ON SOME FUNCTIONAL EQUATIONS ARISING IN THE COMMUNICATION NETWORKS

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Numerous researchers have investigated various examples of functional equations that are of the general form

$$C_1(x,y)P(x,y) = C_2(x,y)P(x,0) + C_3(x,y)P(0,y) + C_4(x,y)P(0,0), \tag{1}$$

where C_i, $i = 1,2,3,4$, are given functions in two complex variables x, y and the unknown function P is a probability generating function (PGF). So, P is defined and analytic in the closed unit disc of the complex plane. The equations have many important applications in the queuing theory and in the communication networks (see, e.g., [1, 2, 6, 7, 9, 12, 13, 14]).

A quite popular technique of solving those equations is a reduction to a boundary value problem (cf. [3, 4, 5, 8]). Unfortunately, that approach is not always sufficiently effective and there is no universal efficient solving method known for such equations, so far.

The lecture contains some general remarks concerning the issue of solving the equations, with some examples. In particular, we discuss the cases of the following two functional equations

$$y(x - A(x,y))P(x,y) = A(x,y)[(\overline{\gamma}xy + \gamma x - y)P(0,y) + \gamma x(y-1)P(0,0)], \tag{2}$$

and

$$\left[(1+\alpha+\beta)xy - \alpha y - \beta x - x^2y^2\right]P(x,y) = \beta x(y-1)P(x,0) + \alpha y(x-1)P(0,y), \tag{3}$$

where the unknown function P is a PGF, and therefore must be of the form

$$P(x,y) = \sum_{m,n=0}^{\infty} p_{m,n}x^m y^n,$$

with some sequence of nonnegative real numbers $p_{m,n}$ $(m, n = 0, 1, 2, \ldots)$ satisfying the normalization condition

$$\sum_{m,n=0}^{\infty} p_{m,n} = 1.$$

The first equation (2) arises in [10] (see also [11]) in a performance analysis of an ATM (Asynchronous Transfer Mode) buffered switch transmitting two-class traffic over unreliable channels. The port is modeled as two logical queues with one server offering two service rates, 1 and γ, with r_1, r_2 being the arrival rates of class-1 and class-2 packets, respectively, and

$$A(x,y) = \Big(1 - \frac{r_1+r_2}{N} + \frac{1}{N}(r_1x + r_2y)\Big)^N,$$

where N is the number of input/output ports and $\overline{\gamma} = 1 - \gamma$.

The second equation (3) was obtained in [6], in a double queue model, where the arriving customers simultaneously place two demands handled independently by two servers, with service times rates α, β and the stability condition $1 < \alpha \leq \beta$.

REFERENCES

[1] I. Adan, O.J. Boxma, J. Resing, Queueing models with multiple waiting lines. Queueing Systems 37 (2001), 65–98.

[2] J.W. Cohen, On the asymmetric clocked buffered switch. Queueing Systems 30 (1998), 385–404.

[3] J.W. Cohen, O.J. Boxma, Boundary Value Problems in Queueing System Analysis. Elsevier, 2000.

[4] G. Fayolle, R. Iasnogorodski, Two coupled processors: the reduction to a Riemann-Hilbert problem. Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete 47 (1979), 325–351.

[5] G. Fayolle, R. Iasnogorodski, V.A. Malyshev, Random Walks in the Quarter-Plane: Algebraic Methods, Boundary Value Problems and Applications. Springer, 1999.

[6] L. Flatto, S. Hahn, Two parallel queues created by arrivals with two demands I. SIAM Journal on Applied Mathematics 44 (1984), 1041–1053.

[7] F. Guillemin, J.S. van Leeuwaarden, Rare event asymptotics for a random walk in the quarter plane. Queueing Systems 67 (2011), 1–32.

[8] V. Malyshev, An analytical method in the theory of two-dimensional positive random walks. Siberian Mathematical Journal 13 (1972), 917–929.

[9] H. Nassar, Two-dimensional queueing model for a LAN gateway. WSEAS Transactions on Communications 5 (2006), 1585–1590.

[10] H. Nassar, M.A. Ahmed, Performance analysis of an ATM buffered switch transmitting two-class traffic over unreliable channels. International Journal of Electronics and Communications (AEÜ) 57 (2003), 190–200.

[11] H. Nassar, J. Carpinelli, F. Nada, Queueing analysis of an ATM multichannel switch routing two-class multimedia traffic with two service rates. IEICE Transactions in Communication E87 (2004), 1505–1513.

[12] H. Nassar, Y. Fouad, Analysis of two-class discrete packet queues with homogenous arrivals and prioritized service. Communications in Information and Systems 3 (2003), 101–117.

[13] H. Nassar, H.A. Mahdi, Queueing analysis of an ATM multimedia multiplexer with non-pre-emptive priority. IEE Proceedings-Communications 150 (2003), 189–196.

[14] J. Resing, L. Örmeci, A tandem queueing model with coupled processors. Operations Research Letters 31 (2003), 383–389.

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