Information Control of Allocation Tasks in the Synthetic Manufacturing Environment

R. Bucki, P. Suchánek, and D. Vymětal

***Abstract*—**The paper focuses on the problem of mathematical modeling of the highly complex manufacturing system imitating real production systems. However, for simplicity needs modeling is carried out in the proposed synthetic manufacturing environment. Authors concentrate on introducing extended specification details leading to creating the proper functional model of the potential manufacturing system. The most characteristic thing highlighted here is the way of modeling its flexibility which allows the modeled system to adjust to the required configuration of production stations resulting from customers' demand e.g. the number of stations in the manufacturing line, implemented tools in each production station and a sequence of passing ordered elements to be realized in other available manufacturing plants of identical production possibilities. Orders are accepted on the basis of the order matrix distinguishing customers and their demands. Heuristic algorithms choose the production plant and, subsequently, orders which are to be realized to meet the stated criterion. The operating principle forms the basis for creating the simulator of the modeled manufacturing system. However, there are production strategies which decide about the exact moment of beginning realization of the order matrix. The sample study case justifies the approach presented in the paper.

***Keywords*—**mathematical modeling, optimization, heuristic algorithms, equations of state, manufacturing strategies

I. Introduction

THERE is a competitive approach to the problem of managing manufacturing companies as they concentrate on continuous improvements. In the manufacturing environment this means that factory layouts will often be reviewed to make control and management matters more effective. Production organization requires the best possible solutions. Planning must be supported by proper specification where assumptions are to be simplified in order to enable planners to model the exact manufacturing tasks. Then the model forms the basis for building the simulator imitating flow of products in the course of the manufacturing process. The variety and complexity of production processes can be modeled more appropriately by supplying an integrated view of different modeling approaches.

Depending on an investigated problem, simulation packages may be used or it could be necessary to design and create its own simulation system [1]. It is necessary to apply discrete simulation techniques to study the reception area processes analyzing the performance of the system and investigating alternative configurations and policies for its operations [2]. Modeling and simulation issues of complex production systems requires discrete methods as there are true benefits consisting in the proper rearrangement of production logistics [3]. The use of emulation models for process control systems reduces the time and the cost of testing control systems [4]. In the production process, storages have capability as well as limitations and facilities must be run continuously.

A mathematical model is built, and a rule-based heuristics algorithm is developed to solve the problem. The efficiency of the algorithm is proved by simulated data from a real continuous production process [5]. The performance of heuristics can also be assessed using randomly generated test instances. The decomposition scheme is often able to produce high-quality solutions, while the genetic algorithm achieves results with reasonable quality in a short amount of time [6]. A decision support of logistic system optimizing is successfully provided by software which can model discrete-time system. Comparative analysis of different strategies, qualitative analysis of applicability and relative merits of strategies lead to achieving the best possible solution under the same simulation conditions [7]. Virtual reality techniques use at industrial processes provides a real approach to the product life cycle. For components manual assembly, the use of virtual surroundings facilitates a simultaneous engineering in which variables such as human factors and productivity take a real act. In the actual phase of industrial competition a rapid adjustment to client needs and to market situation is required. Design is optimized applying the methodology together with the use of virtual reality tools [8]. Mathematical modeling of the complex logistic system controlled by a determined heuristic algorithm enables production process optimization by means of the stated criteria respecting defined bounds [9]. Using expert systems in the operational management of production is another helpful way of finding a satisfactory solution in accordance with the stated manufacturing criteria [10]. Decision-making must be supported by a detailed mathematical description of simulation models [11].

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The main problem of establishing equipment replacement decisions rules under specific conditions is to find decision variables that minimize total incurred costs over a planning horizon. Basically, the rules differ depending on what type of production type is used. For batch production organization methods are suitable criteria built on the principle of economies of scale. [12]. Therefore forecast of the diagrams course is significant for short-term and long-term planning of production [13]. An important issue regarding the implementation of cellular manufacturing systems relates to deciding whether to convert an existing job shop into a cellular manufacturing system comprehensively in a single go, or in stages incrementally by forming cells one after the other taking the advantage of the experiences of implementation. The multi-stage programming solves small problems faster than exact algorithms such as branch and bound [14]. In the minimization of tool switches problem there is a need to seek for a sequence to process a set of jobs so that the number of tool switches required is minimized. The computational test results indicate that good results can be obtained by a variation which keeps the best three branches at each node of the enumeration tree, and randomly choose, among all active nodes, the next node to branch when backtracking [15]. Much attention has to be paid to production planning and control (PPC) in job-shop manufacturing systems. However, there is a remaining gap between theory and practice, in the ability of PPC systems to capture the dynamic disturbances in manufacturing process. Since most job-shop manufacturing systems operate in a stochastic environment, the need for sound PPC systems has emerged, to identify the discrepancy between planned and actual activities in real-time and also to provide corrective measures. There is the need to integrate production ordering and batch sizing control mechanisms into a dynamic model and a comprehensive real-time PPC system for arbitrary capacitated job-shop manufacturing. A system dynamics approach is adopted proving to be appropriate for studying the dynamic behavior of complex manufacturing systems [16].

A concrete example which is possible to apply efficaciously the method of multiple criteria programming in dealing with the problem of determining the optimal production plan for a certain period of time is shown in [17]. This work emphasizes: the selection of optimization criteria, the setting of the problem of determining an optimal production plan, the setting of the model of multiple criteria programming in finding a solution to a given problem, the revised surrogate trade-off method, generalized multi-criteria model for solving the production planning problem as well as the problem of choosing technological variants in the metal manufacturing industry. Moreover, this work reflects on the application of the method of multiple criteria programming while determining the optimal production plan in manufacturing enterprises.

The model of the Simplified Production System not only considers the two-dimensional layout of a multiple-production line system under the constrained factory space to achieve the minimal total material transportation flow but also selects a specific site to deploy the production system for achieving the least complexity. The two-staged solving procedure is implemented to solve production tasks. The major objective of SPS models is regarded as a Quadratic Assignment Problem (QAP). Most QAPs consider the one-dimensional layout of a production line, however the size of workstation, the rework process, the layout of multi-production lines, and the finite factory space are seldom mentioned. The space discrete technique is applied to make the infinite deployable positions become finite, and therefore the two-dimensional layout of a production system can be conducted. In addition, the simplified design of a production environment is seldom caught attention. Actually, it is important to develop such a decision support tool that can be duplicated to solve other cases by changing its input parameters only [18].

The scientific-research work will concern logistic systems of transport, manufacturing and storing which realize market orders. Terms, types and quantities of market orders are random. The problem consists in such control of transport, manufacturing and storing (by the computer-aided dispatcher) that the orders are realized with the maximal economical effects. If the accepted random market orders are not realized in time, there is a risk of losses.

There is a need to analyze logistic systems of transport, manufacturing and storing with different structures. They can be arranged in series, in a parallel way, as a tree or arranged in a serial-parallel way. These systems consist of production units, transport sets and stores. Service centers, car assembly lines, rolling mills and high storage stores are examples of such systems.

Modeling of the logistic process consists in proposing discrete equations of real states. These equations use heuristic rules. A logistic process simulation lets economic effects determine - according to the chosen strategy. A strategy is a set of heuristic rules used in accordance with the so-called characteristic states of the process. The simulation procedure consists - at the first stage - in recognizing the real state (as the one belonging to a certain characteristic state). At the second stage, a heuristic rule is chosen on this basis. The subsequent real state is determined from equations of state with the help of this heuristic rule, etc. The simulation of the logistic process course can be carried out in this way beginning with the given initial state up to the allowable final state. The best final state is the optimal state within the accepted optimization criterion.

Discrete equations of the logistic process state lead to the conclusion that the system can be in different real states. The number of real states is enormous in theory as well as in reality. Because of the above, dispatchers distinguish characteristic states only. The vital remains the problem of recognition real states which, in fact, means classifying them to certain characteristic states.

The control strategy is a set of heuristic rules. Dispatchers of logistic systems implement heuristic rules to control them basing these rules on their own experience which is treated as natural intelligence. Heuristic rules are connected with characteristic states. The strategy originates when each state is matched with a certain heuristic rule.

The analysis of control effectiveness strategy requires creating computer-based simulators and carrying out simulation experiments. The aim of the simulation research is forming the decision table which will be filled with the artificial intelligence of the production system dispatcher.

Simulation research for different (random) initial states enables to calculate preferences (probabilities) of heuristics for every heuristic state. The decision table includes knowledge necessary to control production. Rows of this table represent characteristic states of the production system whereas columns represent control strategies. The computer can make appropriate control decisions in the production system or support the dispatcher on the basis of the decision table.

Simulation tools are designed to carry out a modeling process on the basis of simulating discrete tasks. Simulation of discrete events is an analysis method of a complex system behavior by means of carrying out experiments on a computer model. During such a discrete simulation there are no continuous changes in the system – they come into being only if important events occur. Simulation tools enable to prepare a model visualizing examined system and animation of the course of processes in the given system. From the very beginning this model helps to carry out a discussion on the structure of the modeled system between the person responsible for analysis and the manager. The simulation model is understood as an effective tool supporting mutual communication as well as a tool supporting examination of complex phenomena and processes.

II. General Assumption

Let us propose an information system imitating the continuous production process carried out in J work stations arranged in a series. Further, we assume that in each work station there is a machine which can perform I operations. However, only one tool can be adjusted to perform the operation on the order matrix element in each j-th work station. Operations are performed in work stations in sequence. Moreover, we assume there are more than one identical production systems available. Let us assume that A manufacturing plants are arranged in a parallel way:

$$E = [e_\alpha],\ \alpha = 1,...,A$$

where: - e_α the α-th manufacturing plant.

Production decisions are made at the k-th stage, where $k = 0,1,...,K$. Let us introduce the matrix of orders:

$$Z^k = \left[z^k_{m,n}\right]$$

$$m = 1,...,M\ ,\ n = 1,...,N\ ,\ k = 0,1,...,K$$

where: $z^k_{m,n}$ - the n-th order of the m-th customer at the k-th stage (in conventional units).

Let us introduce the vector of charges:

$$W = [w_l],\ l = 1,...,L$$

where: w_l - the l-th charge material.

The assignment matrix of ordered products to charges takes the form:

$$\Omega = \left[\omega_{(m,n),l}\right]\ ,\ m = 1,...,M\quad n = 1,...,N\ ,\ l = 1,...,L\ ,$$

where: $\omega_{(m,n),l}$ - the assignment of the n-th order of the m-th customer to the l-th charge material.

Elements of the assignment matrix take the following values:

$$\omega_{(m,n),l} = \begin{cases} 1 \text{ - if the } n\text{-th order of the } m\text{-th customer is realized from the } l\text{-th charge,} \\ 0 \text{ - otherwise.} \end{cases}$$

We also assume that used charge vector elements are immediately supplemented, which means that we treat them as the constant source of charge material. However, for simplicity reasons, we assume that each n-th order of the m-th customer is made from the universal charge which enables realization of the given element of the order matrix from any l-th charge vector element.

Let us introduce the general structure for realizing the order $z_{m,n}$ in the α-th manufacturing plant:

$$E(m,n) = \left[e_\alpha(m,n)_{i,j}\right]$$

$$\alpha = 1,...,A\ ,,\ i = 1,...,I\ ,,\ j = 1,...,J\ m = 1,...,M\ ,$$
$$n = 1,...,N\ ,$$

where: $e_\alpha(m,n)_{i,j}$ - the i-th tool in the j-th production station in the α-th manufacturing plant able to realize the n-th order of the m-th customer.

At the same time the elements of this structure take the following values:

$$e_\alpha(m,n)_{i,j} = \begin{cases} 1 \text{ - if the order } z^k_{m,n} \text{ is realized in the } j\text{-th work station using the } i\text{-th tool in the } \alpha\text{-th plant,} \\ 0 \text{ - otherwise.} \end{cases}$$

Let us define the route matrix of orders:

$$D = \left[d_\alpha(m,n)_{i,j}\right]$$

$$\alpha = 1,...,A\ ,\ i = 1,...,I\ ,\ j = 1,...,J\ ,\ m = 1,...,M\ ,$$
$$n = 1,...,N$$

where: $d_\alpha(m,n)_{i,j}$ - the number of the i-th tool in the j-th work station in the α-th manufacturing plant to realize the order $z^k_{m,n}$.

A sample run of the route in the α-th manufacturing plant through each subsequent work station is illustrated as follows:

$$w_l \to d_\alpha(m,n)_{i,1} \to \cdots \to d_\alpha(m,n)_{i,j} \to \cdots \to$$
$$\to d_\alpha(m,n)_{i,J} \to z_{m,n}$$

The base life matrix of the complex system for a new brand set of tools used to manufacture elements of the order matrix takes the following form:

$$G = \left[g_{i,j}\right]$$

where: $g_{i,j}$ - the base number of units which can be manufactured by the i-th tool in the j-th work station before the tool in this station is completely worn out and requires immediate replacement (should there be no active work station, then $g_{i,j} = -1$).

Let $\Psi = \left[\psi_{m,n}\right]$ be a matrix of conversion factors determining how many units of each element $z_{m,n}$ of the order matrix can be realized with the use of the i-th tool in the j-th work station, $\psi_{m,n} > 0$, the base coefficient for the calculation purpose $\psi_0 = 1$.

The life matrix element for the $z_{m,n}$ order takes the form:

$$g(m,n)_{i,j} = \psi_{m,n} \cdot g_{i,j}$$

$$i = 1,...,I,\ j = 1,...,J,\ m = 1,..,M,\ n = 1,...,N,$$

Let us now define the life matrix for realizing the order $z_{m,n}$:

$$G(m,n) = \left[g(m,n)_{i,j}\right]$$

$$i = 1,...,I,\ j = 1,...,J,\ m = 1,..,M,\ n = 1,...,N,$$

where: $g(m,n)_{i,j}$ - the base number of the order $z_{m,n}$ conventional units which can be realized by means of the i-th tool in the j-th work station in the each α-th manufacturing plant before the tool is completely worn out.

The system of manufacturing plants is analyzed in case of their state while realizing the product $z_{m,n}$ at the k-th stage in the matrix form:

$$S(m,n)^k = \left[S_\alpha(m,n)^k\right]$$

$$\alpha = 1,...,A,\ k = 1,...,K,\ m = 1,...,M,\ n = 1,...,N$$

where: $S_\alpha(m,n)^k$ - the matrix of state of the α-th manufacturing plant for the product $z_{m,n}$ realization at the k-th stage.

Consequently,

$$S_\alpha(m,n)^k = \left[s_\alpha(m,n)_{i,j}^k\right]$$

where $s_\alpha(m,n)_{i,j}^k$ is the number of conventional units of the product $z_{m,n}$ already realized by the i-th tool in the j-th work station in the α-th manufacturing plant.

The base state of the element of the state matrix in the α-th manufacturing plant is calculated:

$$s_\alpha(0)_{i,j}^k = \frac{\psi_0}{\psi_{m,n}} \cdot s_\alpha(m,n)_{i,j}^k$$

If for another unit (m,n) the state of the station is exceeded, we mark it as $s_\alpha(m,n)_{i,j}^k = -1$. It means no element $z_{m,n}$ of the order matrix can be realized in the manufacturing plant and it triggers the need to carry out the replacement process to resume the production in the discussed work station.

Let $P(m,n)_\alpha^k = \left[p_\alpha(m,n)_{i,j}^k\right]$ be the matrix of the flow capacity of the α-th manufacturing plant for the product $z_{m,n}$ realization at the k-th stage where $p_\alpha(m,n)_{i,j}^k$ is the number of conventional units of the product $z_{m,n}$ which still can be realized with the use of the i-th tool in the j-th work station the α-th manufacturing plant. If the flow capacity of the station does not allow to realize at least one conventional unit (m,n), then $p_\alpha(m,n)_{i,j}^k = -1$. If there is remaining flow capacity in the i-th tool of the j-th work station but the subsequent unit *(m,n)* cannot be realized fully in this station, then the replacement process in this station is carried out automatically.

On the basis of the above assumptions the flow capacity of the i-th tool in the j-th work station for the element $z_{m,n}$ can be determined:

$$p_\alpha(m,n)_{i,j}^k = g(m,n)_{i,j} - s_\alpha(m,n)_{i,j}^k$$

The matrix of production times is defined:

$$T_{m,n}^{pr} = \left[\tau(m,n)_{i,j}^{pr}\right]$$

where: $\tau(m,n)_{i,j}^{pr}$ - the time of realization one conventional unit of the product $z_{m,n}$ with the use of the i-th tool in the j-th work station. If the product $z_{m,n}$ is not realized in the j-th work station with the use of the i-th tool, then $\tau(m,n)_{i,j}^{pr} = -1$.

Throughout the manufacturing process tools get worn out and require replacement for new ones. The manufacturing process is brought to a standstill in the work station in which the tool cannot realize any order and, as a consequence, leads to stopping production activities in preceding work stations. For this reason, the replacement is to be carried out as fast as possible. Let us define the matrix of replacement times for the tools:

$$T^{repl} = \left[\tau_{i,j}^{repl}\right]$$

where: $\tau_{i,j}^{repl}$ - the replacement time of the i-th tool in the j-th work station. If the i-th tool in the j-th work station is not implemented in the production process, then $\tau_{i,j}^{repl} = -1$.

The production rate vector $V = \left[v_{m,n}\right]$ is defined, where: $v_{m,n}$ - the number of units of the product $z_{m,n}$ made in the manufacturing route in the time unit on condition there is no need to carry out the replacement procedure.

The total manufacturing time of the element $z_{m,n}$ is calculated as follows:

$$T = \sum_{\alpha=1}^{A}\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{j=1}^{J} y^{'}(\alpha)_{i,j}^{k}\,\tau(m,n)_{i,j}^{pr} + $$
$$+ \sum_{\alpha=1}^{A}\sum_{k=0}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J} y^{''}(\alpha)_{i,j}^{k}\,\tau_{i,j}^{repl} - \Delta T$$

where: ΔT - the time during which elements are manufactured simultaneously, $y^{'}(\alpha)_{i,j}^{k}$ - the value indicating realizing one conventional unit of the product $z_{m,n}$ with the use of the *i*-th tool in the *j*-th work station in the *α*-th manufacturing plant at the *k*-th stage and $y^{''}(\alpha)_{i,j}^{k}$ - the value indicating replacement of the *i*-th tool in the *j*-th work station in the *α*-th manufacturing plant at *k*-th stage. Moreover:

$$y^{'}(\alpha)_{i,j}^{k} = \begin{cases} 1\text{- if realizing the product } z_{m,n} \text{ with the use of the } i\text{-th tool in the } j\text{-th work station in the } \alpha\text{-th manufacturing plant at the } k\text{-th stage is carried out,} \\ 0 \text{ - otherwise.} \end{cases}$$

$$y^{''}(\alpha)_{i,j}^{k} = \begin{cases} 1\text{- if the replacement procedure of the } i\text{-th tool in the } j\text{-th work station in the } \alpha\text{-th manufacturing plant at the } k\text{-th stage is carried out,} \\ 0 \text{ - otherwise.} \end{cases}$$

III. Equations of State

The state of the complex system of parallel manufacturing plants changes after every decision about production the element $z_{m,n}$ in the *α*-th manufacturing plant:

$$S_{\alpha}(m,n)^{0} \to \ldots\ldots \to S_{\alpha}(m,n)^{k} \to \ldots \to S_{\alpha}(m,n)^{K}$$

The state of the *i*-th tool in the *j*-th work station in case of the product $z_{m,n}$ manufacturing changes consequently:

$$s_{\alpha}(m,n)_{i,j}^{0} \to \ldots \to s_{\alpha}(m,n)_{i,j}^{k} \to \ldots \to s_{\alpha}(m,n)_{i,j}^{K}$$

which can be written in the following form:

$$s_{\alpha}(m,n)_{i,j}^{k} = \begin{cases} s_{\alpha}(m,n)_{i,j}^{k-1} \text{ - if the product } z_{m,n} \text{ is not realized by the } i\text{-th tool in the } j\text{-th work station in the } \alpha\text{-th manufacturing plant at the } k\text{-th stage,} \\ s_{\alpha}(m,n)_{i,j}^{k-1} + x_{\alpha}(m,n)_{i,j}^{k} \text{ - otherwise.} \end{cases}$$

where: $x_{\alpha}(m,n)_{i,j}^{k}$ - the amount of the product $z_{m,n}$ units realized by the *i*-th tool in the *j*-th work station in the *α*-th manufacturing plant at the *k*-th stage.

Let $\rho(\alpha)_{\upsilon,\zeta}$ be the *i*-th tool to be replaced with a new one in the station in the *j*-th work station in the *α*-th manufacturing plant, $1 \le \upsilon \le I$, $1 \le \zeta \le J$. The state of this station in case of replacement of tools changes as follows:

$$s_{\alpha}(m,n)_{i,j}^{k} = \begin{cases} s_{\alpha}(m,n)_{i,j}^{k-1} \text{ - if } i \ne \upsilon \text{ and } j \ne \zeta \text{ at the stage } k\text{-1,} \\ 0 \text{ - otherwise.} \end{cases}$$

The order matrix *Z* changes after every production decision:

$$Z^{0} \to Z^{1} \to \ldots \to Z^{k} \to \ldots \to Z^{K}$$

The order matrix is modified after every decision about production:

$$z_{m,n}^{k} = \begin{cases} z_{m,n}^{k-1} - x_{\alpha}(m,n)_{i,j}^{k} \text{ - if the number of units } x_{\alpha}(m,n)_{i,j}^{k} \text{ is realized,} \\ z_{m,n}^{k-1} \text{ - otherwise,} \end{cases}$$

where: $x_{\alpha}(m,n)_{i,j}^{k}$ - the number of units of the order $z_{m,n}$ realized at the *k*-th stage.

IV. Control

The control of the complex of manufacturing systems consists in implementing heuristic algorithms which choose:

- a manufacturing plant from the set of plants to place the order to be realized,
- an order from the matrix of orders Z^{k}, $k = 0,1,\ldots,K$ for manufacturing.

A. *The algorithm of the maximal flow capacity of the production plant*

This algorithm chooses the *α*-th manufacturing plant for order realization on condition that it is characterized by the maximal coefficient $\xi_{\alpha}^{k} = \sum_{i=1}^{I}\sum_{j=1}^{J} p_{\alpha}(m,n)_{i,j}^{k}$. To determine the λ-th manufacturing plan, where $1 \le \lambda \le \mathrm{A}$, the condition (1) must be met, where $\xi_{\lambda}^{k} = \xi_{\alpha}^{k}$:

$$(q_{\alpha_{\max}}^{k} = \xi_{\lambda}^{k}) \Leftrightarrow \left[\xi_{\lambda}^{k} = \max_{1 \le \alpha \le \mathrm{A}} \xi_{\alpha}^{k} \right] \tag{1}$$

B. *The algorithm of the minimal flow capacity of the production plant*

This algorithm chooses the *α*-th manufacturing plant for order realization on condition that it is characterized by the maximal coefficient $\xi_{\alpha}^{k} = \sum_{i=1}^{I}\sum_{j=1}^{J} p_{\alpha}(m,n)_{i,j}^{k}$. To determine the *λ*-th plant for order realization, where $1 \le \lambda \le \mathrm{A}$, the condition (2) must be met, where $\xi_{\lambda}^{k} = \xi_{\alpha}^{k}$:

$$(q_{\alpha_{\min}}^{k} = \xi_{\lambda}^{k}) \Leftrightarrow \left[\xi_{\lambda}^{k} = \min_{1 \le \alpha \le \mathrm{A}} \xi_{\alpha}^{k} \right] \tag{2}$$

C. The algorithm of the maximal order

This algorithm chooses the order matrix element characterized by the maximal value $\gamma^k_{m,n}$. To produce the order $z^k_{\mu,\eta}$, $1 \le \mu \le M$, $1 \le \eta \le N$ the condition in the form (3) must be met, where $\gamma^k_{m,n} = z^k_{m,n}$.

$$(q^k_{z_{\max}} = z^k_{\mu,\eta}) \Leftrightarrow \left[\gamma^k_{\mu,\eta} = \max_{\substack{1\le m\le M \\ 1\le n\le N}} \gamma^k_{m,n} \right] \qquad (3)$$

D. The algorithm of the minimal order

This algorithm chooses the order matrix element characterized by the minimal value $\gamma^k_{m,n}$. To produce the order $z^k_{\mu,\eta}$, $1 \le \mu \le M$, $1 \le \eta \le N$ the condition in the form (4) must be met, where $\gamma^k_{m,n} = z^k_{m,n}$.

$$(q^k_{z_{\max}} = z^k_{\mu,\eta}) \Leftrightarrow \left[\gamma^k_{\mu,\eta} = \min_{\substack{1\le m\le M \\ 1\le n\le N}} \gamma^k_{m,n} \right] \qquad (4)$$

V. Manufacturing Criteria

The presented criteria are meant to either maximize the production output or minimize the tool replacement time. Let us propose production criteria for the complex system along with their unavoidable bounds.

A. The production maximization criterion

The production maximization criterion in the form shown in (5) is reduced to the tool replacement bound specified in the form (6) and the flow capacity bound (7) where $x^k_{m,n}$ is the number of units of the order $z^k_{m,n}$ realized at the k-th stage and c is the maximal allowable tool replacement time.

$$Q_1 = \sum_{k=1}^{K} q^k_1 = \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N} x^k_{m,n} \to \max \qquad (5)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} y''(\alpha)^k_{i,j}\tau^{repl}_{i,j} \le c \qquad (6)$$

$$\forall_{1\le i\le I}\ \forall_{1\le j\le J}\ y'(\alpha)^k_{i,j}\, p_\alpha(m,n)^k_{i,j} \le g(m,n)_{i,j} \qquad (7)$$

B. The minimal tool replacement time criterion

The minimal tool replacement time criterion in the form (8) is reduced to the flow capacity bound specified in the form (9) and the order bound (10).

$$Q_2 = \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{j=1}^{J} y''(\alpha)^k_{i,j}\tau^{repl}_{i,j} \to \min \qquad (8)$$

$$\forall_{1\le i\le I}\ \forall_{1\le j\le J}\ y'(\alpha)^k_{i,j}\, p_\alpha(m,n)^k_{i,j} \le g(m,n)_{i,j} \qquad (9)$$

$$\sum_{m=1}^{M}\sum_{n=1}^{N} x^k_{m,n} \le z^k_{m,n} \qquad (10)$$

VI. Description of the Operating Principle

The informal description of the operating principle of a potential computer program which will be created on the basis of the above algorithms uses the structural conventions:

i. $Q = Q_1 \vee Q_2$

ii. $k = 0$

iii. Input: $A, E(m,n), D, W, G, \Omega, \Psi, T^{pr}_n, T^{repl}, V$

iv. $k =: k+1$

v. $Z^k_{m,n}, S_\alpha(m,n)^k, q^k_{\alpha_{\max}}, q^k_{\alpha_{\min x}}, q^k_{z_{\max}}, q^k_{z_{\min rel}}$

vi. Production process for:

$$Q \to q^k_\alpha = \begin{cases} q^k_{\alpha_{\max}} \to q^k_z = \begin{cases} q^k_{z_{\max}} \\ q_{z_{\min rel}} \end{cases} \\ q^k_{\alpha_{\min}} \to q^k_z = \begin{cases} q^k_{z_{\max}} \\ q^k_{z_{\min rel}} \end{cases} \end{cases}$$

vii. Choose the best procedure according to Q.

viii. $\forall_m \forall_n z^k_{m,n} = 0$? If yes, go to (ix). If no, go to (iv).

ix. Report

VII. Manufacturing Strategy

Orders are set continually within each stage k, where $k = 0,1,\ldots,K,K+1,\ldots$. We need to assume that once the order matrix realization begins at the stage $k = 0$ in the α-th manufacturing plant on the basis of the decision of the set algorithm, then no element of the n-th type from the m-th customer can be added to this matrix. The order matrix elements are to be realized completely. Because of this reason this order matrix will be shown as Z^0_1. In the meantime, new orders are set. When a new order of the n-th type from the m-th customer appears at the stage k, another order matrix originates. However, its realization begins only then, when the satisfactory number of its elements is reached as follows:

$$\sum_{m=1}^{M}\sum_{n=1}^{N} \zeta(2)^k_{m,n} = \zeta_{sat}$$

where ζ_{sat} - the number of orders igniting the procedure of allocating the order matrix to the α-th manufacturing plant, $0 < \zeta_{sat} < M \cdot N$. At the same time, $\zeta^k_{m,n}$ is the number of the n-th type order set by the m-th customer up till the k-th stage, otherwise $\zeta^k_{m,n} = 0$. As an example, let us assume that the next order is realized beginning with the stage $k-1$ in the λ-th manufacturing plant. The third order matrix realization begins at the stage $K-1$ and the fourth at the $K+1$. The first order matrix realization is completed in the K-th stage but realization of other orders continues. The above assumptions are illustrated by the following scheme:

$$
\begin{array}{cccccccc}
 & 1 & \ldots & \lambda & \ldots & \alpha & \ldots & A \\
 & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
k=0 & & \cdots & & \cdots & Z_1^0 & \cdots & \\
k=1 & & \cdots & & \cdots & Z_1^1 & \cdots & \\
\ldots & & & & & \vdots & & \\
k-1 & & \cdots & Z_2^{k-1} & \cdots & Z_1^{k-1} & \cdots & \\
k & & \cdots & Z_2^{k} & \cdots & Z_1^{k} & \cdots & \\
k+1 & & \cdots & Z_2^{k+1} & \cdots & Z_1^{k+1} & \cdots & \\
\ldots & & & \vdots & & \vdots & & \\
K-1 & & \cdots & Z_2^{K-1} & \cdots & Z_1^{K-1} & \cdots & Z_3^{K-1} \\
K & & \cdots & Z_2^{K} & \cdots & Z_1^{K} & \cdots & Z_3^{K} \\
K+1 & Z_4^{K+1} & \cdots & Z_2^{K+1} & \cdots & & \cdots & Z_3^{K+1} \\
\ldots & \vdots & & \vdots & & & & \vdots
\end{array}
$$

The manufacturing strategy proposed in the paper seems to be adequate only if the production plants are situated within a long distance from each other. It is justified by the need to minimize transport costs. However, if the location of manufacturing plants can be neglected, then we can analyze the strategy in which the elements are distributed proportionately to two or more manufacturing plants. Another thing worth emphasizing is the fact that great losses make the production process more expensive when available production stations are not in use. This should lead to streaming order matrix elements to production stations characterized by available capacity ($p_\alpha(m,n)_{i,j}^k > 0$).

VIII. Sample Case Analysis

Let us verify the correctness of the approach presented above by means of the following case study in which we implement the following data:

$z_{1,1}^0 = 5$, $z_{1,2}^0 = 0$, $z_{2,1}^0 = 0$, $z_{2,2}^0 = 3$; $\underset{1 \le l \le L}{\forall}\ \omega_{(m,n),l} = 1$;

$E = [e_\alpha]$, $\alpha = 1$; $I = 2$, $J = 2$;

$e_1(1,1)_{1,1} = 1$, $e_1(1,1)_{1,2} = 1$, $e_1(1,1)_{2,1} = 0$,

$e_1(1,1)_{2,2} = 0$; $e_1(2,2)_{1,1} = 0$, $e_1(2,2)_{1,2} = 1$,

$e_1(2,2)_{2,1} = 1$, $e_1(2,2)_{2,2} = 0$;

$d_1(1,1)_{1,1} = 1$, $d_2(1,1)_{1,2} = 2$;

$d_1(2,2)_{2,1} = 1$, $d_2(2,2)_{1,2} = 2$;

$g_{1,1} = 3$, $g_{2,1} = 2$, $g_{1,2} = 1$, $g_{2,2} = -1$;

$s(0)_{1,1}^0 = 2$, $s(0)_{2,1}^0 = 0$, $s(0)_{1,2}^0 = 0$;

$\psi_{1,1} = 1$, $\psi_{2,2} = 2$;

$\tau(1,1)_{1,1}^{pr} = 3$, $\tau(1,1)_{1,2}^{pr} = 1$; $\tau(2,2)_{2,1}^{pr} = 2$, $\tau(2,2)_{1,2}^{pr} = 1$;

$\tau_{1,1}^{repl} = 2$, $\tau_{2,1}^{repl} = 2$, $\tau_{1,2}^{repl} = 1$;

It is assumed that all tools are replaced with new ones. The minimal tool replacement criterion is used. After carrying out the scheduling process, we receive the following results:

$$
T = \begin{cases} 28 & \text{for the algorithm of the maximal order,} \\ 26 & \text{for the algorithm of the minimal order.} \end{cases}
$$

The total order realization time is minimized by implementing the algorithm of the minimal order.

IX. Simulation Process

The assumptions proposed in the paper formed the basis for building the required simulator for carrying out manufacturing processes. Figures 1 and 2 show the way of implementing introductory data used in the sample case analysis. Data are implemented separately for both available orders $z_{1,1}^0 = 5$ and $z_{2,2}^0 = 3$. The minimal tool replacement criterion is used.

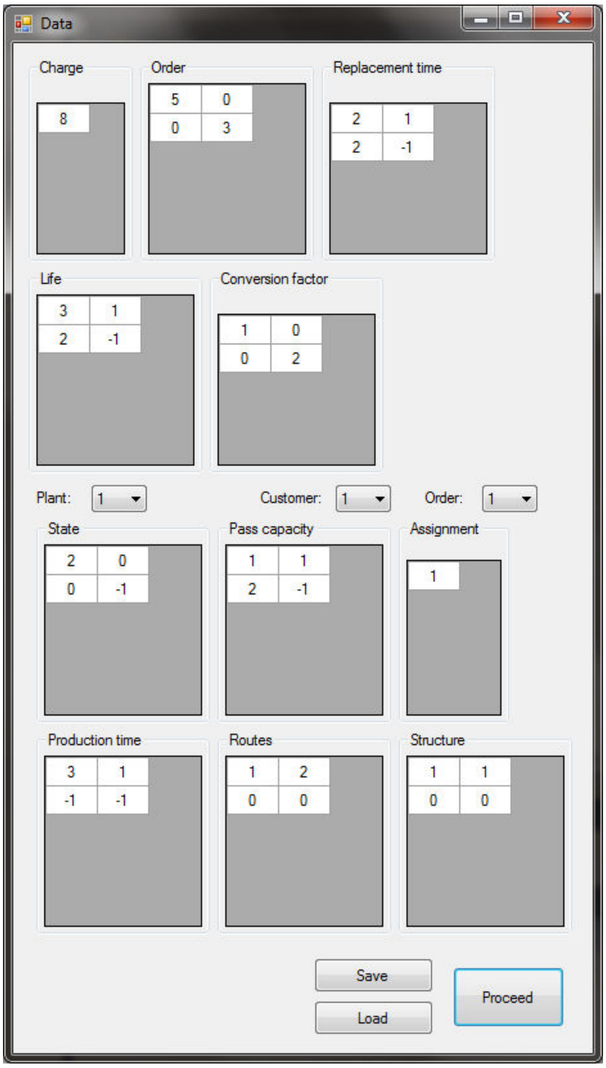

Fig. 1 implementing sample data – order $z_{1,1}^0 = 5$

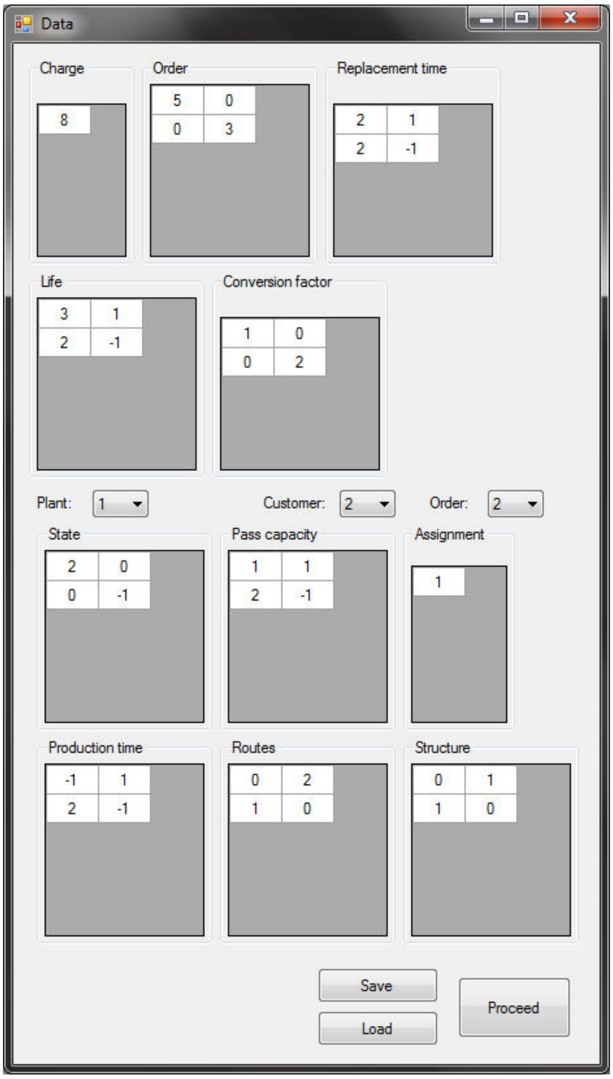

Fig. 2 implementing sample data – order $z^0_{2,2} = 3$

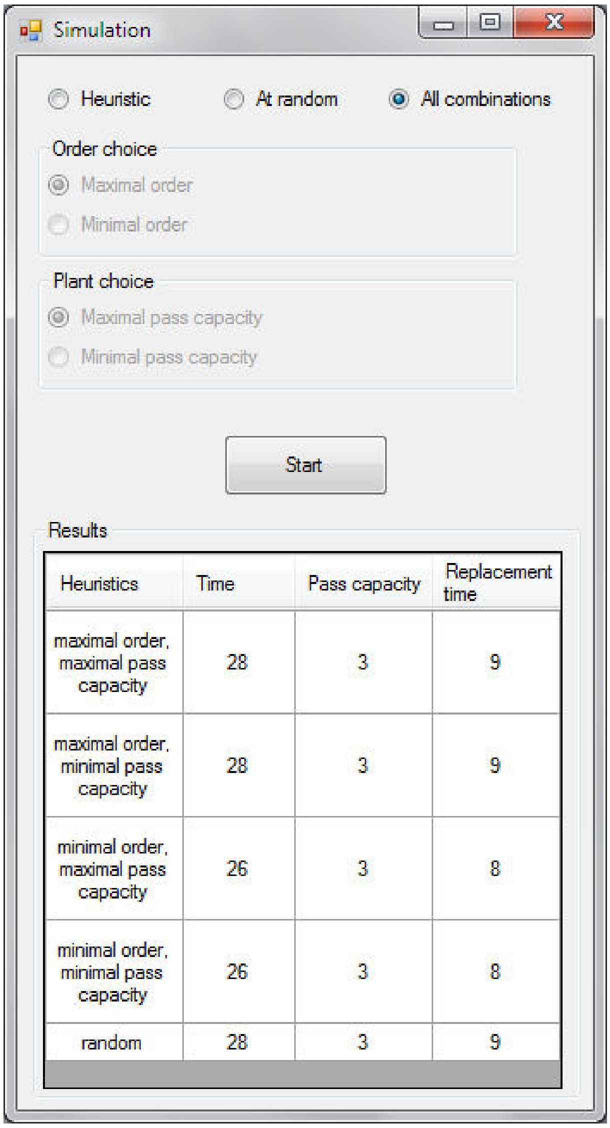

Fig. 3 results – “All combination” case

The simulator enables an operator to use any available combination of heuristics as well as generate the sequence of realizing the order at random. The approach emphasized in this simulation case includes all possibilities. Results are shown in Figure 3. The use of the algorithm of the minimal order and either of algorithms of the flow capacity delivers the best results. Moreover, the lost flow capacity of the manufacturing plant is the same for all possible cases here. The replacement time of tools in the stations is the shortest in case of implementing the algorithm of the minimal order and the algorithms of either the minimal flow capacity of the production plant or the maximal flow capacity of the production plant.

The main aim of the article is to prepare the right specification background of the general model of the hypothetical complex system consisting of parallel manufacturing plants. This is later followed by the modeling procedure. The number of orders can be immense which may lead to big losses during the manufacturing process. Such losses may result from e.g. replacing a tool which has not been worn out completely, the wrong sequence of production decisions and not implementing the control elements properly.

X. Conclusion

There is potentially a big number of orders choosing heuristics which can be put forward as well as heuristics which choose the α-th manufacturing plant for making the order. If they fail to deliver expected satisfactory result data, their combination may lead to obtaining the cost saving procedures. However, this can be achieved only by means of a simulation method. After making a decision about which criterion is the most important one, the solution focusing on the preferred way of minimizing costs should be sought for. Another aspect worth highlighting is the priority of criteria. The priority-based control should be sought for as well in order to adjust the cost reducing procedures to satisfy the need of realizing the order. The analytical way of searching for a satisfactory solution may mean an immense demand for charges in an inconvenient time interval. The work presented in the paper forms the basis for building a simulator which, after its successful validation, lets operators of complex production systems make the proper decisions about production. The simulator created on the basis of the designed model allows both a "screening" or worst-case analysis and more detailed assessments. Searching for the satisfactory solution remains the priority in the subsequent work.

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