Theoretical analysis and experimental verification of moment resistance of steel and timber beams strengthened by external CFRP composites

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***Abstract*—**The paper is focused on problems of the application of external reinforcement based on FRP composites for strengthening steel and timber beams to increase their bending moment resistance. Strengthening steel or timber beams using external bonded CFRP composites can appear the advanced way, but the result effect of this reinforcing can be questionable. In this paper the brief information on some results of the research oriented to the investigation of the actual behaviour and resistance of steel and timber beams strengthened by CFRP reinforcement is presented. With respect to practical usage and requirements the attention is paid to the strengthening by externally bonded carbon lamellas. For this problem solution the theoretical analysis and experimental verification mainly are utilized. This paper shows the results and evaluation of experiments realized so far, to verify the actual behaviour and to obtain the objective resistance of reinforced (i.e. externally strengthened by bonded CFRP lamellas) steel and timber beams in comparison with the objective resistance of non-reinforced steel and timber beams obtained from the tests and also in comparison with the predicted resistances, which have been calculated using general principles of the determination of composite cross-section bending resistance, and based on these results to assess the efficiency of CFRP lamellas strengthening. This research is realized on the author's workplace in co-operation with the company of "PREFA KOMPOZITY Inc." focusing on the development and production of the reinforcement based on glass-fibre or carbon-fibre reinforced composites, among others.

***Keywords*—**Theoretical analysis, experimental verification, resistance, steel, timber, beam, strengthening, carbon fibre-reinforced polymer, composites, external reinforcement, plasticity, elasticity.

I. Introduction

THE usage of FRP composite external reinforcement is very often applied for strengthening concrete structures, because of suitable FRP material parameters in comparison of concrete material properties, that concrete structures strengthened by FRP are widely used in practice, mainly in the case of reconstructions caused by the load increasing necessity, for example.

Also in the field of timber structures the methods of strengthening by external bonded reinforcement based on FRP using glass or carbon fibres are applied, frequently in the case of structural members composed of glued lamella wood, to increase the load-carrying capacity, but in the case of timber structural members composed of grown wood the strengthening utilizing FRP is not applied so widely in the practice.

In the field of steel structures the utilization of these strengthening methods is not so usual, mainly because of relatively low CFRP Young's modulus of elasticity in relation to its high tensile strength and compared with corresponding steel parameters.

Nevertheless, recently the problems of timber and steel structural members strengthening are widely discussed because the practice very often requires increase of existing construction resistance.

Within the framework of the solution focused on strengthening steel beams using the external bonded CFRP lamellas, the theoretical and experimental methods are utilized. The first phase of the research was oriented to the elaboration of comparative and parametric studies. The results of these studies have been used for the prediction of the test specimens' cross-sections for the experimental investigation because of their assumed effective actual behaviour and bending moment resistance. For testing the specimens of such geometrical parameters, which respect real possibilities of practical usage (beam cross-sections and spans typical for floor constructions), have been chosen. Cross-sections and material parameters of carbon-fibre-reinforced lamellas have been used in accordance with the usual assortment produced by co-operating company.

II. Generally – Basic Information

Strengthening flexural members using reinforcement based on fibre-reinforced polymers is given by advanced material properties of used fibres. CFRP composites usually use carbon fibres with very high tensile strength and high modulus of elasticity. Generally it is assumed, strengthening by CFRP can

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be more efficient for timber than steel. To compare the results and to evaluate the reinforcing effectiveness, the research is more widely focused on timber beams and steel beams strengthening, too.

To strengthen steel bending structural members, carbon fibres could be efficient because of their very high tensile strength (thousands MPa), even several times higher than steel strength, but their Young's modulus of elasticity is usually less than Young's modulus of steel. Depending on the production technology, if tensile strength is increasing, then modulus of elasticity is decreasing usually, regardless of the high price of high Young's modulus carbon fibres. So that, though carbon fibre tensile strength can be much higher than steel tensile strength, the modulus of elasticity usually does not reach the value of steel modulus of elasticity.

Within the framework of the research oriented to the strengthening steel and timber beams using the external bonded FRP reinforcement, the theoretical and experimental methods of the solution are utilized. The first phase of the research was directed towards the elaboration of comparative and parametric studies. The results of these studies have been used for the prediction of the test specimens' cross-sections available for the experimental investigation of the actual behaviour and bending resistance. For the test specimens such mechanical and geometrical parameters, which respect real possibilities of practical usage, have been chosen; that means the beams cross-sections and spans have been considered as typical for the beams of floor structures. Cross-sections and material parameters of carbon-fibre-reinforced lamellas have been used in accordance with the usual assortment produced by co-operating company.

A. *Theoretical Analysis of Bending Moment Resistance*

To predict the bending moment resistance, both elastic and plastic approaches have been considered for both type of the beams, that means for steel and timber beams, too.

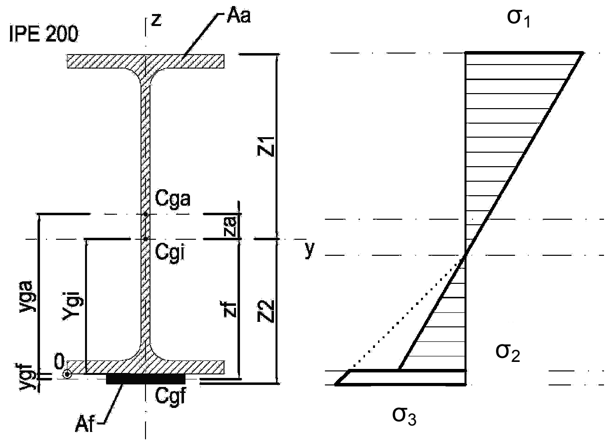

Fig. 1 concept of the moment resistance calculation – stress distribution: elastic principle – $\sigma_3 > \sigma_2$ in the case of $E_f > E_s$ only

For the calculation of the predicted elastic bending moment resistances the concept of the composite cross-section has been applied (see e.g. [1], [2], [24]), that means the substitute cross-section based on the parameter $n = E_s / E_f$, where E_s, E_f are Young's modulus of elasticity of steel beam and CFRP reinforcement, is used. The high strength of CFRP polymer can be effectively utilized if $E_f > E_s$, because in such a case only the stress σ_3 in CFRP is more than the stress σ_2 ($\sigma_3 > \sigma_2$) in the bottom flange of steel beam – see Fig. 1. If Young's modulus of CFRP is less than Young's modulus of steel, the elastic approach does not give moment resistance increasing. Then the plastic approach only is efficient from the viewpoint of the moment resistance increasing – see Fig. 2.

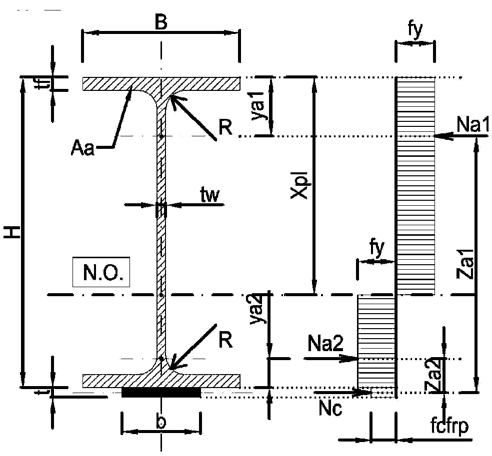

Fig. 2 concept of the moment resistance calculation – stress distribution: plastic principle

In the case of timber beams strengthened by CFRP external reinforcement, the same principles can be used. But here the Young's modulus of elasticity of CFRP is very high compared to Young's modulus of timber, as well as the strength. Then the modulus and strength of CFRP cannot be enough utilized by the elastic behaviour, so that here the elastic approach also does not give the important resistance increasing. The plastic approach only gives the significant increase of the resistance, but tests shown (see below) the actual resistances do not reach by far these values. On that account for the calculation of the timber beams resistance the partial plastic approach neglecting the tensile part of timber beams has been applied (see below).

For the calculation of assumed resistances basic principles mentioned above in accordance with [1], [2], [3], [4], [7] have been used. Practically the bending moment resistance has been calculated using [24], [25], [26].

B. *Experimental Verification of Bending Moment Resistance*

The test specimens (steel and timber ones too) have been simply supported and loaded by forces in the beam thirds.

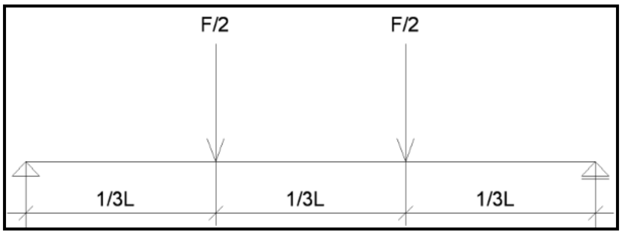

Fig. 1 tested specimen static scheme: four-points bending

The force F applied by hydraulic jack has been introduced to the beam through the stiff steel girder, so that the values of the forces acting in the span thirds were $F/2$. During loading process, the forces F, deflections w at a mid-span and the stresses on the tensile beam edge have been measured through the strain gauges. The same test set-up, in principle, has been used in the case of all specimens.

III. Steel Beams Strengthened by CFRP

Respecting these conditions given above, for the first phase of the experimental verification of CFRP strengthened steel beams test specimens with following material and geometrical parameters have been chosen:

(i) steel beams – for steel beams IPE hot-rolled members with the dimensions of IPE 200, IPE 180, IPE 160, IPE 140, IPE 120 have been used; two beam spans in dependence of cross-section dimensions were used – $L = 4$ m (IPE 200, IPE 180, IPE 160) and $L = 3$ m (IPE 140, IPE 120); steel grade S 235 (actual properties have been measured – see below);

(ii) carbon-fibre-reinforced-polymer composite – the cross-section of CFRP lamellas was 50 x 1.2, tensile strength was of 3 000 MPa, Young's modulus of 155 GPa.

A. *Test Specimens, Test Arrangement, Test Performance*

For the experimental verification the predicted resistance calculation has been performed. At first the preliminary calculation of the elastic resistance moment for strengthened beam of IPE 200 section was made, but for considered steel cross-section and CFRP reinforcement practically no increase of the bending moment resistance occurred. The calculation for other cross-sections (IPE 180, IPE 160, IPE 140, IPE 120) confirmed, that the elastic behaviour gives no significant resistance increasing, hence the plastic behaviour only can give required reserve and significant resistance increasing.

Table I Overview of tested specimens: steel beams

Steel cross-section	IPE without CRP	IPE with CFRP
IPE 200	IPE 200 1 IPE 200 2 IPE 200 3	IPE 200-C 1 IPE 200-C 2 IPE 200-C 3
IPE 180	IPE 180 1	IPE 180-C 1 IPE 180-C 2 IPE 180-C 3
IPE 160	IPE 160 1	IPE 160-C 1 IPE 160-C 2 IPE 160-C 3
IPE 140	IPE 140 1	IPE 140-C 1 IPE 140-C 2 IPE 140-C 3
IPE 120	IPE 120 1	IPE 120-C 1 IPE 120-C 2 IPE 120-C 3

So far 15 steel beams (cross-sections IPE 200, IPE 180, IPE 160, IPE 140, IPE 120 – 3 specimens for each group) strengthened by CFRP lamellas and 7 non-strengthened steel beams (the same cross-sections – 1 specimen for each cross-section) have been tested. The overview of the specimens tested so far is shown in Table I. The actual values of material parameters of steel beams – yield strength and Young's modulus of elasticity (for each section separately) have been measured and are viewed in Table II.

Table II Mechanical properties of steel

Steel cross-section	Yield strength f_y [MPa]	Young's modulus E_s [GPa]
IPE 200	302.0	193.0
IPE 180	315.4	209.4
IPE 160	290.0	202.0
IPE 140	347.5	215.0
IPE 120	344.7	206.2

One of the specimens without CFRP lamella (IPE 200) was loaded very speedily because of the test equipment defect, so that this result is not relevant and was not considered to the evaluation. The behaviour of one of the first test specimens (IPE 200) has been influenced by lateral buckling (see Fig. 8). But this problem has not been solved here and for the next specimen testing the test set-up has been completed adding horizontal supporting to eliminate lateral buckling effect.

Fig. 4 test arrangement and realization – steel beams with CFRP

Fig. 5 test realization – steel beams with CFRP: load introducing to the specimen through the stiff girder

Fig. 6 strain gauges on bottom flange of steel beam and on CFRP

Fig. 7 typical failure of CFRP strenghtened steel beam

Illustrations of the tests arrangement and realization are shown in Figs. 4 and 5, the strain and failure mechanisms of test specimens are seen in Fig. 7, as well as CFRP lamella and strain gauge of the bottom flange in Fig. 6.

B. *Test Results*

Graphs in Figs. 8, 9, 10, 11, 12 present "*M* – *w*" diagrams, i.e. relation between moments $M_{u,exp}$ and deflections w in the mid-span for all 22 tested specimens (without and with CFRP strengthening). For each cross-section always 3 tests of CFRP strengthened beam and 1 test of non-strengthened beam have been performed, for the comparison. In graphs the objective ultimate bending moments $M_{u,exp}$ from the tests in comparison with the moment resistances $M_{u,el}$, $M_{u,pl}$ (for strengthened steel beams) calculated based on the theoretical analysis (for more see above) for measured properties are illustrated.

The overview of the bending moment resistances $M_{u,exp}$ obtained from the tests and their comparison with predicted values $M_{u,el}$, $M_{u,pl}$ calculated based on the elastic and plastic behaviour is shown in Table III. The calculations (see e.g. [8], [10], [14] and below) indicated that plastic bending moment resistances of CFRP strengthened beams in comparison with elastic moment resistances should be about by 35 % (IPE 200) up to 45 % (IPE 120) higher and should have the increasing trend with the cross-section height decrease. However, the tests show that the actual bending moment resistances of strengthened beams practically reach the values of elastic moment resistances or are higher by from 0 % to 15 % usually and have the decreasing trend with the cross-section height decrease. Probably it is caused due to the better activation of CFRP lamellas bonded on the higher beams than lower ones, because of the height and stiffness of the beam.

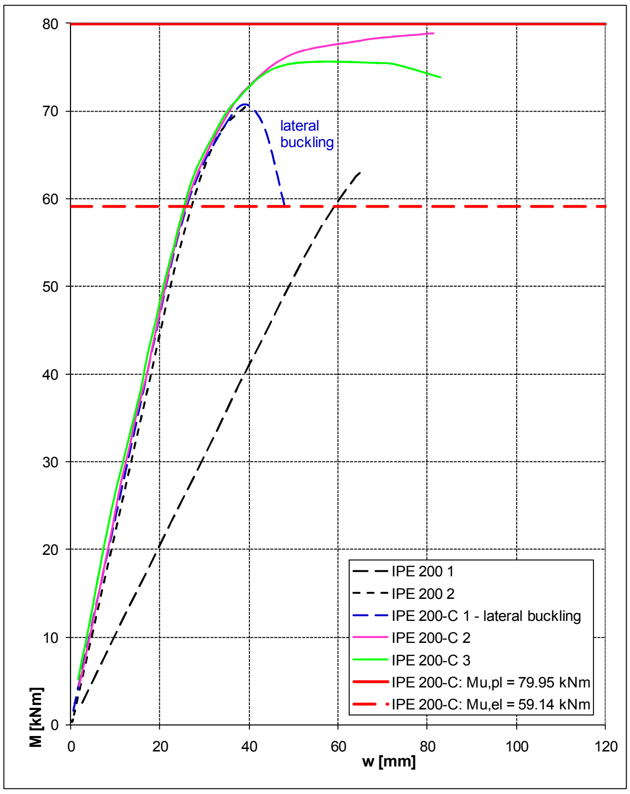

Fig. 8 "$M_{u,exp}$ – w" diagrams for IPE 200: experiment vs. calculation

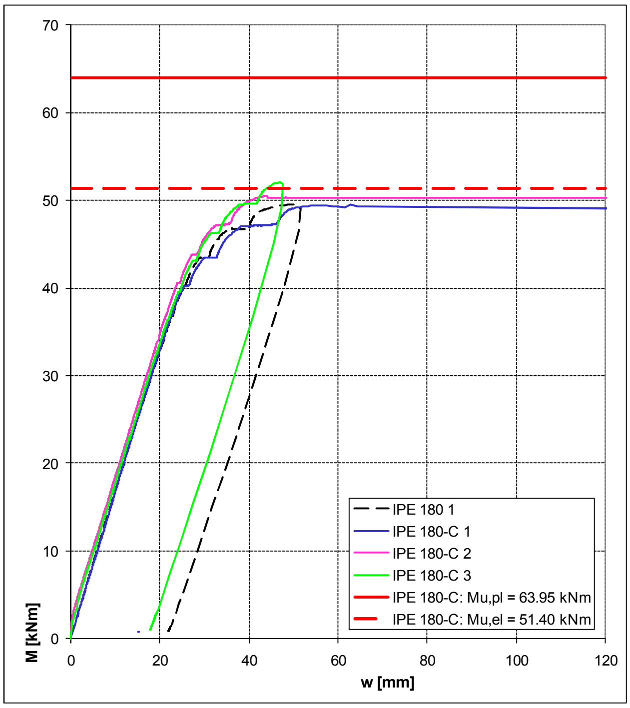

Fig. 9 "$M_{u,exp} - w$" diagrams for IPE 180: experiment vs. calculation

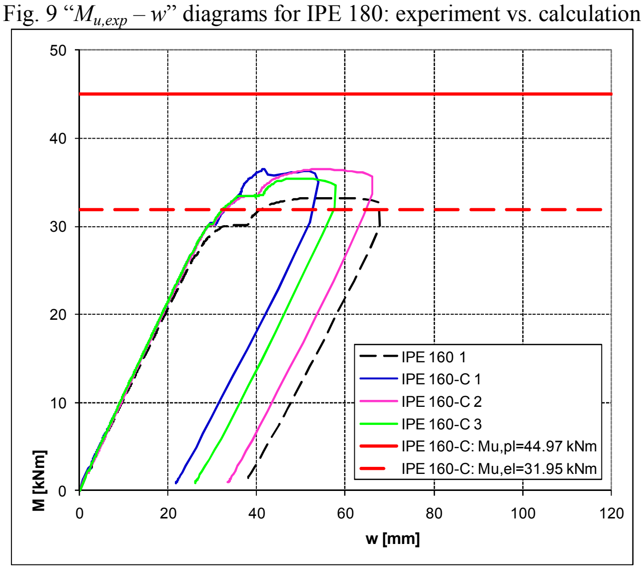

Fig. 10 "$M_{u,exp} - w$" diagrams for IPE 160: experiment vs. calculation

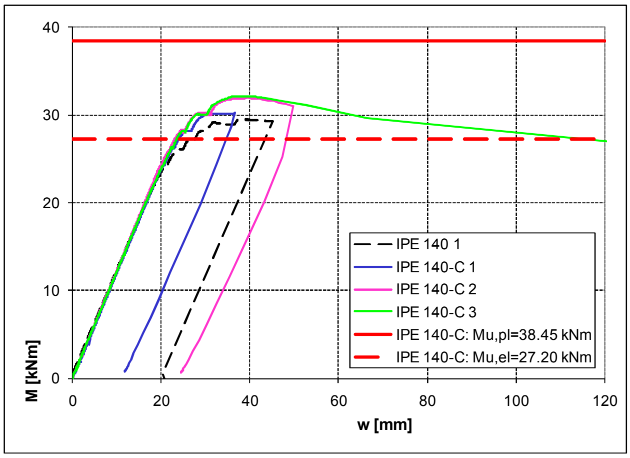

Fig. 11 "$M_{u,exp} - w$" diagrams for IPE 140: experiment vs. calculation

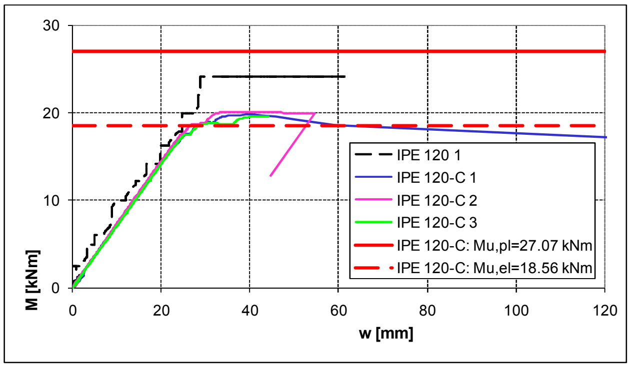

Fig. 12 "$M_{u,exp} - w$" diagrams for IPE 120: experiment vs. calculation

In the case of one cross-section dimension (IPE 180) the resistance increase was about 2.5 % only, but it is probably caused by the particular random behaviour of the specimen, which deviates of to the typical behaviour and cannot be taking into account as representative.

Table III Comparison of the bending moment resistances: steel beams – experiment vs. calculation

Cross section	Bending moment resistances M_u			Difference [%]
steel only	exper.	$M_{u,exp}$ [kNm]		$M_{u,exp\text{-}C}$ to $M_{u,exp}$
steel with CFRP	exper.	$M_{u,exp\text{-}C}$ [kNm]		$M_{u,exp\text{-}C}$ to $M_{u,el}$
	calcul.	$M_{u,el}$	$M_{u,pl}$	$M_{u,exp\text{-}C}$ to $M_{u,pl}$
IPE 200	exper.	63.12, 73.33, 76.56		+8.5
IPE 200-C	exper.	70.84, 82.28, 77.88		+30.2
	calcul.	59.14	79.95	-4.2
IPE 180	exper.	49.49		+2.5
IPE 180-C	exper.	49.53, 50.51, 52.07		-1.4
	calcul.	51.40	63.95	-20.7
IPE 160	exper.	33.24		+8.7
IPE 160-C	exper.	36.48, 36.52, 35.43		+13.1
	calcul.	31.95	44.97	-19.6

IPE 140	exper.	29.56		+6.3
IPE 140-C	exper.	30.21, 31.93, 32.15		+15.6
	calcul.	27.20	38.45	-18.3
IPE 120	exper.	18.74		+5.9
IPE 120-C	exper.	19.80, 20.11, 19.63		+5.4
	calcul.	18.56	27.07	-26.7

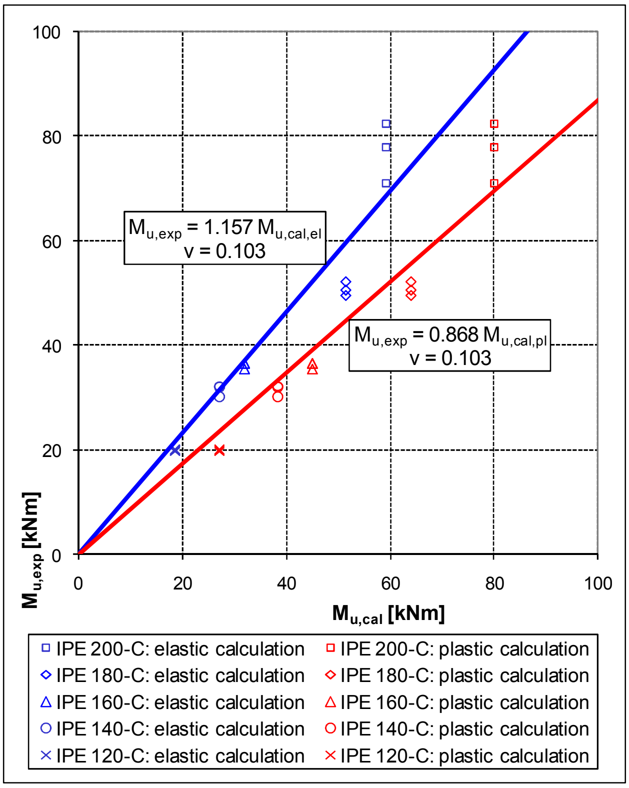

Fig. 13 comparison of experimental and theoretical moment resistances for steel beams: elastic and plastic calculation

C. Bending Moment Resistance Evaluation

Test results of the tests of CFRP reinforced steel beams have been evaluated with regards to the approaches of the moment resistance calculation. Experimental resistances and theoretical resistances based on elastic and plastic behaviour have been compared with respect to the suitability of the calculation method and variability of the differences between test and calculated values. The comparison of the resistances is expressed in Fig. 13 showing the relation of experimental to theoretical resistances $M_{u,exp}$ vs. $M_{u,cal}$, including variation coefficients of the ratios of $M_{u,exp} / M_{u,cal}$.

IV. Timber Beams Strengthened by CFRP

For the first phase of the experimental verification of CFRP strengthened timber beams test specimens with following material and geometrical parameters have been chosen:

(i) timber beams – members of rectangular cross-sections of 100/220, 100/200, 100/180, 100/160, 100/140, 100/120 have been used; two beam spans in dependence of cross-section dimensions were used – $L = 4$ m (100/220, 100/200, 100/180) and $L = 3$ m (100/160, 100/140, 100/120); the grade of used wood was declared as the class of C 24 (characteristic strength of 24 MPa); the quality of timber has been verified by material tests, that characteristic strength 24 MPa has been confirmed.

(ii) carbon-fibre-reinforced-polymer composite – the same CFRP lamellas as for steel beams, of cross-section 50 x 1.2 with tensile strength of 3 000 MPa and Young's modulus of 155 GPa have been used.

A. Test Specimens, Test Arrangement, Test Performance

Predicted bending moment resistances have been calculated using EN 1994-1-1 [24] and EN 1995-1-1 [25]. Because of the actual behaviour of timber members, besides elastic and full-plastic calculations the plastic calculation with neglected tensile timber in cross-section has been used, too (see below).

Table IV Overview of tested specimens: timber beams

Timber cross-section	Timber without CRP	Timber with CFRP
100/220	100/220 1 100/220 2 100/220 3	100/220-C 1 100/220-C 2 100/220-C 3
100/200	100/200 1 100/200 2 100/200 3	100/200-C 1 100/200-C 2 100/200-C 3
100/180	100/180 1 100/180 2 100/180 3	100/180-C 1 100/180-C 2 100/180-C 3
100/160	100/160 1 100/160 2 100/160 3	100/160-C 1 100/160-C 2 100/160-C 3
100/140	100/140 1 100/140 2 100/140 3	100/140-C 1 100/140-C 2 100/140-C 3
100/120	100/120 1 100/120 2 100/120 3	100/120-C 1 100/120-C 2 100/120-C 3

So far 18 CFRP-strengthened timber beams (cross-sections 100/220, 100/200, 100/180, 100/160, 100/140, 100/120 – 3 specimens for each group) and 18 non-strengthened timber beams (the same cross-sections – 3 specimen for each cross-section) have been tested. The overview of the specimens tested so far is shown in Table IV.

Illustration of the test arrangement and realization is shown in Fig. 14, the examples of strain and failure mechanisms of test specimens are seen in Figs. 16 and 17, as well as CFRP lamella and strain gauges of the bottom edge in Fig. 15.

Fig. 14 test arrangement and realization – timber beams with CFRP

Fig. 15 strain gauges on bottom edge of timber beam and on CFRP

Fig. 16 failure of CFRP strengthened timber beam – lamella rupture

Fig. 17 failure of CFRP strengthened timber beam – timber failure

B. *Test Results*

Graphs in Figs. 18, 19, 20, 21, 22 show "$M - w$" diagrams, i.e. moment $M_{u,exp}$ and deflections w in the mid-span for all 36 tested specimens without and with CFRP strengthening. For each cross-section always 3 tests of CFRP strengthened beam and 3 tests of non-strengthened beam have been performed. In the graphs objective ultimate bending moments $M_{u,exp}$ obtained from the tests in comparison with moment resistances $M_{u,el}$, $M_{u,pl}$ (for strengthened timber beams) calculated based on the theoretical analysis (for more see above) for characteristic values of material properties are illustrated.

The overview of the bending moment resistances $M_{u,exp}$ obtained from the tests and their comparison with predicted values $M_{u,el}$, $M_{u,pl}$ calculated based on the elastic and plastic behaviour is shown in Table V.

The comparison of the resistances obtained from the tests with assumed resistances calculated using elastic and plastic concepts (seeable from Figs. 18 to 23 or from Table V) gives the following information: the elastic moment resistances $M_{u,el}$ determined using substitute cross-section based on the elastic behaviour assumption are importantly less than the objective ultimate resistances obtained from the tests (about 60 % of test values, in average). On the other hand, the plastic moment resistances $M_{u,pl,\mathrm{II}}$ determined using plastic stress distribution in all cross-section parts (CFRP, compression and tension timber, too) based on full-plastic behaviour assumption are significantly more than the ultimate resistances obtained from the tests (about by 40 % higher than test values, in average).

Arising from the facts described above, the plastic moment resistance $M_{u,pl,\mathrm{I}}$ determined using plastic stress distribution in CFRP and compression timber only, that means taking into account tension CFRP and compression part of timber and neglecting tension part of timber, seems to be the most exact for the expression of the actual bending moment resistance.

C. *Bending Moment Resistance Evaluation*

Test results of the tests of CFRP reinforced timber beams have been evaluated with regards to the approaches of the moment resistance calculation. Experimental resistances $M_{u,exp}$

and theoretical resistances $M_{u,el}$, $M_{u,pl,\mathrm{I}}$ have been compared with respect to the suitability of the calculation method and variability of the differences between test and theoretical values. The comparison of the resistances is shown in Fig. 24 by the relation of experimental to theoretical resistances $M_{u,exp}$ vs. $M_{u,el}$, $M_{u,pl,\mathrm{I}}$ including variation coefficients of the ratios of experimental to calculated resistances.

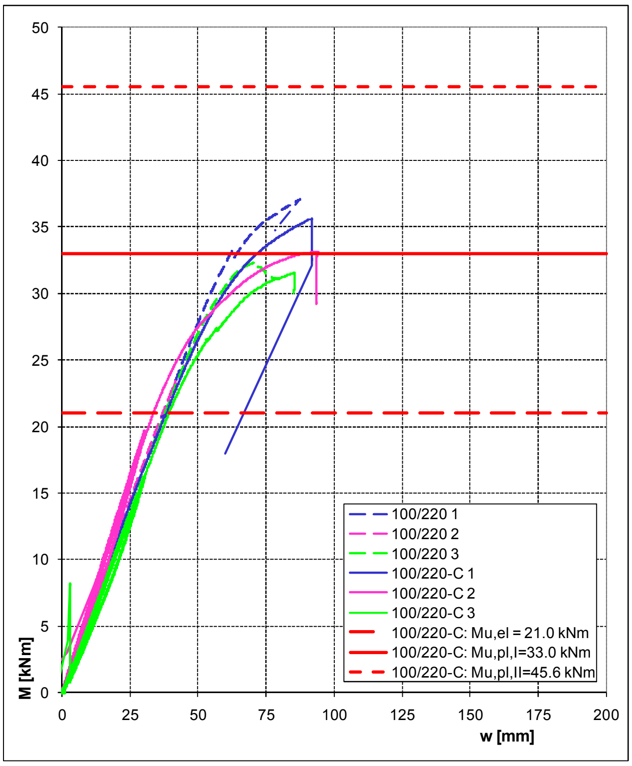

Fig. 18 “$M_{u,exp} - w$” diagrams for 100/220: experiment vs. calculation

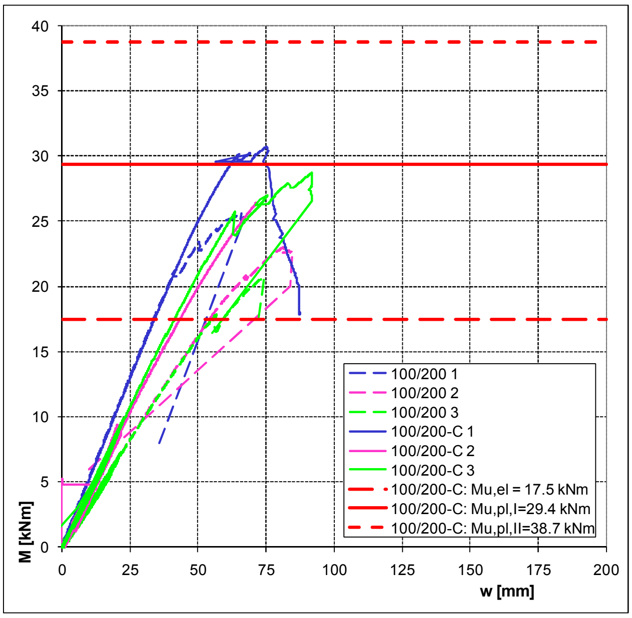

Fig. 19 “$M_{u,exp} - w$” diagrams for 100/200: experiment vs. calculation

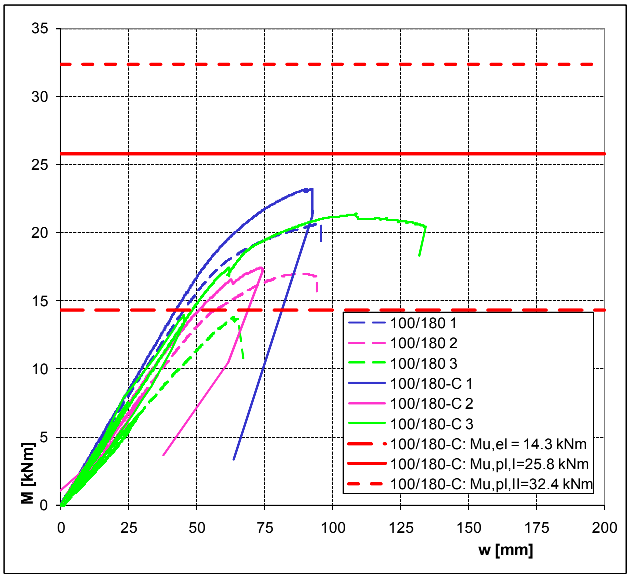

Fig. 20 “$M_{u,exp} - w$” diagrams for 100/180: experiment vs. calculation

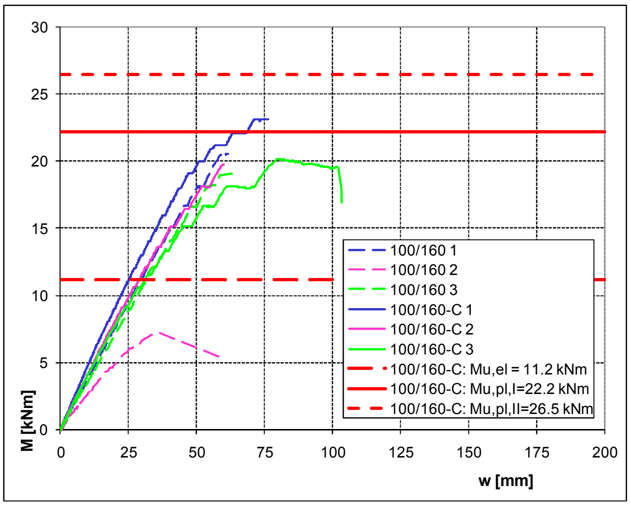

Fig. 21 “$M_{u,exp} - w$” diagrams for 100/160: experiment vs. calculation

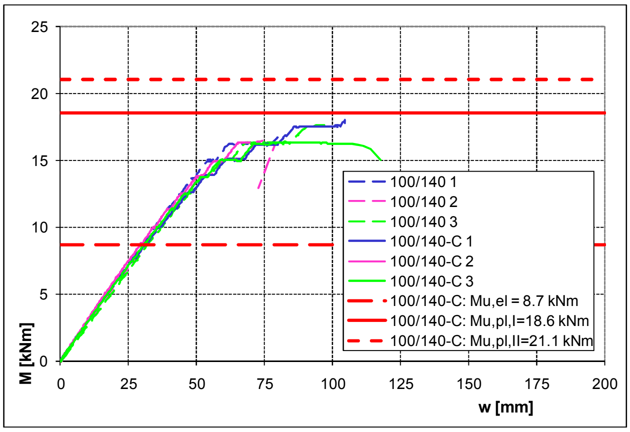

Fig. 22 “$M_{u,exp} - w$” diagrams for 100/140: experiment vs. calculation

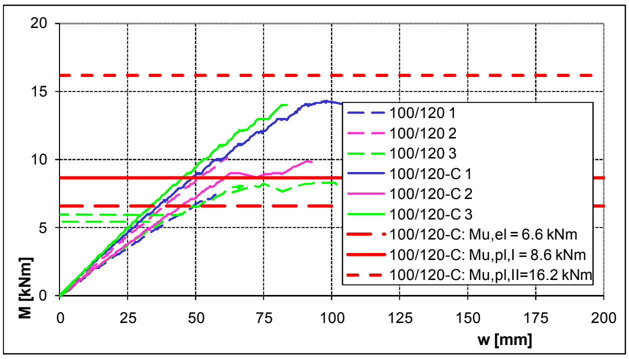

Fig. 23 "$M_{u,exp} - w$" diagrams for 100/120: experiment vs. calculation

Table V Comparison of the bending moment resistances: timber beams – experiment vs. calculation

Cross section	Bending moment resistances M_u			Difference [%]
timber only	exper.	$M_{u,exp}$ [kNm]		$M_{u,exp\text{-}C}$ to $M_{u,exp}$
timber with CFRP	exper.	$M_{u,exp\text{-}C}$ [kNm]		$M_{u,exp\text{-}C}$ to $M_{u,el}$
	calcul.	$M_{u,el}$	$M_{u,pl,I}$	$M_{u,exp\text{-}C}$ to $M_{u,pl,I}$
100/220	exper.	37.08, 31.27, 32.32		+0.5
100/220-C	exper.	36.44, 33.14, 31.54		+60.5
	calcul.	21.00	33.00	+2.2
100/200	exper.	25.64, 23.04, 20.75		+24.0
100/200-C	exper.	30.71, 26.64, 28.73		+63.9
	calcul.	17.50	29.40	-2.4
100/180	exper.	20.67, 16.96, 20.69		+8.8
100/180-C	exper.	24.69, 17.42, 21.37		+48.0
	calcul.	14.30	25.80	-18.0
100/160	exper.	20.54, 7.21, 19.09		+34.6 (+6.0)
100/160-C	exper.	23.13, 19.76, 20.13		+87.6
	calcul.	11.20	22.20	-5.4
100/140	exper.	16.79, 16.47, 16.65		+1.6
100/140-C	exper.	18.00, 16.35, 16.37		94.4
	calcul.	8.70	18.60	-9.1
100/120	exper.	7.37, 10.13, 8.30		+48.1
100/120-C	exper.	14.32, 9.88, 14.02		+93.0
	calcul.	6.60	8.60	+48.1

The evaluation of the bending moment resistance in the meaning Figs. 13 and 24 is based on the comparison of test results and calculated assumed resistances. It is aimed to obtain the suitable procedure for the determination of the resistance, which corresponds with the actual behaviour and gives the most exact results as possible. Mainly the variation coefficients of experimental to calculated resistance ratios are taken as the indicators of the calculation method exactness and thus of the suitability of the applied beam behaviour approach.

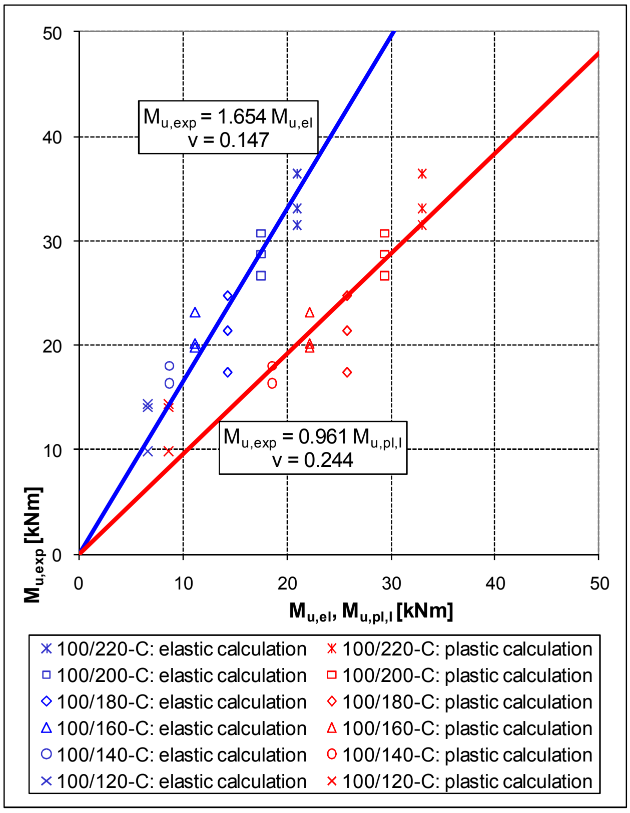

Fig. 24 comparison of experimental and theoretical moment resistances for timber beams: elastic and plastic calculation

V. Conclusions

The bending moment resistances verified experimentally unfortunately did not confirm assumed (calculated) resistance increasing, neither in the case of steel beams, nor in the case of timber beams not quite. Contrary to calculation results, the actual resistances are smaller than predicted ones.

Based on the experimental results and verification of the theoretical analysis some particular concluding remarks can be formulated:

(i) Steel beams strengthened by CFRP lamellas:

- The actual bending moment resistance of the strengthened steel beam obtained from the tests is higher than the elastic moment resistance determined by the calculation with measured material properties, but not significantly.
- The actual moment resistance obtained from the tests does not reach the plastic moment resistance calculated with actual material properties.
- According to the test results the moment resistance determined by elastic calculation is more exact; but according to variation coefficients, which are the same for elastic and plastic calculation (both ones are v = 0.103), plastic calculation is probably useable for the resistance determination, too.
- The effect of CFRP strengthening to the steel beams resistance increase is not very significant if used the lamellas of given geometrical and material parameters

(with relatively low Young's modulus).

- To increase the moment resistance more significantly, it will be necessary to apply CFRP reinforcement of the different material properties that means with higher Young's modulus mainly.
- It is a question of the justified usage of the plastic calculation; according to Fig. 13 formulas for both elastic and plastic approach correspond with the actual resistances – the variation coefficients in both cases are the same, but it can be given randomly due to the small test number.

(ii) Timber beams strengthened by CFRP lamellas:

- The actual bending moment resistance of the strengthened timber beam obtained from the tests is significantly higher than the elastic moment resistance determined by the calculation with measured material properties.
- The actual moment resistance obtained from the tests so far does not reach the calculated full-plastic moment resistance.
- The moment resistance determined by plastic calculation not taking into account tension timber is more exact; but according to the variation coefficient (v = 0.244), which is higher than ones for elastic calculation (v = 0.147), elastic calculation is probably more exact for the resistance determination.
- The effect of CFRP strengthening to the timber beams resistance increase is significant related to the elastic behaviour only, if used the lamellas of given geometrical and material parameters; in this case probably the usage of glass-fibre-reinforced polymers (GFRP) could be more effective because of the more suitable material parameters and thus their more efficient utilization.
- From the viewpoint of the cross-section behaviour it is necessary in both cases (steel and timber beams, too), using strain gauges outputs, to investigate the stresses in steel or timber cross-section and CFRP reinforcement to verify the stress introduction in the cross-section and the interaction between both section parts; the evaluation of the stresses was not performed in detail so far, it is under evaluation recently.

In connection with the conclusions the research oriented to the usage of composite materials of various structure and configuration – except of FRP composites [8], [10], [13] also fibre-reinforced concrete [13], [14], fibre-cement composites [9], [14] and other high-performance materials [11], [16], [17], [20], as well as the experimental and theoretical analysis of the resistance [9], [11], [12], [13], [15], [18], [19], [21] – is continued, to investigate the positive effect to the increase of the resistance of the usual materials structural members.

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