Applying Genetic Algorithm and Fourier Series to WQI of Tha Chin River in Thailand

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Abstract—Water pollution is the main problem that effects on the community in Thailand. The Pollution Control Department in Thailand has indexed the water quality using eight parameters. In this paper, the main propose is to reduce eight parameters of water quality index(WQI) to four parameters which are dissolved oxygen (DO), total solid (TS), biological oxygen demand (BOD), and suspended solid (SS). The factor analysis, correlation analysis and fourier series are used to simulate the data. The data obtained from Tha Chin river during 2002 - 2007 is used in this model. The genetic algorithm is applied to find the weight of each parameters. The result shows that the modified WQI using four parameters provides the same result obtained from the model using eight parameters.

Keywords— water quality index, water pollution, fourier series, genetic algorithm, factor analysis, mathematical model.

I. INTRODUCTION

ATER is one of the most important things for human life, animal, plants, and microbe. Water is also the main factor for the development of the country. Nowadays, every country has to deal with the water pollution problem. The polluted water come from many sources such as waste water from household, industry, agriculture, and businesses. The impact of the water pollution can harm wildlife, fish, human life and whole ecosystems [1], [2].

Many researchers in the world have tried to treat and improve the water quality. Some researchers do in the experiments but some use the simulation. The mathematical models are used to simulate the problems. Iordache et al. [3] used computer simulation to improve the municipal waste water treatment system in Romania. The new solution for water treatment process was proposed in this work. The new method used to treat the waste water in the sewage system was also proposed in [4]. The mathematical modeling and numerical simulation were presented in this paper. The results show that the aerobic and anaerobic areas should have alternate around 60 meter. The nonlinear continuous-time model of the waste water biodegradation process identification based on neural networks was presented in [5]. The results show that the back propagation neural network can be used to solve the waste water problems of the nonlinear systems.

The water quality problems have widely studied in many countries. Wai Cheung IP and Heung Wong [6] applied new formula to evaluate water quality of Hang Jiang river, China. They focused on Wuhan segment during years 1992 to 1998. This method provides finer grading of the water quality. Mohaned Fahim Hassan [7] studied the water quality control in streams with time delays and system constraints. The parallel processing was used to save the execution time. The results show that this technique can solve the water quality control problems.

The physical model and water quality model were coupled together for the simulation of the Satilla river estuary, Georgia [8]. The impact of physical, chemical and biogeochemical processes were investigated in this work. Stefano Marsili-Libelli and Elisabetta Giusti [9] proposed the water quality model for small river basins. The model in a small river is often considered from economic viewpoint. Moreover, there are many researchers have used the mathematical modelling to study the water quality problems [10], [11], [12], [13].

The water quality of the Landzu River was studied in Nigeria [14]. The results show that the Water Quality Index (WQI) was lower than drinking water standard. The water quality index is the useful tool for water quality management. In India, the water quality index was used to measure the quality of the Ravi River [15]. A modified water quality index was proposed based on several parameters to compute WQI by monthly data during three years. It was found that the main factor that effect the WQI in Ravi River is the dissolved oxygen. Moreover, the software was developed to analyze the quality of water in Ramganga River, India using WQI[16]. The WQI applied eight parameters which are pH, total hardness, BOD, total alkalinity, dissolved oxygen, total solids, total suspended solids and chloride. It was found that water quality of the upstream is better than downstream. Furthermore, there are many other researches have been studied based on WQI in order to measure the quality of water [17], [18], [19], [20], [21].

In this work, we consider the water quality of the Tha Chin River, Nakhonpathom, Thailand. The quality of water degenerates all of it parts. It was found that the water quality is lower than the water quality standard of the surface water type 2, 3 and 4 that note in the Tha Chin river upper part, middle part and lower part, respectively. It is indicated that the Tha Chin river have got the pollution from people living near the river. The growing aquatic industry and agro-industry

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are also effect to the quality of water.

In general, nine parameters are used to compute the WQI which are pH, dissolved oxygen (DO), total solid (TS), fecal coliform bacteria (FCB), nitride, total phosphorus (TP), suspended solids (SS), temperature and biological oxygen demand(BOD). However, the temperature of water and air in Thailand has a small change. Therefore, the Pollution Control Department of Thailand applies only eight parameters to index the quality of water. This paper aims to compute the WQI in order to identify the water quality standard. The mathematical model is used to reduce parameters of the WQI. The factor analysis technique and fourier series are applied to reduce parameters. Genetic algorithm is used to compute the weight of each parameter of the index that most fit to the real data.

This paper aims to construct a completed mathematical model to predict the WQI. The rest of the paper is organized as follows. Section 2 describes the materials and methods: factor analysis, correlation analysis, fourier series, and genetic algorithm. The results are also presented in this section. Finally, Section 3 describes the conclusions.

II. MATERIALS AND METHODS

In this section, the factor analysis is applied to reduce the number of parameters from eight to four. The selected parameters are analyzed based on the real data set collected from Tha Chin river between 2002 and 2007. Three seasons are considered, which are

- 1. Summer, start from mid-February to mid-May.
- 2. Rainy, start from mid-May to mid-October.
- 3. Winter, start from mid-October to mid-February.

The season information described above refers to Thai Meteorological Department in 2007.

In order to compute the WQI, the mathematical model is developed based on the genetic algorithm. The final formulas are used to compute the WQI and compared with the values computed by the Pollution Control Department of Thailand.

A. Factor Analysis

This technique is used to group the variables. The variables that have strong relations will be grouped together in the same factor. Their relations can be positive or negative. However, all variables in the same factor will have the same direction (positive or negative). There are weak or no relations between variables in different factors. The computation steps are shown below.

step 1 : Apply the KMO (Kaiser-Meger-Olkin) equation shown below to compute the degree of relations among parameters.

$$KMO = \frac{\sum r_i^2}{\sum r_i^2 + \sum (partial \ correlation)^2}, \qquad (1)$$

where $0 \le KMO \le 1$ and r_i is the coefficient of correlation. If KMO is greater than 0.5 then the factor analysis technique can be used.

TABLE I FACTOR LOADING VALUES USED TO GROUP THE VARIABLES.

	component				
	1	2	3	4	
$NO_3^- (mg/l)$	0.985	0.015	0.043	0.024	
TP (mq/l)	0.980	0.077	-0.050	0.024	
BOD (mq/l)	0.136	0.891	0.170	0.033	
TS (mg/l)	-0.065	0.875	0.001	-0.125	
pH	0.220	0.082	0.903	-0.144	
DO (mg/l)	-0.313	0.112	0.798	0.273	
SS (mg/l)	-0.083	-0.362	-0.084	0.841	
FCB $(MPN/100ml)$	-0.214	-0.434	-0.227	-0.675	

step 2 : Analyze the factors using principal component analysis (PCA) method. In this step, we will get the factor loading (I_{ij}) .

step 3 : Consider the absolute value of the factor loading (I_{ij}) . If a variable has the factor loading value close to one then this variable is set to that factor. If a variable has the factor loading value in between 0.4-0.6 then the factor rotation technique will be used. The orthogonal rotation using Varimax method is applied in this work.

step 4 : Consider the value of I_{ij} obtained from step 2–3, several factors will be formed. These factors will be used in next Section.

The Tha Chin river data set used in this study is collected from 2002 to 2007. The factor loadings computed from this data set are shown in Table I. From Table I, we can organize variables into four groups :

Group 1 : NO_3^- and TP Group 2 : DO and pH Group 3 : TS and BOD Group 4 : FCB and SS.

In the next step, the representation of each group is selected based on the value of coefficients of decision (R^2) . This value shows the relation between water quality index and each parameter. The parameter that has the strongest relation will be selected.

B. Correlation Analysis

The degree of relationship between two variables can be compute by using the correlation analysis method. In this work, the coefficients of decision (R^2) are computed based on [22]. The parameter that has the maximum value of R^2 will be selected in each group. The results are shown in Table II. Therefore, the representation of group 1 is TP, which has R^2 46.6 %. DO is the representation of group 2, which has R^2 29.8 %. BOD is the representation of group 3, which has R^2 2.8 % and group 4 belongs to SS, which has R^2 14.3 %.

There are several techniques used to forecast coefficients of the parameters in the model. One of those techniques is fourier series which will be used in this paper.

TABLE II The value of ${\cal R}^2$ of each parameter.

ParameterGroups	TP (1)	$NO_{3}^{-}(1)$	DO (2)	pH (2)	BOD (3)	TS (3)	FCB (4)	SS (4)
${R^2 \over \%}$	0.466	0.413	0.298	0.185	0.028	0.019	0.005	0.143
	46.6	41.3	29.8	18.5	2.8	1.9	0.5	14.3

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C. Fourier Series

The fourier series are the series which are applied in many applications. In this research, we use this series to predict the trend of the data. The fourier series equation is shown below.

$$f(x) = a_0 + \sum_{i=1}^{\infty} a_i \sin(wx) + b_i \cos(wx),$$
 (2)

where $a_0, a_1, a_2, ..., b_1, b_2, b_3, ...$ are the coefficients computed by non-linear least squares method. In this work, we set i = 1, ..., 6 to equation 2. and let x = 1, 2, 3, ... which is the index that represents the season of each year i.e. 1 represents the winter of year 2002–2003, 2 represents the summer of year 2003, 3 represents the rainy year 2003,... consequently. Equation 2 can be reduced to

$$f(x) = a_0 + \sum_{i=1}^{6} a_i \sin(wx) + b_i \cos(wx).$$
(3)

From the real data of the Tha Chin river between 2002–2007, the Pollution Control Department of Thailand analyzed and computed the coefficient of equation 3 as explained in table III. After fourier series are applied, R^2 and RMSE are calculated and shown in Table IV. The relation level between the calculated values and the real values is very high as described in Figure 1–4. From all the above information, we can conclude that the predicted values of four parameters be accurate and correct as described in table III

 TABLE III

 PARAMETERS USED IN THE FOURIER SERIES.

	DO	BOD	TP	SS
a_0	2.207	2.11	0.3637	43.83
$a_1 \\ a_2$	0.05041	0.05985	-0.1749	-11.64
	0.1134	-0.4699	0.1403	-25.01
$a_3 \\ a_4$	0.1525	-0.4331	-0.01149	20.46
	-0.09492	1.064	-0.07036	3.121
a_5 a_6	0.05234	-0.4552 0.0	0.0961	-23.54 9.485
b_1	-0.4723	0.06906	0.06241	-31.20
$b_2 \\ b_3$	0.1806	1.398	-0.1766	13.57
	0.1403	-1.528	0.1742	15.79
b_4	0.08196	-0.6309	-0.1341	-15.49
b_5	-0.3059	1.218	0.1097	4.571
$b_6 \\ w$	0	0	0	15.2
	0.484	0.9087	0.4737	1.939

The RMSEs shown in Table IV are low. This means that the predicted values are very close to the real data. Therefore,

TABLE IV : THE R^2 and RMSE of DO, BOD, TP, and SS.

	DO	BOD	TP	SS
R^2	0.9491	0.9542	0.9891	0.999998
RMSE	0.2725	0.3508	0.07649	0.0659

TABLE V : The predicted values of DO, BOD, TP and SS since rainy season year 2007 until winter year 2010.

	DO(mg/l)	BOD(mg/l)	TP(mg/l)	SS(mg/l)
Jan.07–Sep07	2.171241	2.982873	0.166228	68.57532
Oct.07–Jan.08	1.135449	0.833389	0.257767	13.7175
Feb.08-May.08	1.734325	2.578293	0.082124	19.87823
Jun.08-Sep.09	2.13859	2.936531	0.646132	16.7814
Oct.09-Jan.10	1.62073	0.915038	1.229692	48.44357
Feb.10-May.10	2.555251	2.242338	0.428152	21.0307
Jun.10-Sep.10	2.649974	2.145068	0.173967	39.81221
Oct.10–Jan.11	2.664743	3.83332	0.377013	33.81221

we will apply these four parameters to calculate the WQI in the next step.

D. Water Quality Index, WQI

In Thailand, the WQI is set as a standard index. It is set by the Department of Environmental Quality Promotion (DEQP), Ministry of Natural Resources and Environment. The criteria used to classify the index are described in Table VI. Table VII shows the taking advantage of water resources in each category. The WQI scores computed from the data using eight parameters are transformed to values in between 0–100 using the formulas shown in table VIII. The equation that is used to calculate the score is shown below

$$WQI = \sqrt[8]{(pH)(DO)(TS)(FCB)(NO_3^-)(TP)(SS)(BOD)}.$$
(4)

In this paper, the WQI is calculated using only four parameters selected using the factor analysis technique described in previous sections. The model is created using the equation shown below.

$$mWQI = w_0 + w_1DO + w_2BOD + w_3TP + w_4SS$$
, (5)



Fig. 1. The DO values calculated from the proposed model compare to the real data.



Fig. 2. The BOD values calculated from the proposed model compare to the real data.

TABLE VII

The taking advantage of water resources in each category (Source: The Department of Environment Quality, Promotion, Ministry of Natural Resources and Environment, Thailand).

Type of water quality	Detail
1	Water can be used to propagate the basic nature of life and conservation of aquatic ecosystems.
2	Water can be used to conserve the fishery, swimming and water sports. The consumer must be sterile.
3	Water can be used for agriculture. The consumer must be sterile and generally improve water quality through the process before
4	Water for industrial use. The consumer must be sterile and generally improve water quality through the process before.
5	Water can be used for transportation.



Fig. 3. The SS values calculated from the proposed model compare to the real data.



Fig. 4. The TP values calculated from the proposed model compare to the real data.

TABLE VI THE CRITERIA USED TO MEASURE THE QUALITY OF WATER COMPARED TO THE SCORE OF THE WQI.

WQI value	Water quality	Type of water quality
85-100	Excellent	1
70-84	Very good	2
50-69	Good	3
20-49	Bad	4
0–19	Very bad	5

where w_0, w_1, w_2, w_3, w_4 are the weight of the parameters. These weights are computed using the genetic algorithm explained in the next section.

E. Genetic Algorithm

The genetic algorithm method can be divided into five parts. 1. Finding the pattern of chromosome that shows genetic representation. The pattern of chromosome in this work is the weight values.

2. Initial generation or population, in this case, it is parameters of the WQI.

3. Evaluation of fitness function, in this case, it is the group of weight values that provide the lowest RMSE.

Parameters	Condition	Formula
pH	<= 1.6 1.6-7.1 7.1-12.45 > 12.45	$\begin{array}{c} 2\\ -0.089pH^3 + 3.0018pH^2 - 3.533pH + 0.2901\\ -0.2286pH^4 - 10.45pH^3 - 174.05pH^2 + 1231.6pH - 3038.7\\ 2\end{array}$
DO(mg/l)	< 0.105 0.105-4.15 4.15-7.69 7.69-13.8 > 13.8	$\begin{array}{c} 2\\ 0.2677DO^3-1.4538DO^2+14.496DO+0.5\\ -0.3171DO^3+3.7511DO^2+2.915DO\\ 0.1506DO^3-4.7806DO^2+40.856DO\\ 2\end{array}$
TS(mg/l)	0–120 120–500 > 500	$\begin{array}{r} -0.0014TS^2 + 0.1863TS + 80 \\ -0.00001TS^2 - 0.1338TS + 97.5 \\ 20 \end{array}$
FCB(MPN/100ml)	0-1200 1200-12000 12000-100000 > 100000	$\begin{array}{r} 2\cdot10^{-5}FCB^2-0.0563FCB+100\\ 2\cdot10^{-7}FCB^2-0.005FCB+66.417\\ -4\cdot10^{-14}FCB^3+9\cdot10^{-9}FCB^2-0.0008FCB+43.367\\ 2\end{array}$
TP(mg/l)	0-1.48 > 1.48	$\frac{100 \cdot e^{-2.5TP}}{2}$
$NO_3^-(mg/l)$	0–23 > 23	$\frac{100 \cdot e^{-0.16NO_3^-}}{2}$
SS(mg/l)	0-4 4-75 75-230 >= 230	$\begin{array}{c} 100\\ 0.0043SS^2 - 1.1687SS + 105.03\\ 0.0007SS^2 - 0.4647SS + 72.208\\ 2\end{array}$
BOD(mg/l)	0–27.1 > 27.1	$-0.0069BOD^3 + 0.4766BOD^2 - 11.464BOD + 100$ 2

TABLE VIII Formula used to calculate the score of each parameter.

4. Generation Operation for the next generation, which are selection, crossover, and mutation.

5. Define parameter values used to find the genetic results. Figure 5 shows the best fitness function and the mean fitness function of each chromosome. The weight values computed using the genetic algorithm are $w_0 = 3.571$, $w_1 = 0.585$, $w_2 = 0.273$, $w_3 = 0.108$ and $w_4 = 0.171$.

Table IX shows the WQI collected between October 2003 and May 2007 from the Tha Chin River compared to the computed mWQI from the algorithm. Figure 6 shows the percentage of the error between WQI and mWQI. The results show that the predicted values are very close to the real data. It can be concluded that our proposed method provides high efficiency and can be used to predict the WQI.

III. CONCLUSION

From our proposed technique, the water quality index of Tha Chin river in Thailand can be reduced to four parameters. By this way, the process used to check the water quality is improved. The factor analysis, correlation analysis, and fourier series are applied to predict the trend of the data. The data used in this process is collected from Tha Chin river during years 2002 to 2007. It is found that the parameters that effect the WQI are DO, BOD, TP and SS. The genetic algorithm is used to find the weight of each parameter in the WQIs. The results show that modified WQI can be used to check the quality of water in Thailand. Moreover, the water quality of Tha Chin river is found to be in a good level. However, the water quality need to be improved continually.

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REFERENCES

- [1] I. Pepper, C. Gerba, and M. Brusseau, *Environmental and pollution science*. Rev.ed of Pollution science, 1996.
- J. L. Schnoor, Environment Modeling: fate and transport of pollutants in water, air and soil. A Wiley - Interscience publication, 1996.
- [3] S. Iordache, P. Nicolae, C. Necula, and G. Busuioc, "Municipal wastewater treatment improvement using computer simulation," in *Proceedings* of 4th WSEAS International Conference on Waste Management, Water Pollution, Indoor Climate, 2010, pp. 61–66.
- [4] D. Robescu, R. Mocanu, and D. Robescu, "New method for wastewater pre-treatment in the sewage system," in *Proceedings of the 10th WSEAS international conference on Mathematics and computers in biology and chemistry*, 2009, pp. 153–156.
- [5] D. Sendrescu, D. Popescu, C. Ionete, and M. Roman, "Neural networks identification of wastewater biodegradation process," in *Proceedings* of the 4th WSEAS international conference on Energy, Environment, Ecosystems and Sustainable Development, 2008, pp. 298–302.

Season	DO(mg/l)	BOD(mg/l)	TP(mg/l)	SS(mg/l)	WQI real data	mWQI
Oct.02–Jan03	26.1845	82.6191	57.6950	65.7350	58.9123	58.91569
	28.6668	64.8082	56.3246	62.9420	60.6231	54.87983
Feb.03–May.03		000			0010202	
Jun.03-Sep.03	14.7594	90.8833	63.0313	81.0237	54.5523	57.6788
Oct.03–Jan.04	23.3524	73.9279	68.8612	83.2994	60.4554	59.09571
Feb.04-May.04	26.5695	75.3800	11.0277	76.9896	51.6061	54.04908
Jun.04-Sep.04	22.3121	75.8860	6.0350	62.6211	50.5102	48.70041
Oct.04-Jan.05	31.2725	78.1842	56.1624	83.7600	64.2108	63.59819
Feb.05-May.05	34.7342	75.9138	53.3036	71.8244	60.5893	62.65374
Jun.05-Sep.05	32.9787	75.7578	42.0350	57.9884	56.0329	58.00124
Oct.05-Jan.06	36.5849	81.5251	72.8574	73.4149	66.0807	67.65203
Feb.06-May.06	26.1793	74.8474	35.7801	80.6298	58.3529	56.97119
Jun.06-Sep.06	30.6273	86.2296	57.2162	9.8890	53.0640	52.89903
Oct.06–Jan.07	31.7600	78.3224	43.0095	79.6386	62.3375	61.79585
Feb.07-May.07	31.5689	78.3224	47.2367	62.2786	59.3131	59.17203

TABLE IX WQI AND MWQI OF THA CHIN RIVER.

- [6] W. C. Ip, H. Wong, and B. Q. Hu, "River water quality evaluation in china," in *Proceedings of the WSEAS International Conference on ENVIRONMENT, MEDICINE and HEALTH SCIENCES*, 2010, pp. 13– 16.
- [7] M. Hassan, "A partially closed loop distributed controller for constrained water quality control in streams with time-delays," in *Proceedings* of the 2006 IASME/WSEAS International Conference on Energy and Environmental Systems, 2006, pp. 165–172.
- [8] L. Zheng, C. Chen, and F. Y. Zhang, "Development of water quality model in the satilla river estuary, georgia," *Ecological Modelling*, vol. 178, pp. 457–482, 2004.
- [9] S. Marsili-Libelli and E. Giusti, "Water quality modelling for small river basins," *Environmental Modelling and Software*, vol. 23, pp. 451–463, 2008.
- [10] B. Tyagi, S. Gakkhar, and D. S. Bhargava, "Mathematical modelling of stream do-bod accounting for settleable bod and periodically varying bod source," *Environmental Modelling and Software*, vol. 14, pp. 461–471, 1999.
- [11] K. Yin, Z. Lin, and Z. Ke, "Temporal and spatial distribution of dissolved oxygen in the pearl river estuary and adjacent coastal waters," *Continental Shelf Research*, vol. 24, pp. 1935–1948, 2004.
- [12] S. Su, J. Zhi, L. Lou, F. Huang, X. Chen, and J. Wu, "Spatio-temporal patterns and source apportionment of pollution in qiantang river(china) using neural-based modeling and multivariate statistical techniques," *Physics and Chemistry of the Earth*, p. In press, 2010.
- [13] L. Prathumratana, S. Sthiannopkao, and K. W. Kim, "The relationship of climatic and hydrological parameters tosurface water quality in the lower mekong river," *Environment International*, vol. 34, pp. 860–866, 2008.
- [14] J. Yisa and T. Jimoh, "Analytical studies on water quality index of river landzu," *American Journal of Applied Sciences*, vol. 7, no. 4, pp. 453– 458, 2010.
- [15] K. Ashwani and D. Anish, "Water quality index for assessment of water quality of river ravi at madhopur (india)," *Global Jouranl of environmental sciences*, vol. 8, no. 1, pp. 49–57, 2009.
- [16] M. Alam and J. Pathak, "Rapid assessment of water quality index of ramganga river western uttar pradesh (india) using a computer programme," *Nature and Science*, vol. 8, no. 11, pp. 1–8, 2010.
- [17] S. Abbasi, "Qualidex a new solfware for generating water quality indice," *Environmental Monitoring and Assessment*, vol. 119, pp. 201– 231, 2006.
- [18] J. Cho, K. Sung, and S. R. Ha, "A river waternext term quality management model for optimizing regional wastewater treatment using a genetic algorithm," *Journal of Environmental Management*, vol. 73, pp. 229–242, 2004.
- [19] B. Pimpunchat, W. Sweatman, W. Triampo, G. Wake, and A. Parshotam, "Modelling river pollution and removal by aeration," *International Congress on Modelling and Simulation*, pp. 2431–2437, 2007.
- [20] B. Pimpunchat, W. Sweatman, G. Wake, W. Triampo, and A. Parshotam, "A mathematical model for pollution in a river and its remediation by aeration," *Applied Mathematics Letters*, vol. 22, no. 3, pp. 304–308, 2009.

- [21] V. Rankovic, J. Radulovic, I. Radojevic, A. Ostojic, and L. Comic, "Neural network modeling of dissolved oxygen in the grua reservoir, serbia," *Eological Modelling*, vol. 221, no. 8, pp. 1239–1244, 2010.
- [22] A. Bluman, *Elementary Statistics: A Step by Step Approach*, 6th ed. Boston, USA: McGRAW-Hill, 2007.



Fig. 5. The fitness values of the genetic algorithm.



Fig. 6. The percentage error between WQI and mWQI.