

BOUNDARY VALUE PROBLEMS AND A CLASS OF FUNCTIONAL EQUATIONS
ARISING IN THE QUEUING THEORY

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During the past five decades numerous researchers investigated a class of functional equations (FEs), with many important applications (e.g., in communication and networks). The general form of equations from that class is

$$C_1(x, y)P(x, y) = C_2(x, y)P(x, 0) + C_3(x, y)P(0, y) + C_4(x, y)P(0, 0), \quad (1)$$

where $C_i(x, y)$, $i = 1, 2, 3, 4$, are given functions in two complex variables x, y . The main unknown function $P(x, y)$ is a probability generating function (PGF) and therefore it is defined and analytic in the unit disc of the complex plane.

Particular examples of functional equations of form (1) arise, e.g., in some models of a 2×2 switch [1], an asymmetric switch [2], a queueing system which has applications in the inventory control of database systems [6], the wireless networks [7], a gateway linking two ethernet local area networks [9], a switch transmitting two-class traffic over unreliable channels [10], a multimedia multiplexer [11], and of some other systems [12].

Unfortunately, there is no universal solution method known for such FEs, so far. The most popular technique is a reduction to a boundary value problem, which was pioneered by Malyshev [8] (cf. [4]). An ample discussion on related issues can be found in [3, 5].

For instance, the equation in [9] has the form

$$\begin{aligned} (M(x, y) - xy)P(x, y) &= (1 - y)(M(x, 0) + \bar{r}_1 \xi_2 xy)P(x, 0) \\ &\quad + (1 - x)(M(0, y) + \bar{r}_2 \xi_1 xy)P(0, y) \\ &\quad - (1 - x)(1 - y)M(0, 0)P(0, 0) \end{aligned} \quad (2)$$

with

$$P(x, y) = \sum_{m, n=0}^{\infty} p_{m, n} x^m y^n, \quad x, y \in \bar{D},$$

being PGF of a sequence of nonnegative real numbers $p_{m, n}$ ($m, n = 0, 1, 2, \dots$) with the normalization condition

$$\sum_{m, n=0}^{\infty} p_{m, n} = 1, \quad (3)$$

and

$$M(x, y) = (\bar{r}_1 + r_1 \bar{s}_1 y + \xi_1 xy)(\bar{r}_2 + r_2 \bar{s}_2 x + \xi_2 xy),$$

where $0 < r_j, s_j, \xi_j < 1$ for $j = 1, 2$ are fixed real numbers and $\bar{q} = 1 - q$ for every q .

The lecture concerns possible descriptions (involving also boundary value problem techniques) of solutions to some of equations of type (1); in particular, solutions to (2).

REFERENCES

- [1] I. Adan, O.J. Boxma, J. Resing, Queueing models with multiple waiting lines. *Queueing Systems* 37 (2001), 65–98.
- [2] J. Cohen, On the asymmetric clocked buffered switch. *Queueing Systems* 30 (1998), 385–404.
- [3] J.W. Cohen, O.J. Boxma, *Boundary Value Problems in Queueing System Analysis*. Elsevier, 2000.
- [4] G. Fayolle, R. Iasnogorodski, Two coupled processors: the reduction to a Riemann-Hilbert problem. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* 47 (1979), 325–351.

- [5] G. Fayolle, R. Iasnogorodski, V.A. Malyshev, Random Walks in the Quarter-Plane: Algebraic Methods, Boundary Value Problems and Applications. Springer, 1999.
- [6] L. Flatto, S. Hahn, Two parallel queues created by arrivals with two demands I. SIAM Journal on Applied Mathematics 44 (1984), 1041–1053.
- [7] F. Guillemin, J.S. van Leeuwen, Rare event asymptotics for a random walk in the quarter plane. Queueing Systems 67 (2011), 1–32.
- [8] V. Malyshev, An analytical method in the theory of two-dimensional positive random walks. Siberian Mathematical Journal 13 (1972), 917–929.
- [9] H. Nassar, Two-dimensional queueing model for a LAN gateway. WSEAS Transactions on Communications 5 (2006), 1585–1590.
- [10] H. Nassar, Y. Fouad, Analysis of two-class discrete packet queues with homogenous arrivals and prioritized service. Communications in Information and Systems 3 (2003), 101–117.
- [11] H. Nassar, H.A. Mahdi, Queueing analysis of an ATM multimedia multiplexer with non-pre-emptive priority. IEE Proceedings-Communications 150 (2003), 189–196.
- [12] J. Resing, L.Örmeci, A tandem queueing model with coupled processors. Operations Research Letters 31 (2003), 383–389.

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Present permanent employment: Department of Mathematics, Pedagogical University, Kraków, Poland; position of professor

1983 – Master of Science in Mathematics, Jagiellonian University, Kraków, Poland

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Major research interests: functional equations and inequalities with their applications, Ulam's type stability (e.g., of difference, differential, functional, integral and operator equations), real and functional analysis, fixed point theory.

Author of over 100 papers that are already printed or accepted for publication.

Chairman of the Scientific Committee of the series of conferences: *International Conference on Functional Equations and Inequalities* (ICFEI) (<http://uatacz.up.krakow.pl/icfei/15ICFEI/>)

Chairman of the Organizing Committees of 10th (2005), 11th (2006), 12th (2008), 13th (2009), 14th (2011), 15th (2013), and 16th (2015) ICFEIs (<http://uatacz.up.krakow.pl/icfei/15ICFEI/prev.php>)

Chairman of the Scientific and Organizing Committees of the conference: *Conference on Ulam's Type Stability*, Ustroń (Poland), June 2-6, 2014 (<http://cuts.up.krakow.pl/>)

Member of the Programm or Scientific Committees of several other international conferences

Editor (jointly with Th.M. Rassias) of the monograph *Functional Equations in Mathematical Analysis* (nearly 750 pages; collection of 47 papers of 67 authors), volume 52 (2013) of *Springer Optimization and Its Applications* series, dedicated to the 100th anniversary of S.M. Ulam

Lead Editor of Banach Center Publications volume 99 (2013) titled: *Recent Developments in Functional Equations and Inequalities. Selected Topics*

Lead Guest Editor of Abstract and Applied Analysis annual special issues: *Ulam's Type Stability* (<http://www.hindawi.com/journals/aaa/type.stability/>) in the years 2012, 2013

Lead Guest Editor of Journal of Function Spaces (formerly: Journal of Function Spaces and Applications) special issue: *Ulam's Type Stability and Fixed Points Methods* (<http://www.hindawi.com/journals/jfs/si/329604/cfp/>)

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Supervisor of four promoted PhD students.

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