

# Phase Diversity Method in Mobile Communication Systems

Michael Bank, Y. Barish, S. Tapuchi, Miriam Bank, B. Hill, M. Haridim

**Abstract**—Phase diversity techniques are used in many communication systems, including PAL TV system, FBS method, and the Alamouty algorithm for MIMO. Phase diversity algorithms have proved effective in mitigating phase distortion in wireless channels. In this article we present a new method of phase diversity processing that is based on phase arithmetical summing.

**Keywords**—FBS, MIMO, Doppler, OFDMA. PALTV, Phase summing

## I. INTRODUCTION

Phase Diversity is a well-known technique for improving the performance and reliability of wireless communications over fading channels. In this approach, signals are transmitted through multiple channels, each having an independent fading, and coherently combine them at the receiver. Different types of diversity (frequency, time, space, phase) are widely used today in modern communication systems.

Generally, it is known that repeating the information bits is an effective means for reducing the error rate. But, diversity is not a simple repeating of the signal. In this case, signals are transmitted with different parameters and/or different conditions. For example, in case of frequency diversity the signal is transmitted twice or more but in different frequency bands. The desired signals are summed using an arithmetical method, whereas noise components are summed using an RMS method. As a result the signal to noise ratio is improved (similar to spread spectrum systems). In the same manner, in space diversity, information signals are summed using an arithmetical method. However, independent fading is only partially compensated. Signal to noise ratio may be improved by means of patch diversity and Rake receiver, where the latter is synchronized with each patch separately. In this method, as in above mentioned methods, the receiver picks up some identical signals with different noise levels.

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This paper is dedicated to phase diversity methods, for example in PAL TV, OFDMA and MIMO systems [1 - 7].

## II. PHASE DIVERSITY IN PAL TV SYSTEM

The influence of the channel impairments including Doppler Effects [3 – 5], can be significantly reduced by transmitting each symbol twice; first the original symbol, and then its phase-conjugate symbol. In the receiver, the phase-conjugate symbol is first returned to its original phase and then added to the original signal. In this manner, any phase distortion introduced by the channel is cancelled out. This phase diversity method was first proposed by Walter Bruch for PAL TV systems [6].

Fig. 1 shows the phase shift cancellation in PAL TV. Let's assume that the transmitted signal has a phase  $\phi$  and the channel shifts the signal phase by an unknown amount of  $\Delta$ , such that the received signal's phase is  $\phi + \Delta$ .

If, as mentioned above, we transmit the signal twice, first with  $\phi$  and then with  $-\phi$ , the received phases will be  $\phi + \Delta$  and  $-\phi + \Delta$ , respectively. The receiver reverses the phase of the second signal, such that upon adding the phases of the two signals, the phase shift  $\Delta$  cancels out.

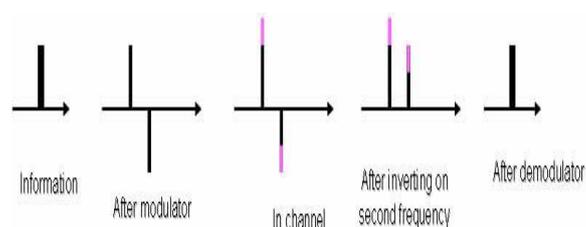


Fig.1- Phase shift compensation in PAL TV system

This method significantly improved the quality of TV signals reception. This phase diversity method has been applied and in other systems too, although not always have a reference to the PAL TV.

This is phase diversity method. The phase is transmitting two times with opposite value (+ and -). This allows improving receiving quality.

### III. THE EFFECT OF DOPPLER SHIFT ON THE OFDMA SIGNALS

In a multi-path channel the duration of a received symbol is frequency changes by the same factor  $k_d$  the number of carrier cycles in each symbol does not change, regardless of how fast the Transmitters (Tx) or Receivers (Rx) move. So in case of Doppler Shift, the frequency  $f$  and symbol time  $T$  are varied by a factor of  $k_d$  given by [3]

$$k_d = 1 + \frac{V}{c} \cos \varphi, \quad (1)$$

where  $V$  is the moving object velocity,  $c$  is the light velocity and  $\varphi$  is the angle between the directions of signal propagation and the moving objects. Note that under these circumstances, the orthogonal condition prevails:

$$[(f_{i+1} - f_i) \cdot k_d] \cdot [T / k_d] = 1$$

In real situations, however, the orthogonal property of the subcarriers will be violated to some extent. Since the receiver's synchronization system cannot react instantly to changes in the symbol period or in the FFT parameters, we may assume that the Rx symbol time set by the symbol synchronization system remains at a fixed value. A Doppler shift of  $\Delta\omega$  introduced an a phase shift of  $\Delta\omega t$  in the received signal, and an additional phase shift due to incompatibility between carrier frequencies and the spectral components after FFT,

Further influences include ICI (Inter-carrier interferences) due to partial loss of the orthogonality, changes of the pilot signals parameters, and a time delay (or forestalling)

subject to changes due to the variations in the length of the signal propagation path. The change of the signal duration will be denoted hereafter by factor  $k_d$ . However, since the

between adjacent symbols due to incompatibility between of the received symbol duration and Rx clock synchronization system [3].

Fig. 2 depicts an example of a typical HAP system using the OFDMA technology system [9] with symbol duration 0.1ms, frequency difference between carriers ( $\Delta F$ ) 10kHz, central frequency in the 2GHz L band range and from 5.85 to 7.075 GHz allocated by the ITU for the future HAPS. These higher frequency bands are required especially for broadband mobile radio, vehicles at speed of up to 300 km/h, and even more for aircrafts. The angle shift change as function of the distance between the HAP and the vehicle. ?

Using equation (1) the Doppler shift is estimated as  $\pm 2.8$  kHz or 30% of  $\Delta F$ . It should be noted that also the reflected signals undergo a Doppler shift.

To mitigate the ICE effects of a randomly changing Doppler frequency shift, OFDMA uses pilot signals, which leads to a significant increase in power, and system redundancy [8,9]. The HAP systems, on the other hand, are characterized by the favorable Rician distribution as the higher altitude and the resulting transmission angles produce a line of sight. This leads to a lower level of fading and Doppler effects. In addition, the higher operation ranges of a few hundred km results in a significant reduction of hand over steps. For similar scenarios HAPS will be more advantageous than Satellites due to their significantly less dispersion losses and lower cost.

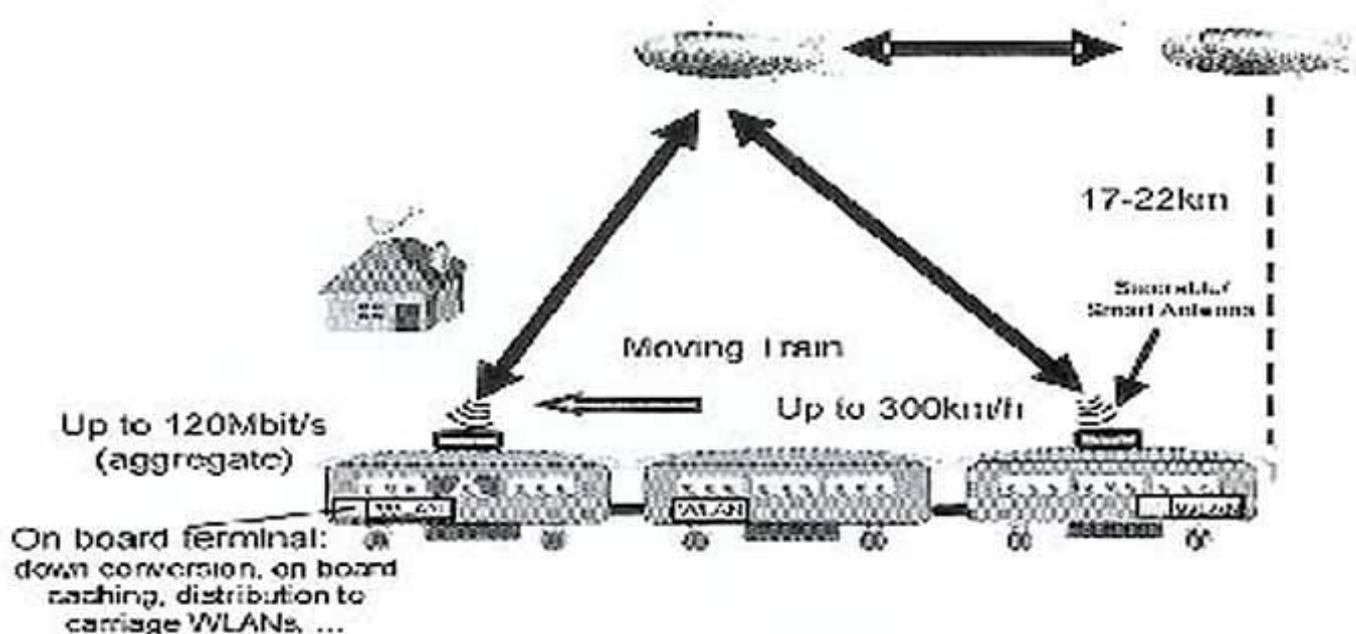


Fig. 2- Scenario of the radio communication link between HAP and a mobile high speed train

We now proceed with the mathematical description of the OFDMA signal variations due to Doppler shift. The discrete Fourier transform (DFT) of  $\{x_j\}_{j=0}^{N-1}$  is given by

$$X_k = \sum_{j=0}^{N-1} x_j e^{-i\frac{2\pi}{N}kj}, \quad k = 0, \dots, N-1 \quad (2),$$

and the inverse discrete Fourier transform is given by

$$x_j = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i\frac{2\pi}{N}kj}, \quad j = 0, \dots, N-1 \quad (3).$$

Here  $x_j$ ,  $j = 0, \dots, N-1$  are  $N$  samples in time domain, and  $X_k$ ,  $k = 0, \dots, N-1$  are  $N$  spectral components. Defining:  $t_j = \frac{T}{N}j$ ,  $\omega_k = \frac{2\pi}{T}k$ , Eq.s (2)

and (3) can be rewritten as

$$X_k = X(\omega_k) = \sum_{j=0}^{N-1} x(t_j) e^{-i\omega_k t_j}, \quad (4)$$

$$k = 0, \dots, N-1$$

$$x_j = x(t_j) = \frac{1}{N} \sum_{k=0}^{N-1} X(\omega_k) e^{i\omega_k t_j}, \quad (5).$$

$$j = 0, \dots, N-1$$

In general,  $X_k$  is a complex number whose absolute value  $|X_k|$  and angle  $\angle X_k$  are the amplitude and phase of the  $k^{\text{th}}$  frequency component. Since  $\{x_j\}_{j=0}^{N-1}$  is a real signal, its

DFT is symmetric, i.e.  $X_{N-k} = X_k^*$  (where  $X_k^*$  denotes the complex conjugate of  $X_k$ ).

As stated above, the Doppler shift leads to spectrum variations [8, 10].

Due to the Doppler frequency shifts, the frequency of each component is shifted to  $\tilde{\omega}_k = \omega_k + \Delta\omega_k$ ,

where  $\Delta\omega_k = \frac{2\pi}{T}\delta_k$  and  $\delta_k$  denotes the relative shift of the  $k^{\text{th}}$  component. In the time domain the new signal is

$$\begin{aligned} \tilde{x}_j &= \tilde{x}(t_j) = \frac{1}{N} \sum_{k=0}^{N-1} X(\omega_k) e^{i\tilde{\omega}_k t_j} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} X(\omega_k) e^{i(\omega_k + \Delta\omega_k) t_j} \\ &= \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i\frac{2\pi}{N}(k + \delta_k)j} \\ &= \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{i\frac{2\pi}{N}(m + \delta_m)j} \end{aligned} \quad (6)$$

Note that since  $\{\tilde{x}_j\}_{j=0}^{N-1}$  is a real signal,  $\Delta\omega_{N-k} = -\Delta\omega_k$ , i.e.  $\delta_{N-k} = -\delta_k$ .

The spectrum of the new signal is

$$\begin{aligned} \tilde{X}_k &= \sum_{j=0}^{N-1} \tilde{x}_j e^{-\frac{2\pi i}{N}kj} \\ &= \sum_{j=0}^{N-1} \left( \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{i\frac{2\pi}{N}(m + \delta_m)j} \right) e^{-\frac{2\pi i}{N}kj} \\ &= \sum_{m=0}^{N-1} X_m \sum_{j=0}^{N-1} \frac{1}{N} e^{i\frac{2\pi}{N}(m + \delta_m)j} e^{-\frac{2\pi i}{N}kn} \\ &= \sum_{m=0}^{N-1} X_m \frac{1}{N} \sum_{j=0}^{N-1} e^{i\frac{2\pi}{N}(m + \delta_m - k)j} \end{aligned}$$

Therefore

$$\tilde{X}_k = \sum_{m=0}^{N-1} a_{km}(\delta_m) X_m \quad (7),$$

where

$$a_{km}(\delta_m) = \frac{1}{N} \sum_{j=0}^{N-1} e^{i\frac{2\pi}{N}(m + \delta_m - k)j} \quad (8)$$

In other words, we can obtain the new spectrum vector  $\tilde{X}$  of length  $N$  by multiplying the  $N \times N$  matrix  $a$  by the original spectrum vector  $X$  of length  $N$ . Matrix element  $a_{km}(\delta_m)$  shows how the original  $m^{\text{th}}$  spectral component affects the new  $k^{\text{th}}$  spectral component. The matrix elements can be computed using Eq. (8).

If  $\delta_m = 0$ , then

$$a_{km}(0) = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases} \quad (9.1).$$

For small  $\delta_m \neq 0$  ( $0 < |\delta_m| < 1$ ), using

$$\sum_{j=0}^{N-1} q^j = \frac{1-q^N}{1-q}, \quad q \neq 1,$$

for  $q = e^{i\frac{2\pi}{N}(m+\delta_m-k)}$ , we obtain

$$\begin{aligned} a_{k,m}(\delta_m) &= \frac{1}{N} \sum_{j=0}^{N-1} e^{i\frac{2\pi}{N}(m+\delta_m-k)j} \\ &= \frac{1}{N} \cdot \frac{1 - e^{i2\pi(m+\delta_m-k)}}{1 - e^{i\frac{2\pi}{N}(m+\delta_m-k)}} \\ &= \frac{1}{N} \cdot \frac{1 - e^{i2\pi\delta_m}}{1 - e^{i\frac{2\pi}{N}(m+\delta_m-k)}} \\ &= \frac{1}{N} \cdot \frac{1 - e^{i2\pi(m+\delta_m-k)}}{1 - e^{i\frac{2\pi}{N}(m+\delta_m-k)}} \\ &= \frac{1}{N} \cdot \frac{1 - e^{i2\pi\delta_m}}{1 - e^{i\frac{2\pi}{N}(m+\delta_m-k)}} \end{aligned} \quad (9.2)$$

$\{a_{k,m}(\delta_j)\}_{k,m=0}^{N-1}$  is a  $N \times N$  matrix whose entries are

complex numbers depending on  $\{\delta_m\}_{m=0}^{N-1}$ . If  $\delta_m = 0$  for all  $m = 0, \dots, N-1$ , then (9.1) implies that

$\{a_{k,m}(\delta_m)\}_{k,m=0}^{N-1}$  is the identity  $N \times N$  matrix and in this

case  $\tilde{X}_k = X_k$  for all  $k = 0, \dots, N-1$ . In other words, if there are no frequency shifts, then the spectrum is not changed, as expected [8].

The relation  $\delta_{N-k} = -\delta_k$  implies

$$a_{N-k, N-m}(\delta_{N-m}) = (a_{k,m}(\delta_m))^*.$$

Note that the spectral components affected by the Doppler effect can be computed by Eq.s (7), (8). Remarkably, these numerical simulation and calculations do not require FFT size increasing. We believe that this can be a significant contribution to simulation design.

#### IV. FBS METHOD

The FBS method is based on a combination of the PAL phase compensation method with the Walsh functions [3]. Walsh functions are orthogonal sequences. As an example, Table 1, shows the fourth order Walsh function (Walsh Hadamard Matrix - WHM).

Table 1- Fourth order WHM

1	1	1	1
1	1	-1	-1
1	-1	1	-1
1	-1	-1	1

WHM is used in CDMA systems for transmitting different signals over the same frequency band. We'll illustrate the operation of the FBS method using a simple example of an OFDMA signal with MPSK modulation. We take four subcarriers with phases  $\varphi_1 \varphi_2 \varphi_3 \varphi_4$  (see Fig. 3)

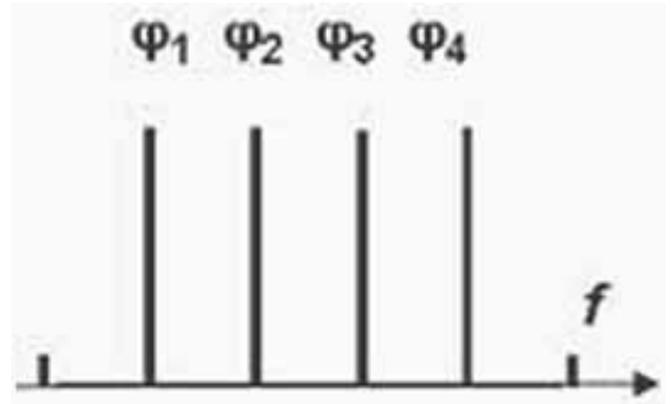


Fig. 3 The spectrum of OFDMA with four carriers

According to the FBS method, each signal, e.g. the signal with phase  $\varphi_1$ , is transmitted four times on all four subcarriers, where the phases of the signals are modified according to the WHM of Table 1. Fig. 4 shows the phases of the transmitted signal.

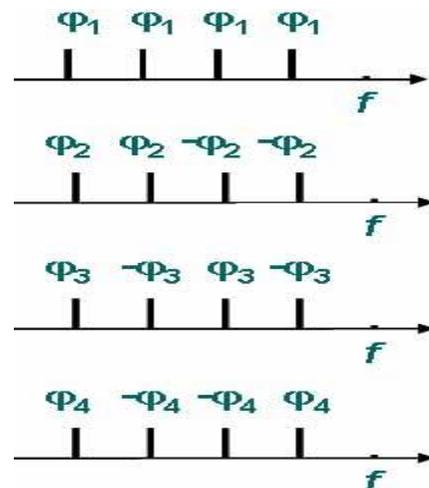


Fig. 4- FBS with four carrier spectrum

For transmitting N signals on N carriers, the FBS signals are:

$$S_{(kl),FBS-1} = E_l \sum_{k=0}^{N-1} e^{j[2\pi f_k t + (-1)^{W_{kl}} (\theta_l + \beta_l)]}, \quad (10)$$

where

$E_l$  - component magnitude,  $l = 0, \dots, N-1$  or Walsh-Hadamard matrix lines

$\theta_l$  - initial phase, chosen for a certain signal. For example, it is either  $45^0$  or other

$\beta_l$  - information symbol of the  $l^{\text{th}}$  FBS signal (BPSK or QPSK representation),

$f_k = f_0 + k\Delta f$  - FBS carrier frequencies,  $k = 0, \dots, N-1$  or Walsh-Hadamard matrix columns

$W_{kl}$  - sequence of phases of the  $l^{\text{th}}$  FBS carrier pattern.

To receive one of these signals, for example signal number 2, the following algorithm must be implemented: the  $S_2$  receiver receives all signals together, makes FFT, obtains four spectral components (amplitudes and phases), changes the phase signs of components 3 and 4 (corresponding to the third signal).

The main advantage of the FBS is its ability to compensate fast phase changes in fading channels and Doppler shift influence without using any pilot signals [3, 4].

## V. ALAMOUTI MIMO ALGORITHM

MIMO systems usually refer to a patch diversity system. In these systems the signals on both the transmitting and the receiving antennas are combined in such a way that the probability of deep fading becomes smaller, but not zero. In the case of a 2X2 MIMO system, for example, such situation exists if the two transmitted signals have the same amplitudes and opposite phases.

second row of the Walsh-Hadamard matrix in Table 1.). The result will be equal to the phase sum divided by four. The phase sum of the other 3 signals should be equal to zero. We calculate arithmetical (not vector) sum  $\varphi_k$  of all signals on each carrier and use amplitude corresponding to: Fig. 6 illustrates the main advantage of using the FBS-1 method.

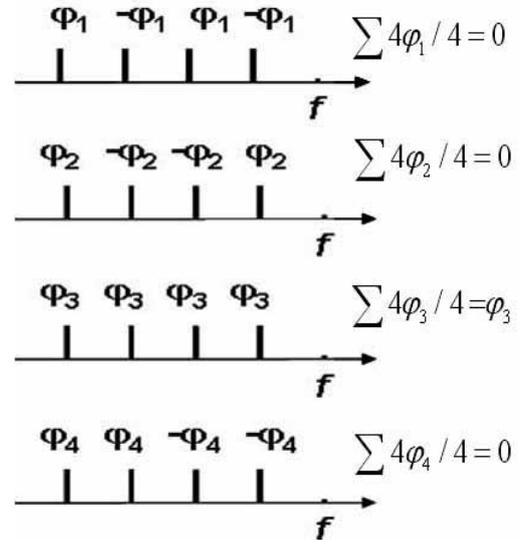


Fig. 5- FBS signal after phase changing for receiving the

The well-known Alamouti algorithm allows eliminating these cases by transmitting each symbol twice, namely the original signal and its phase conjugate [7]. Actually, this algorithm is a kind of phase diversity scheme.

In Alamouti algorithm, the symbol and its phase conjugate are transmitted. For example, signals  $S_0$  and  $S_1$  are transmitted on two different antennas,  $A_0$  and  $A_1$ , during two "tact's".  $A_0$  transmits  $S_0$  and  $A_1$  transmits  $S_1$  during first the "tact" (see Fig. 6).

During the second "tact"  $A_0$  transmits  $-S_1^*$  and  $A_1$  transmits  $S_0^*$ . Then  $A_0$  transmits  $S_3$  and  $A_1$  transmits  $S_4$  during the 3rd "tact". During the 4th tact  $A_0$  transmits  $-S_3^*$  and  $A_1$  transmits  $S_4^*$  (see Fig. 7)

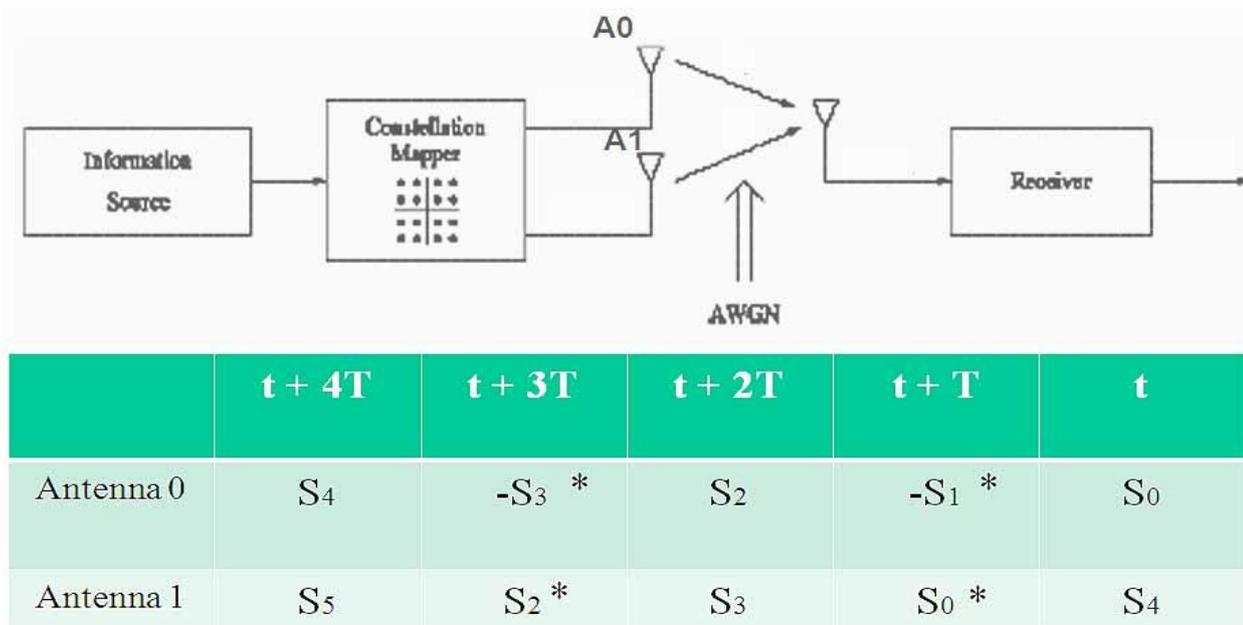


Fig. 6. the Alamouti algorithm

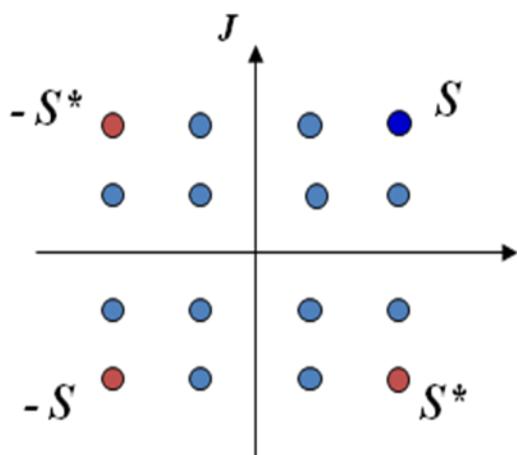


Fig. 7- Signals S after changing polarity and phase

In the Alamouti scheme the receiver receives the sum of two signals, propagating in different paths. It is possible to show that even in case that the received signals have opposite phases; the decoder can extract the correct signal.

That is, we can assume that the Alamouti algorithm is another example of the use of phase diversity.

#### VI. ARITHMETICAL PHASE PROCESSING IN PHASE DIVERSITY SYSTEM

In this section we'll show the arithmetical phase processing by its implementation in the case of the FBS system. As shown in Fig. 3, each data phase is transmitted on several carriers as derived from the Walsh matrix dimension. The FBS method is based on arithmetical sum of the data phases multiplexed by Walsh code before OFDM modulation.

The main challenge during the research was to find a solution for a phase that exceeds  $2\pi$  range. The first solution

that was tested used amplitude changing in order to "mark" the limits of the phase, for example:  $0 - 2\pi, 2\pi - 4\pi \dots$

$$\text{if } \pi m \leq |\varphi_k| < \pi(m + 1) \quad (11)$$

$$\text{then: } E_k = 1 + A_m.$$

$$\varphi_k = (|\varphi_k| - 2\pi m) \times \text{sgn}(\varphi_k)$$

$E_k$  - is the real transmitted amplitude

$A_m$  - is an index none by the receiver for any specific phase cycle.

$\varphi_k$  - the corresponded phase value inside the range of  $0 - 2\pi$

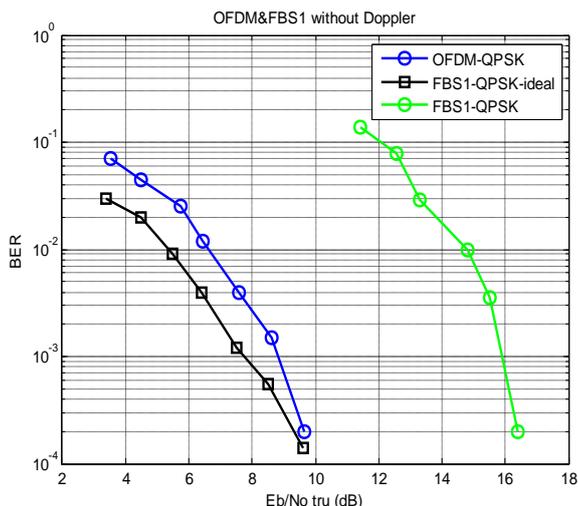


Fig. 8- QPSK in OFDM & FBS1

Fig. 8 shows the FBS1 and OFDM performance in an AWGN channel without Doppler effects. The black line refers to FBS1 with no problem of phase exceeding; this can be achieved only in simulation that controls the transmitted data. The green line is FBS1 performance with amplitude changing. The FBS1-Ideal performance gives good potential for the new method but it is well shown that the use of amplitude changing is not relevant in term of  $E_b/N_0$ . This method is problematic because amplitude identification is sensitive to noise and it creates new problems of frequency disorders. In order not to exceed the phase range of  $0 - 2\pi$  during the arithmetical procedure, we shall use higher orders of MPSK modulation and implement only part of the phases for data coding. Using this high order modulation prevents phase overflow. Obviously, the disadvantage of this method is lower efficiency in terms of BER vs.  $E_b/N_0$  performance. The example compares the OFDM standard method using Pilot Signals and FBS method without Pilot signals. The Matlab simulation includes AWGN channel and Doppler Effects. The simulation implements 4 OFDM carriers. The OFDM is based on QPSK modulation whereas the FBS system uses 16PSK modulation. For implementing 4 carriers in FBS, only 4 phases are used during modulation:

$$\varphi_1 = \frac{-3\pi}{16}; \varphi_2 = \frac{-\pi}{16}; \varphi_3 = \frac{\pi}{16}; \varphi_4 = \frac{3\pi}{16}$$

Fig. 10 shows the FBS performance compared to OFDM in the absence of Doppler effects. The FBS system, in this case, needs 5-6dB more power in order to maintain the OFDM performance. This FBS disadvantage is expected since 16PSK needs more power in order to obtain the QPSK performance.

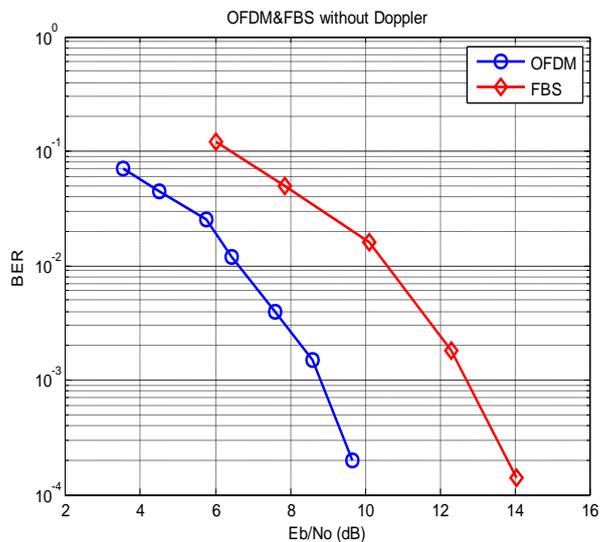


Fig. 9- FBS performance compared to OFDM

Fig.10 shows the FBS and OFDM performance in an AWGN channel including Doppler effects. The Doppler-% parameter refers to frequency change that is proportional to the frequency gap between two OFDM subcarriers. It can be seen that the performance of the FBS method is almost independent on the Doppler interference, while the OFDM performance is significantly degraded.

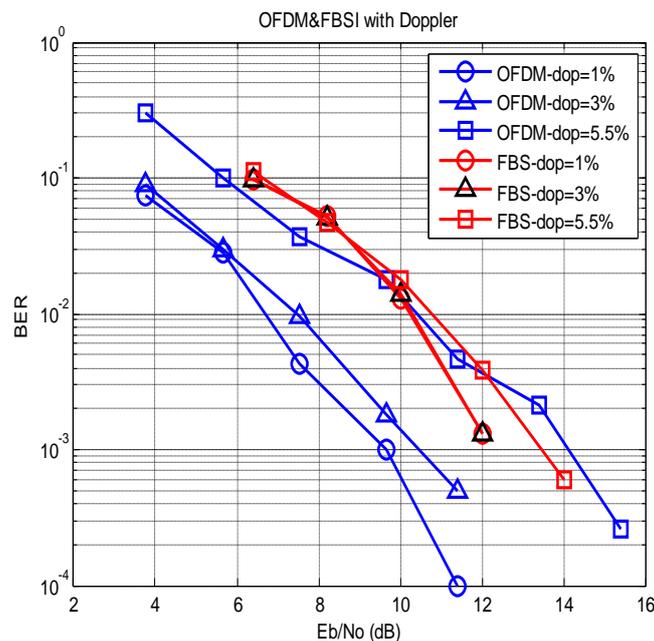


Fig. 10- FBS performance compared to OFDM in a Doppler channel

The FBS1 signal without Pilot signal has a benefit in peak to average ratio.

OFDM	EBS1_QPSK	FBS1_16PSK
2.1	2.4	~1.5

Fig. 10 and peak to average ratio shows the comparison between FBS and OFDM. It is well shown that FBS1 performance is very good in that case. The signal without Pilot Signal utilizes both the bandwidth and power, more efficiently.

## VII. CONCLUSION

The current cellular system must operate in a channel with very difficult options. This is due to high frequencies, a

large number of reflected signals and traffic at high speed. The solution of these problems can help phase diversity method, including the FBS system and Alamouti algorithm.

In these cases, it may be applied the phase arithmetic summation method proposed here.

An important consequence of this method is the possibility of unwanted phase shift compensation in OFDM channel without using Pilot Signals. Doppler Effect influence is violating orthogonality condition. In this case pilot signals are a major cause of increasing the number of errors (BER).

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