

# Computational Kinematics Sensitivity Analysis of Eccentric Reciprocating Slider Mechanism

Waluyo Adi Siswanto, Mohd Norihan Ibrahim

**Abstract**—Kinematics behavior of an eccentric reciprocating slider mechanism depends on its geometrical dimensions. When a shock effect at the slider point is intended for a particular purpose such as in compactor application, an eccentricity of the slider's axis is required. The kinematics evaluation of the slider considers velocity and acceleration functions at the slider point. This paper provides kinematics sensitivity functions at the slider point based on vector analysis of the eccentric slider mechanism. The functions have three design variables: eccentricity, length of connecting rod and the radius of crank rotation with an additional rotational variable for plotting purpose. The sensitivity functions are coded in SMATH to do all calculations and graphical plotting. Validation results show that the functions calculate correctly of known example problems. For a test case, the kinematics sensitivity functions are employed to obtain a feasible optimum design of eccentricity value to achieve maximum peak of acceleration in a slider mechanism.

**Keywords**— eccentric slider, kinematics, sensitivity, vector analysis.

## I. INTRODUCTION

SLIDER mechanism is used to transform rotational motion into translational reciprocating motion by means of a rotating body, a connecting rod and a sliding body or a slider. The common applications are in combustion engines where the slider is a piston. In engine applications, smooth kinematic behavior at the piston point is an important requirement so that the design is always concentric.

There are many works have been done in concentric and eccentric slider crank mechanisms, such as the investigation of transmission angle assessment [1], dynamics behavior of slider with clearance [2, 3], and dynamic modeling of the slider mechanism [4, 5]. Slider-crank mechanism learning tools have been developed for educational purposes. One of these educational tools is published by Campbell and Cheng [6], and the most recent one is reported by Petuya et al. [7].

A reciprocating slider-crank mechanism can be used as the motion engine of a compactor machine. For this particular application, a shock effect with high peak acceleration is

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required to generate high compacting force. Introducing an eccentricity to the slider's axis gives higher peak accelerations. This type of eccentric slider mechanism also applied in cutting machines. Research works related to modeling and analysis of eccentric slider mechanism can be found in cutting applications [8, 9].

In this present work, kinematics sensitivity functions of the slider in a compactor mechanism based on a vertical eccentric slider mechanism as illustrated in Fig. 1. In the eccentric slider mechanism, the motion of the slider does not go through the center rotation of the crank  $A$ , but eccentrically shifted at  $ec$ . The rotation of the crank can be represented by its angular velocity  $\omega$  and the angular acceleration  $\alpha$ . The distance  $AB$  is the radius rotation of the crank whereas  $BC$  is the length of the connecting rod.

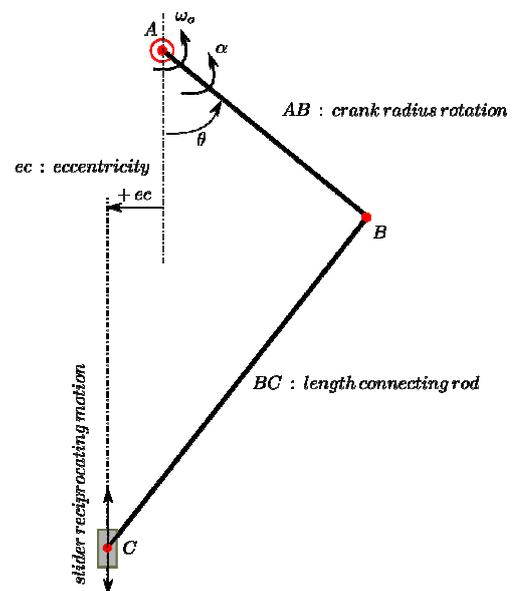


Fig. 1 Vertical eccentric slider mechanism

The kinematics sensitivity functions are mathematical expressions defining the kinematic behaviors of velocity and acceleration which are related to the design parameters of  $AB$ ,  $BC$  and  $ec$ . The functions also relates to the rotational history of the crank expressed as the angular motion. The variables  $AB$ ,  $BC$ ,  $ec$  and  $\theta$  are then defined as the independent variables whereas the velocity and the acceleration of the slider are the dependent parameters characterizing the

kinematics behaviors of the slider.

The sensitivity functions can be used to analyze the kinematics behavior of the slider in terms of its motion represented by the velocity and acceleration and also to optimize the design to achieve the optimum slider's acceleration. The sensitivity functions are defined from vector analysis approach.

II. COMPUTATIONAL VECTOR APPROACH

A. Vector analysis of crank AB

A slider mechanism is considered as one of two-dimensional planar rigid body mechanisms [9]. In classical kinematics theory, rigid body motion follows a combination of rotation and translation [10, 11].

The slider mechanism as illustrated in Fig. 1 consists of two rigid bodies. One is crank represented by a rigid line connecting the center of crank A to the connector pin B; and the other rigid body is connecting-rod depicted as a rigid line connecting the connector pin B to the slider point C. Considering a rigid body can always be interpreted as a combination of translation and rotation, therefore the motion of the rigid body (represented by line AB or BC) can also be described as a combination of translational and rotational vectors.

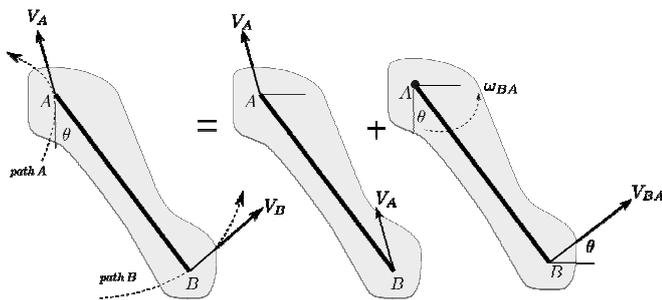


Fig. 2 Velocity vector diagram of crank AB

A general representation of crank AB motion is illustrated in Fig. 2. Since point A is the center of rotation of crank A, the velocity vector equation can be directly obtained from point B.

$$V_B = V_A + V_{BA} \tag{1}$$

substituting  $V_A = 0$ ,  $\omega_{BA} = \omega$  and calculating the direction angle  $\theta$  into the equation, the velocity motion vector at point B can be determined by:

$$V_B = \begin{Bmatrix} V_{Bx} \\ V_{By} \end{Bmatrix} = \begin{Bmatrix} AB \omega \cos(\theta) \\ AB \omega \sin(\theta) \end{Bmatrix} \tag{2}$$

A similar translation and rotation approach is also applicable for acceleration. For the acceleration, however, the present of normal acceleration ( $AB \omega_{BA}^2$ ) and tangential

acceleration ( $AB \omega_{BA}^2$ ) must be accommodated. The general representation of the acceleration vectors is drawn in Fig. 3.

The vector equation obtained from point B is then expressed as:

$$a_B = a_A + (a_{BA})_n + (a_{BA})_t \tag{3}$$

Substituting  $a_A = 0$ ,  $\omega_{BA} = \omega$ ,  $\alpha_{BA} = \alpha$  and using the direction angle  $\theta$  for projecting the vectors, the acceleration motion vector at point B can be rewritten as:

$$a_B = \begin{Bmatrix} a_{Bx} \\ a_{By} \end{Bmatrix} = \begin{Bmatrix} -AB\omega^2 \sin(\theta) \\ AB\omega^2 \cos(\theta) \end{Bmatrix} + \begin{Bmatrix} AB\alpha \cos(\theta) \\ AB\alpha \sin(\theta) \end{Bmatrix} \tag{4}$$

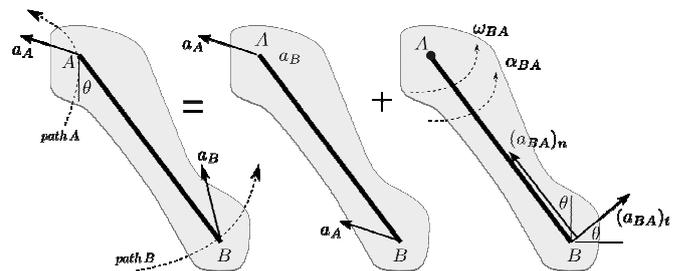


Fig. 3 Acceleration vector diagram of crank AB

B. Vector analysis of connecting rod BC

When crank AB rotates, the angle variables  $\delta$  and  $\phi$  follow the angle  $\theta$ . Therefore, the angles  $\delta$  and  $\phi$  can be expressed as a function  $\theta$ . The positions of the angle parameters  $\delta$ ,  $\phi$  and  $\theta$  are shown in Fig. 4.

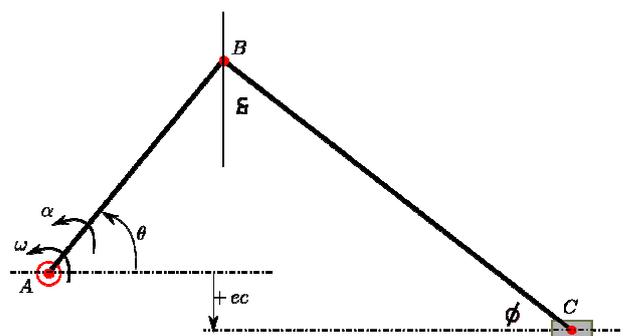


Fig. 4 Angle parameters (rotated slider 90°)

Using a simple trigonometric relationship, the angles  $\delta$  and  $\phi$  which depending on  $\theta$  can be calculated from the following equations:

$$\delta(\theta) = \text{acos} \left( \frac{ec + AB \sin(\theta)}{BC} \right) \tag{5}$$

$$\phi(\theta) = \text{asin} \left( \frac{ec + AB \sin(\theta)}{BC} \right) \tag{6}$$

and the position of the slider  $C$  follows

$$Y(\theta) = AB \cos(\theta) + BC \cos(\phi) \quad (7)$$

Implementing the general plane motion (rotation and translation) concept to the connecting rod, the velocity and acceleration vectors can be drawn. The illustration of the velocity and the acceleration vectors are shown in Fig. 5 and Fig. 6, respectively.

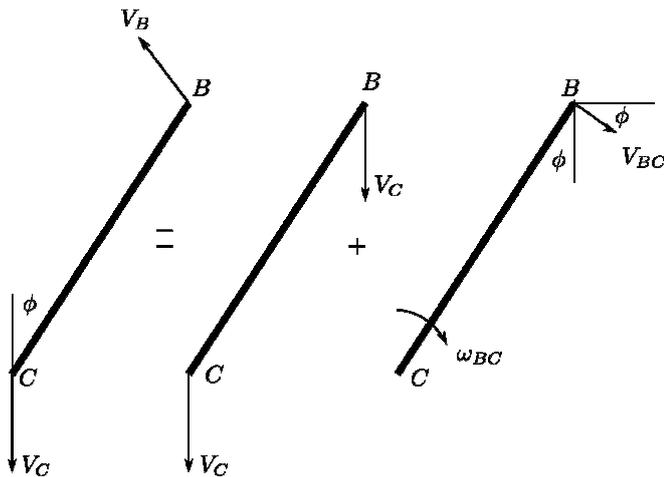


Fig. 5 Velocity vector diagram of connecting rod

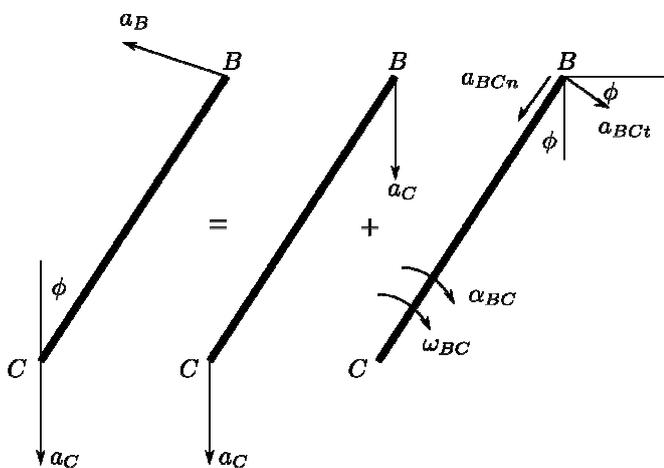


Fig. 6 Acceleration vector diagram of connecting rod

The velocity vector equation obtained from point  $B$  of Fig. 5 is expressed as:

$$V_B = V_C + V_{BC} \quad (8)$$

In a case when the crank rotates with acceleration, the angular velocity of the crank  $\omega$  is not constant but as a function of  $\theta$ , or written as  $\omega(\theta)$ . Recalling the velocity vector of point  $B$  as written in Eq. 2 and knowing that the velocity of the slider is only in vertical direction (y-axis), the

vector equation as stated in Eq. 8 can be rewritten in a complete form as a function of the angle  $\theta$ :

$$\begin{Bmatrix} AB \omega(\theta) \cos(\theta) \\ AB \omega(\theta) \sin(\theta) \end{Bmatrix} = \begin{Bmatrix} 0 \\ -V_C(\theta) \end{Bmatrix} + \begin{Bmatrix} BC \omega_{BC}(\theta) \cos(\phi(\theta)) \\ -BC \omega_{BC}(\theta) \sin(\phi(\theta)) \end{Bmatrix} \quad (9)$$

By looking at the first component of the vector in Eq. 9, the only unknown variable  $\omega_{BC}$  can be calculated directly

$$\omega_{BC}(\theta) = \frac{AB \omega(\theta) \cos(\theta)}{BC \cos(\phi(\theta))} \quad (10)$$

Substituting the result of  $\omega_{BC}$  into the second component of Eq. 9, the velocity of the slider  $V_C$  as a function of  $\theta$  can be obtained:

$$V_C(\theta) = -AB \omega(\theta) \sin(\theta) - BC \omega_{BC}(\theta) \sin(\phi(\theta)) \quad (11)$$

The acceleration vector equation problem is drawn from point  $B$  of Fig. 6.

$$a_B = a_C + (a_{BC})_t + (a_{BC})_n \quad (12)$$

Substituting the acceleration at point  $B$  as written in Eq. 4 and considering that the acceleration of the slider only in vertical direction into Eq. 12, the complete acceleration vector equation can be rewritten in terms of  $x$  and  $y$  components:

$$\begin{Bmatrix} -AB \omega^2 \sin(\theta) \\ AB \omega^2 \cos(\theta) \end{Bmatrix} + \begin{Bmatrix} AB \alpha \cos(\theta) \\ AB \alpha \sin(\theta) \end{Bmatrix} = \begin{Bmatrix} 0 \\ -a_C(\theta) \end{Bmatrix} + \begin{Bmatrix} BC \alpha_{BC}(\theta) \cos(\phi(\theta)) \\ -BC \alpha_{BC}(\theta) \sin(\phi(\theta)) \end{Bmatrix} + \begin{Bmatrix} -BC \omega_{BC}^2 \sin(\phi(\theta)) \\ -BC \omega_{BC}^2 \cos(\phi(\theta)) \end{Bmatrix} \quad (13)$$

Solving the first component (x-axis) will get the angular acceleration of the connecting rod:

$$\alpha_{BC}(\theta) = \frac{\alpha_{BC1}(\theta) + \alpha_{BC2}(\theta) - \alpha_{BC3}(\theta)}{\alpha_{BC4}(\theta)} \quad (14)$$

where

$$\alpha_{BC1}(\theta) = -AB \omega(\theta)^2 \sin(\theta) \quad (15)$$

$$\alpha_{BC2}(\theta) = AB \alpha \cos(\theta) \quad (16)$$

$$\alpha_{BC3}(\theta) = -BC \omega_{BC}(\theta)^2 \sin(\phi(\theta)) \quad (17)$$

$$\alpha_{BC4}(\theta) = BC \cos(\phi(\theta)) \quad (18)$$

Substituting the result into the y component, the acceleration of the slider:

$$a_C(\theta) = a_{C1}(\theta) + a_{C2}(\theta) - a_{C3}(\theta) - a_{C4}(\theta) \quad (19)$$

with the constants

$$a_{C1}(\theta) = -AB \omega(\theta)^2 \cos(\theta) \quad (20)$$

$$a_{C2}(\theta) = -AB \alpha \sin(\theta) \quad (21)$$

$$a_{C3}(\theta) = -BC \alpha_{BC}(\theta) \sin(\phi(\theta)) \quad (22)$$

$$a_{C4}(\theta) = -BC \omega_{BC}(\theta)^2 \cos(\phi(\theta)) \quad (23)$$

### C. Sensitivity Functions

The design parameters of the eccentric slider mechanism are the radius of crank rotation  $AB$ , the length of the connecting rod  $BC$ , and the eccentricity  $ec$ . The input motion of the slider is defined by the angular velocity  $\omega$  and the angular acceleration  $\alpha$  of the rotating crank.

For a plotting purpose, an additional function  $\theta$  is included to provide complete information in rotational variable. Therefore, the kinematics sensitivity variables are defined as functions of three independent design variables;  $AB$ ,  $BC$ ,  $ec$ , with an additional one plotting variable  $\theta$ .

The core of kinematics sensitivity functions are then defined as:

$$V_C(\theta, ec, AB, BC) \quad (24)$$

$$a_C(\theta, ec, AB, BC) \quad (25)$$

### D. Implementation in SMATH

The mathematical equations and functions are written in a symbolic mathematical tool SMATH. In this environment, mathematical expressions are conveniently written in their original forms just like writing them in an equation editor. However, all mathematical expressions must follow a common programming logic. The information should follow a structured manner from top to bottom. Graphical results can also be viewed by plotting the functions.

The angular velocity  $\omega$  or the rotation of the crank is not constant if an angular acceleration  $\alpha$  presents. The angular velocity is changing as the crank rotates with acceleration, or mathematically coding as:

$$\omega(\theta) := \sqrt{\omega_0^2 + 2 \cdot \alpha \cdot \theta}$$

where  $\omega_0$  is the initial angular velocity.

The angle parameters  $\delta$  and  $\phi$  and the slider's position  $Y$  are written in the following codes:

$$\delta(\theta, ec, AB, BC) := a \cos\left(\frac{ec + AB \cdot \sin(\theta)}{BC}\right)$$

$$\phi(\theta, ec, AB, BC) := a \sin\left(\frac{ec + AB \cdot \sin(\theta)}{BC}\right)$$

$$Y(\theta, ec, AB, BC) := AB \cdot \cos(\theta) + BC \cdot \cos(\phi(\theta, ec, AB, BC))$$

The velocity and acceleration of the crank at point  $B$  are derived from Eq. 2 and Eq. 4, respectively. The velocity and the acceleration at point  $B$  are only depending on the angle  $\theta$  and the length of the crank ( $AB$ ).

$$V_B(\theta, AB) := \begin{bmatrix} AB \cdot \omega(\theta) \cdot \cos(\theta) \\ AB \cdot \omega(\theta) \cdot \sin(\theta) \end{bmatrix}$$

$$a_B(\theta, AB) := \begin{bmatrix} -AB \cdot \omega(\theta)^2 \cdot \sin(\theta) \\ AB \cdot \omega(\theta)^2 \cdot \cos(\theta) \end{bmatrix} + \begin{bmatrix} AB \cdot \alpha \cdot \cos(\theta) \\ AB \cdot \alpha \cdot \sin(\theta) \end{bmatrix}$$

After the velocity and the acceleration at crank point  $B$  have been calculated, then the connecting rod  $BC$  transfers them to the slider point  $C$ . The vector analysis results written in Eq. 10, Eq. 11, Eq. 14, and Eq. 15 are then coded in SMATH for the calculation and simulation purposes.

The equations are all extended to accommodate all independent variables; one for the rotation of the crank  $\theta$ , and others for the design parameters: eccentricity ( $ec$ ), crank length ( $AB$ ), and the length of the connecting rod ( $BC$ ).

The velocity functions are coded as:

$$\omega_{BC}(\theta, ec, AB, BC) := \frac{AB \cdot \omega(\theta) \cdot \cos(\theta)}{BC \cdot \cos(\phi(\theta, ec, AB, BC))}$$

$$V_C(\theta, ec, AB, BC) := AB \cdot \omega(\theta) \cdot \sin(\theta) + BC \cdot \omega_{BC}(\theta, ec, AB, BC) \cdot \sin(\phi(\theta, ec, AB, BC))$$

The angular acceleration function is written as:

$$\alpha_{BC}(\theta, ec, AB, BC) := \frac{\alpha_{BC1}(\theta, ec, AB, BC) + \alpha_{BC2}(\theta, ec, AB, BC)}{\alpha_{BC4}(\theta, ec, AB, BC)} + \frac{-\alpha_{BC3}(\theta, ec, AB, BC)}{\alpha_{BC4}(\theta, ec, AB, BC)}$$

where

$$\alpha_{BC1}(\theta, ec, AB, BC) := -AB \cdot \omega(\theta)^2 \cdot \sin(\theta)$$

$$\alpha_{BC2}(\theta, ec, AB, BC) := AB \cdot \alpha \cdot \cos(\theta)$$

$$\alpha_{BC3}(\theta, ec, AB, BC) := -BC \cdot \omega_{BC}(\theta, ec, AB, BC)^2 \cdot \sin(\varphi(\theta, ec, AB, BC))$$

$$\alpha_{BC4}(\theta, ec, AB, BC) := BC \cdot \cos(\varphi(\theta, ec, AB, BC))$$

The acceleration function of the slider is coded as:

$$a_c(\theta, ec, AB, BC) := a_{c1}(\theta, ec, AB, BC) + a_{c2}(\theta, ec, AB, BC) - a_{c3}(\theta, ec, AB, BC) - a_{c4}(\theta, ec, AB, BC)$$

where

$$a_{c1}(\theta, ec, AB, BC) := -AB \cdot \omega(\theta)^2 \cdot \cos(\theta)$$

$$a_{c2}(\theta, ec, AB, BC) := -AB \cdot \alpha \cdot \sin(\theta)$$

$$a_{c3}(\theta, ec, AB, BC) := -BC \cdot \alpha_{BC}(\theta, ec, AB, BC) \cdot \sin(\varphi(\theta, ec, AB, BC))$$

$$a_{c4}(\theta, ec, AB, BC) := -BC \cdot \omega_{BC}(\theta, ec, AB, BC)^2 \cdot \cos(\varphi(\theta, ec, AB, BC))$$

### III. RESULTS AND DISCUSSION

#### A. Analysis and Visualization Tool

All vector equations are all coded in SMath. The calculation procedures and codes are presented in window snippets that can be opened or hidden. These coding snippets are useful for the improvement purpose in the future. Users are provided with working areas where they can use the available functions and plot the results.

The main window provides the basic information of the rotating system: the rotation of the crank ( $\omega_0$ ) and the angular acceleration ( $\alpha$ ) as seen in Fig. 7. Other design parameters,  $AB$ ,  $BC$  and  $ec$  can be independently entered by users in the functions. After this window, there are three user working areas defined as step 1, step 2 and step 3. Prior to each working area (step) there is a coding snippet.

The coding snippet contains the calculation formulas to define functions in the step. The working area is the area where users can plot the functions with certain values of design parameters.

The order of the snippets and the working areas are shown in Fig. 8. The snippets can be either hidden or open by simply toggling the snippet icon bar (+) to open or (-) to hide. This snippet can be set a password so that only authorized users can modify the codes.

Working areas are open areas. Charts, codes, texts can be written in the working areas. By default information of the available functions are written and several example plots are provided. The usage of functions follows the programming

logic. Functions in working area step 2 cannot be used step 1 but can be used in the working area step 3.

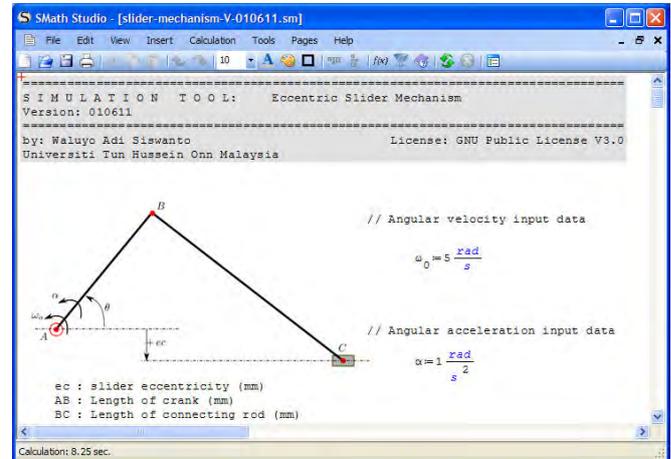


Fig. 7 Main window of analysis tool

Main information window (see Fig. 7)		
Snippet Step 1:	Codes to define functions for step 1	hide (-) / open (+)
Working Area Step 1:	Area to plot, to use available functions in Step 1 or to write codes using the available functions: $\omega(\theta)$	Always open
Snippet Step 2:	Codes to define functions for step 2	hide (-) / open (+)
Working Area Step 2:	Area to plot, to use available functions in Step 2 or to write codes using the available functions: $V_B(\theta, AB)$ ; $a_B(\theta, AB)$ ; $V_{Bmag}(\theta, AB)$ ; $a_{Bmag}(\theta, AB)$ ; $X(\theta, ec, AB, BC)$	Always open
Snippet Step 3:	Codes to define functions for step 3	hide (-) / open (+)
Working Area Step 3:	Area to plot, to use available functions in Step 3 or to write codes using the available functions: $\omega_{BC}(\theta, ec, AB, BC)$ ; $V_c(\theta, ec, AB, BC)$ ; $\alpha_{BC}(\theta, ec, AB, BC)$ ; $a_c(\theta, ec, AB, BC)$	Always open

Fig. 8 Snippets, working areas and available functions

For general usage, there are two operations that will be required: writing calculation function and plotting calculation function.

As an example, to calculate the acceleration of the slider for the design parameter: 0.010 m eccentric, length  $AB=0.125$  m and length  $BC=0.30$  m, the acceleration function must be written in working area step 3. The method to write the function is just like writing using an equation editor. For the index C, a dot command must be previously invoked. To write  $a_C$  in SMath: type a then press period button of the keyboard followed by typing capital C. To calculate the function just press equal button (=) of the keyboard. It should write like this:  $a_C(60\text{deg},0.010,0.125,0.3)$ . After pressing the equal button, the result will appear next to the equal symbol with unit.

When plotting the function, the chart area will be created from the menu Insert > Plot > 2D. An empty chart will be generated (can be resized) with an empty function at lower left corner. The first variable ( $\theta$ ) of the functions in working areas listed in Fig. 8 must be always written as x, because in SMath the horizontal axis variable is always assigned with x variable, for example  $a_C(x,0.010,0.125,0.3)$ . This will plot acceleration of the slider versus the angle of the crank continuously, with the design parameters written in argument variable 2, 3 and 4.

### B. Verification of Functions

To verify and to validate the functions, two example problems from a textbook [10] of sample problems 5/9 and 5/15 are used. The first example problem is the validation test case to calculate the angular velocity of the connecting rod  $\omega_{BC}$  and the slider velocity  $V_C$ , whereas the second problem to calculate the angular acceleration  $\alpha_{BC}$  and the acceleration of the slider  $a_C$ .

Since both cases are concentric, the eccentricity can be simply defined as zero value or these are eccentric problems with  $ec = 0$ . This demonstrates the general capability of the functions to be used in any cases including concentric problem.

#### Test Case 1

The crank with length of rotation 125 mm has a constant clockwise rotational speed of 1500 rpm. The length of connecting rod is 350 mm. It is intended to find  $\omega_{BC}$  and  $V_C$  when the crank at the position of  $60^\circ$ .

The SMath codes defining all calculation functions are then employed. The basic input data for this test case for the slider mechanism:  $\omega_0 = 157.08$  rad/s and  $\alpha = 0$  rad/s<sup>2</sup> for constant rotation.

$$\omega_{BC}(60\text{deg},0,0.125,0.350) = 29.5$$

$$V_C(60\text{deg},0,0.125,0.350) = -20.2$$

The calculated velocity of the slider is -20.2 m/s. The negative direction indicates that the slider moving towards point A. The velocity of the slider as well as the angular velocity of the connecting rod is the same with those from the textbook [10]. The velocity calculation result using the sensitivity function is therefore verified

#### Test Case 2

The example problem has similar geometrical data and input condition. The crank with length of rotation 125 mm has a clockwise rotational speed of 1500 rpm. The length of connecting rod is 350 mm. It is intended to find the angular acceleration of the connecting rod ( $\alpha_{BC}$ ) and the acceleration of the slider ( $a_C$ ) when the crank position at  $60^\circ$ .

The calculations are performed by directly writing the functions in SMath:

$$\alpha_{BC}(60\text{deg},0,0.125,0.350) = -7740$$

$$a_C(60\text{deg},0,0.125,0.350) = -994$$

The results above are also the same with the calculation results found in textbook [10]. The magnitude angular acceleration is 7740 rad/s<sup>2</sup> and the calculated slider acceleration magnitude is 994 m/s<sup>2</sup>. The negative acceleration value means the slider is in deceleration. If the velocity has positive value and affected by deceleration, the slider is slowing down. However when the velocity value is in negative value, the velocity will keep on decreasing in negative manner. As a result, the velocity magnitude of the slider is therefore increased or in another word, the slider is increasing the speed.

### C. Sensitivity Analysis Application

The kinematics performance of the slider is represented by the velocity and acceleration. In compactor design, the acceleration is more important since it can directly provide high force.

For a test case study, the sample problem 5/15 [10] is extended involving eccentricity ( $ec \neq 0$ ). The eccentricity value must be searched to get the acceleration peak as high as possible.

When the eccentricity is introduced, the velocity is shifted according to the eccentricity value. Fig. 9 shows that the magnitude can be increased. By trying an eccentricity value of 0.15 m, the maximum velocity magnitude is increased by 33%. Because the eccentricity is only in half side, the high peak velocity duration in one side is shorter than that in another half.

As for the acceleration, the behavior in different eccentricity can be seen in Fig. 10. The peak acceleration is increased by 97% when the eccentricity value is set 0.15 m.

The velocity and acceleration sensitivity functions can be conveniently used to try the eccentricity values to evaluate the velocity and acceleration as demonstrated in Fig. 9 and Fig. 10. This indicates that design trials using the sensitivity

functions can be easily conducted by directly plotting the functions with certain parameter values. This provides comprehensive information to the design engineers to see the kinematics behaviors.

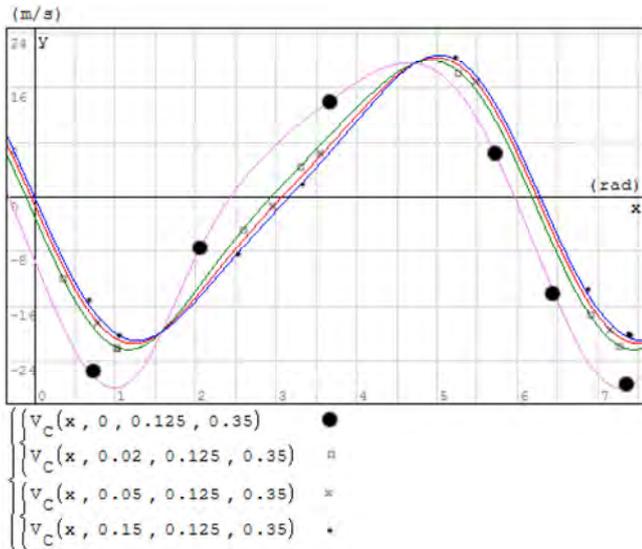


Fig. 9 Velocity of the slider in various eccentricity values

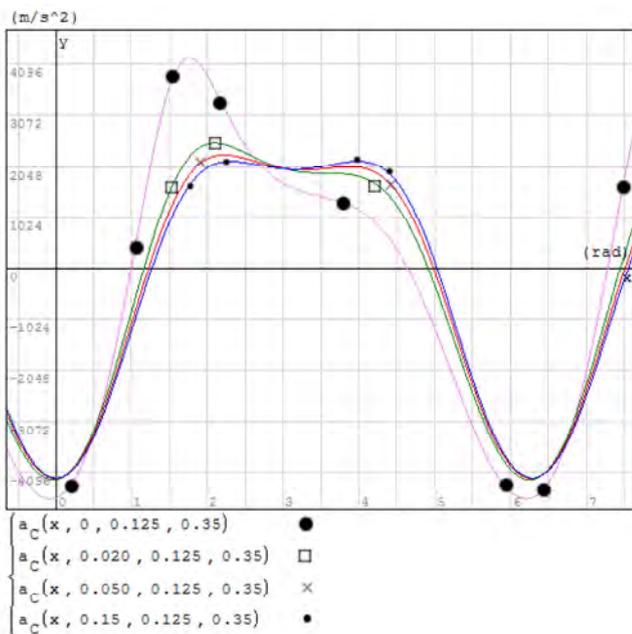


Fig. 10 Acceleration history in various eccentricity values

Since the variable is independent each other, similar method can be used, for example, to evaluate the acceleration in various length of connecting rod at a specific design of eccentricity and crank radius. The plotting syntax could be written the following forms.

$$a_C(x, 0.1, 0.125, 0.250)$$

$$a_C(x, 0.1, 0.125, 0.300)$$

$$a_C(x, 0.1, 0.125, 0.350)$$

These plotting commands will compare acceleration cycles of with three different design trials of length  $BC$  (0.250 m, 0.300 m and 0.350 m) if the slider mechanism has a radius of crank of 0.125 m with eccentricity of 0.1 m.

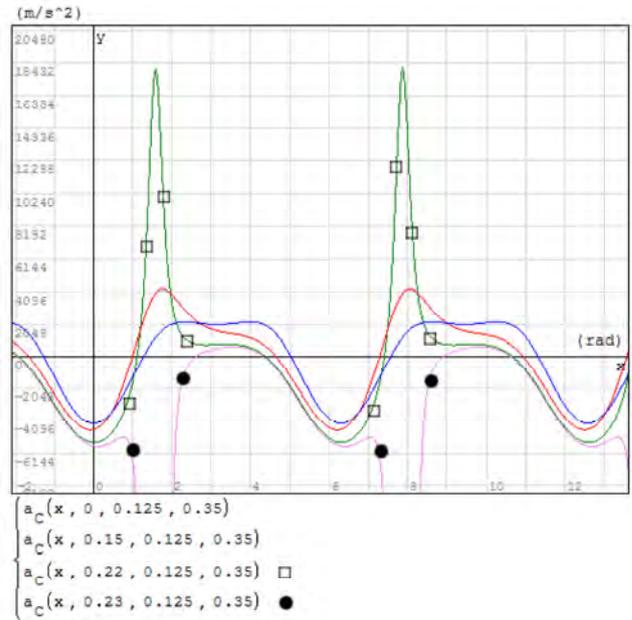


Fig. 11 Unfeasible design detection (acceleration function)

The kinematics sensitivity functions, either velocity or acceleration can be used to find the optimum eccentricity design to obtain the maximum acceleration peak value.

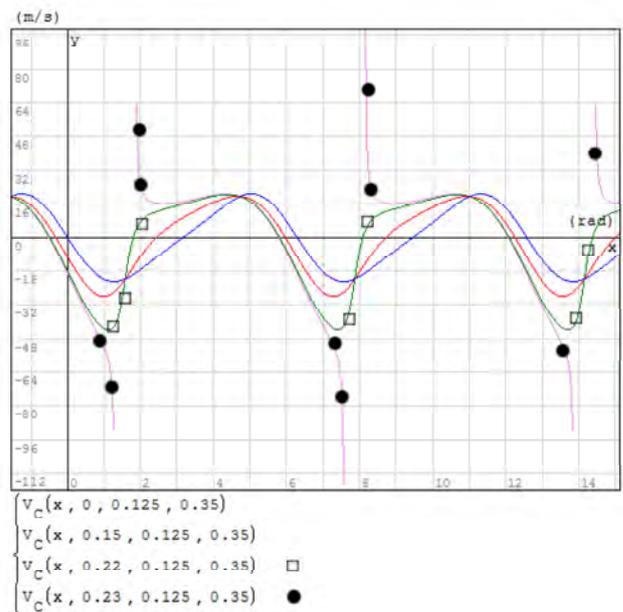


Fig. 12 Unfeasible design detection (velocity function)

In this test case problem, the highest peak of acceleration

can be obtained when the eccentricity parameter at 0.22 m as depicted in Fig. 11. If one wants to increase the peak acceleration by setting the eccentricity to 0.23 m, the functions become discontinued. The discontinuity indicates a unfeasible design. The crank cannot continuously rotating but stuck at the angle where the discontinuity occurs. The discontinuity functions can be clearly shown in acceleration plot as depicted in Fig. 11.

The unfeasible design can also be detected using the velocity sensitivity function. If the acceleration sensitivity function shows discontinuity when  $ec = 0.23$  as seen in Fig. 11, the velocity function in Fig. 12 will show similar discontinuity.

#### IV. CONCLUSION

Two important kinematics sensitivity functions, i.e velocity and acceleration have been defined and coded in SMath. The functions consider three independent design parameters in eccentric reciprocating slider mechanism: eccentricity ( $ec$ ), length of connecting rod ( $BC$ ) and the radius of the crank ( $AB$ ). One additional variable  $\theta$  has been made available for plotting purpose.

The sensitivity functions can be used to calculate a discrete position of a crank represented by the angular position angle  $\theta$  or to plot the cyclic kinematics behavior of the slider.

An optimum eccentricity design can be performed by plotting the combination of in its rotational variable  $\theta$ . Performing trial calculation can be conducted by increasing the eccentricity value while examining either velocity or acceleration function. The unfeasible design can be detected when the function is discontinued.

The developed sensitivity functions coded in SMath are easy to use and can be used to design an eccentric reciprocating slider mechanism. The functions can also be used in concentric slider mechanism design by setting the eccentric value,  $ec = 0$ .

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