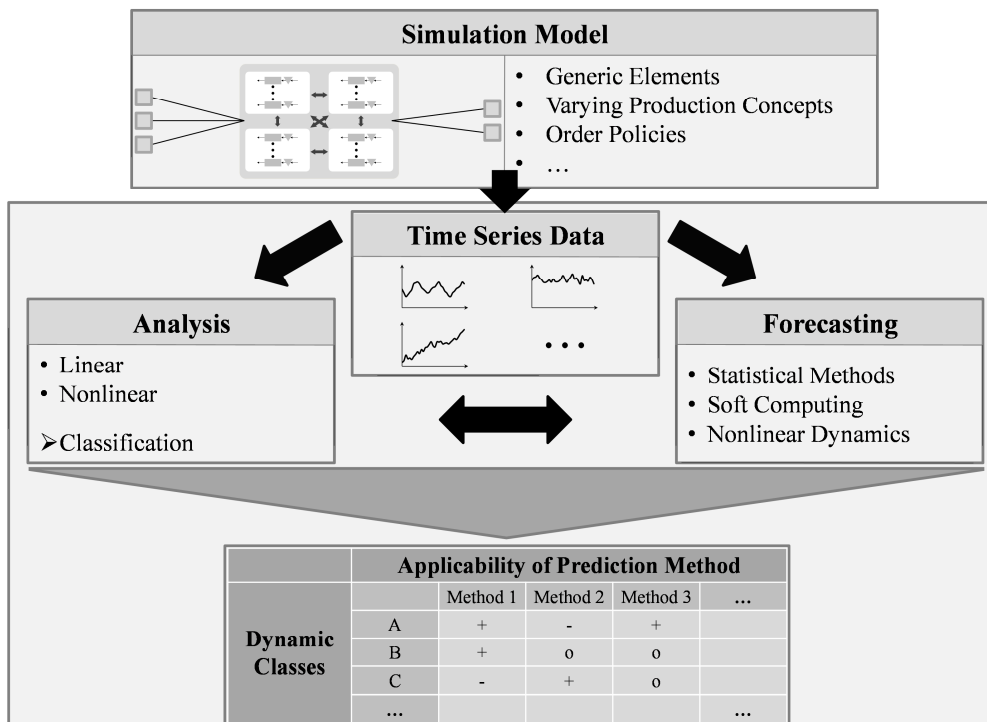


Fig 5 Course of Action Within the Research Project (in Accordance to [6])

priority rules, technical restrictions, order policies, re-entrant flows within working plans of different product types, different numbers of products or varying delivery times creates the opportunity to cover numerous different production cases, thus achieving representative results. Various production programs cover effects of external dynamics. Model input data are product orders of second-level customers to a simulated first-level customer of a forecasting company. Model output data are the desired time series representing demand of a forecasting company's direct customer. For a detailed description of the time series generation by the mentioned simulation model see [44]. For more information on the modeling of job-shop systems and influencing effects see [45], [46].

The generated time series will be analyzed in terms of their properties applying methods of linear and nonlinear time series analysis. Furthermore, these time series will be subject to the extensive application of various forecasting methods including statistical methods and local models of nonlinear dynamics. This step is followed by the assignment of dynamic properties within the time series to those forecasting methods which deliver significant prediction results.

In order to consider stochastic effects, the generated time series will be combined with a varying signal error representing inaccuracy of measurement and non-deterministic processes. The predictability of these manipulated time series will be determined by the Fokker-Planck-equation [47], [48].

Finally, the observed results will be merged into a data base applicable to choose suitable forecasting methods for diverse different cases and to adjust them in terms of their parameters.

VI. CONCLUSION AND OUTLOOK

Due to increasing dynamics, complexity, and volatility of today's markets, the prediction of customer demands often turns out to be difficult. The paper at hand described the research idea of the project "Forecasting in Production Considering Prediction Models of Nonlinear Dynamics". The overall goal of this project is to improve forecasting by deriving recommendations considering the choice and adjustment of suitable forecasting methods depending on the dynamic properties of the incoming time series of past customer orders. Here, the paper presented various forecasting methods and possibilities to analyze and characterize the dynamic properties of time series. In particular, a special prediction algorithm applying local models that use results of delay time embedding of nonlinear time series analysis was described. The project's overall course of action was presented. The simulation-based generation of representative time series will be subject to further work. Here, different production scenarios will be covered within a generic simulation model and a wide variation of several system parameters shall ensure adaptability of the results. The created time series will be analyzed and characterized by methods presented in this paper. The described prediction models will be applied to forecast the time series in order to identify suitable methods related to the time series' properties. By applying data of real-production scenarios the simulation model will be validated and furthermore used to test the selected forecasting methods.

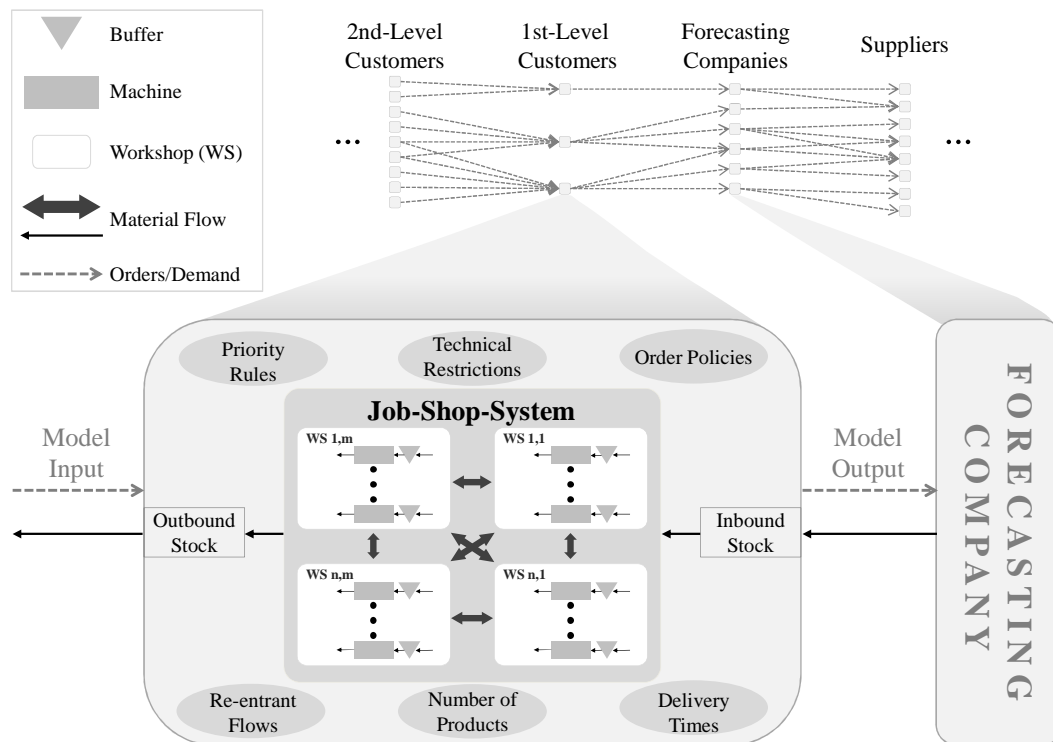


Fig 6 Parameters Within the Discrete-Event Simulation Model (in Accordance to [44])

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