# Fuzzy Concept Mining based on Formal Concept Analysis

Kyoung-Mo Yang, Eung-Hee Kim, Suk-Hyung Hwang, Sung-Hee Choi

***Abstract*—**Data Mining(also known as Knowledge Discovery) is defined as the non-trivial extraction of implicit, previously unknown, and potentially useful information from data. It includes not only methods for extracting information from the given data, but also visualizing the information. Formal Concept Analysis(FCA) is one of Data mining research fields, and it has been applied to a number of areas such as medicine, psychology, library, information science, and software re-engineering and others. FCA is based on a mathematical order theory for data analysis, which extracts concepts and builds a conceptual hierarchy from given data. In order to analyze vague data set of uncertainty information, Fuzzy Formal Concept Analysis(Fuzzy FCA) incorporates fuzzy set theory into FCA. In this paper, we introduce basic notions of FCA and Fuzzy FCA, and developed the Fuzzy FCA-Wizard, that supports Fuzzy FCA's features. We demonstrate the process for discovering knowledge from uncertain data with Fuzzy FCA-Wizard.

***Keywords*—** Data Mining, Knowledge Discovery, Fuzzy Set, Formal Concept Analysis

## I. Introduction

Besides the ordinary set, fuzzy set theory permits uncertainty information that is directly represented by membership value in the range of [0, 1](Fig 1). The membership value which is taken through membership function indicates the grade of membership of set elements. If an element is mapped to the value 0, the element is not included in the fuzzy set, and 1 describes a fully included element[1]. In decision and organization sciences, fuzzy set theory has a great impact in preference modeling and multi-criteria evaluation. Applications can be found in many areas such as management, production research, and finance[2]. In order to analyze vague data set of uncertainty information, Fuzzy Formal Concept Analysis(Fuzzy FCA) incorporates fuzzy set theory into FCA. It extracts useful information with a unit of fuzzy concept from given fuzzy formal context with a confidence threshold, and

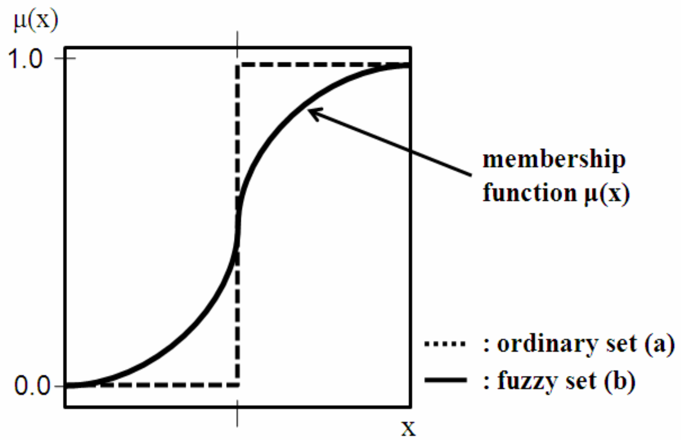


Fig. □ The ordinary set and the fuzzy set

constructs fuzzy lattice by order relations between the fuzzy concepts[3].

Formal Concept Analysis(FCA)[4] is one of Data Mining research fields, and it has been applied to a number of areas such as medicine, psychology, library, information science, and software re-engineering and others[5-10]. There are so many implementation of FCA such as ConExp[11], FCA-Wizard[12], Galicia[13] and ToscanaJ[14](Table I). ConExp is an open-source project and it has been extended several times, because it is easy to use and has powerful visualization system. FCA-Wizard is an extension of ConExp, adding various scales. Galicia is an open-source platform for creating, visualizing and storing concept and Galois lattices. ToscanaJ is a reimplementation of a classic FCA tool called Toscana. FCA is based on a mathematical order theory for data analysis, which extracts concepts and builds a conceptual hierarchy from given

Table. I FCA Tools

| | One-valued context | Lattice visualization | Multi-valued context | Scaling |
|---|---|---|---|---|
| **ConExp** | O | O | X | X |
| **FCA-Wizard** | O | O | O | O |
| **Galicia** | O | O | O | O |
| **ToscanJ** | O | O | O | O |

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data which is represented with a formal context. The basic structure of FCA is the formal context which is binary-relation between a set of objects and a set of attributes. The formal context is based on the ordinary set, which elements have on the two values, 0 or 1(Fig 1).

Although there is well defined Fuzzy FCA theory, it is impossible for human to manually analyze huge amounts of data without appropriate Fuzzy FCA tools. Therefore, we need assistances in the well defined Fuzzy FCA tool's functionalities. In this paper, we introduce basic notions of FCA and Fuzzy FCA, and develop the Fuzzy Formal Concept Analysis Wizard(Fuzzy FCA-Wizard). In order to evaluate usefulness of Fuzzy FCA-Wizard, we experiment on our tool with users and tags data extracted from the online social tagging system such as BibSonomy.

The rest of the paper is organized as follows: in Section II, we introduce some basic notions of the formal concept analysis and the fuzzy formal concept analysis. In Section III, we describe our tool and the experiment to evaluate our tool. Lastly, we conclude the paper with a summary and explain our future directions of our research in Section IV.

## II. Basic notions of FCA and FFCA

### A. Formal concept Analysis

We introduce the basics of formal concept analysis that is given in [4].

**Definition 1.** A formal context is a triple K := (G, M, *I*) consists of two finite set of objects G and set of attributes M, and a binary-relation *I* between the objects and the attributes(i.e., $I \subseteq G \times M$). In order to express that an object $g \in G$ has an attribute $m \in M$, we write $gIm$ or $(g, m) \in I$.■

The formal context can be easily represented by a cross-table as shown in Table II. In this example, the header of columns is a set of attributes as M = {a, b, c, d}, and the header of rows is a set of objects as G = {O1, O2, O3, O4}. The binary-relation *I* is represented by putting "X" in the cross-table. For example, object "O1" has two attributes "a" and "d".

A formal concept has extension and intension which are subset of objects and attributes. The extension and the intension are derived by two functions, which are defined as;

$$intent(A)=\{m\in M \mid \forall g\in A:(g,m)\in I\} \quad \text{for } A \subseteq G,$$

$$extent(B)=\{g\in G \mid \forall m\in B:(g,m)\in I\} \quad \text{for } B \subseteq M.$$

**Definition 2.** Let (G, M, *I*) be a context, a formal concept is defined as a pair(A, B) with $A \subseteq G$ is called extension, $B \subseteq M$ is called intension and $intent(A)=B \wedge extent(B)=A$.

For example, intension of {O1, O3} is {a} and extension of {a} is {O1, O3}, therefore, ({O1, O3}, {a}) is a formal concept. We can extract every formal concept in Table II by definition 2. The set of all concepts are represented at Table III.

The concepts are partially ordered by inclusion of extension(and intension). For example, extension of C6 include

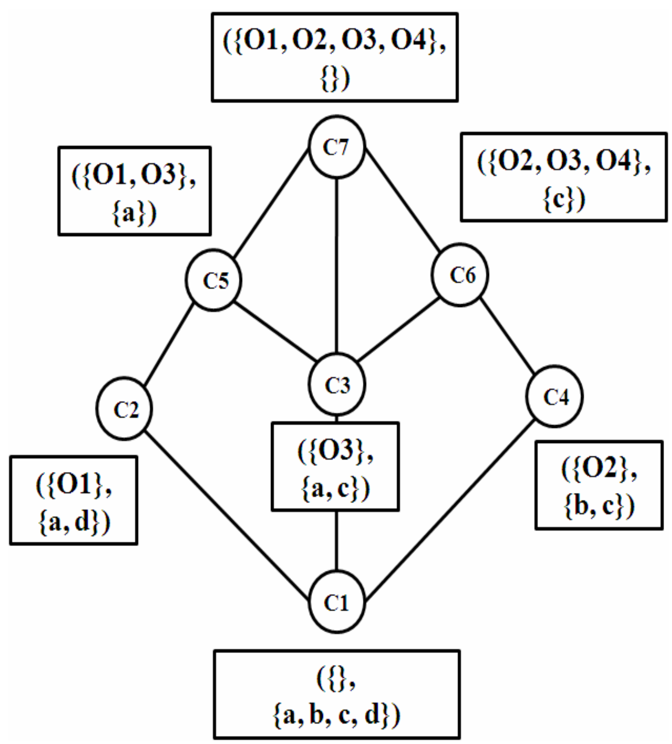


Fig.  Concept lattice of the Table II

Table. II Formal context

| | a | b | c | d |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| O1 | X | | | X |
| O2 | | X | X | |
| O3 | X | | X | |
| O4 | | | X | |

Table. III Formal concept of Table II

| | Extensions | Intensions |
|---|---|---|
| C1 | {} | {a, b, c, d} |
| C2 | {O1} | {a, d} |
| C3 | {O3} | {a, c} |
| C4 | {O2} | {b, c} |
| C5 | {O1, O3} | {a} |
| C6 | {O2, O3, O4} | {c} |
| C7 | {O1, O2, O3, O4} | {} |

extension of C4 as {O2} ⊆ {O2, O3, O4} and intension of C4 include intension of C6 as {c} ⊆ {b, c}.

**Definition 3.** Let $(A_1, B_1)$ and $(A_2, B_2)$ be two formal concepts of a formal context (G, M, *I*). $(A_1, B_1)$, $(A_2, B_2)$ are partially ordered by super-sub relation which is formalized by $(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2$ $(\Leftrightarrow B_1 \subseteq B_2)$.

**Definition 4.** A concept lattice of a formal context K is a set B(C) of all formal concepts of K with the partial order ≤, denoted as L:=(B(C),≤).

In Fig 2, a formal concept C6 ({O2, O3, O4}, {c}) has two direct sub-concepts such as C3({O3}, {a, c}) and C4({O2}, {b, c}). The concept C4 has a direct sub-concept C1({}, {a, b, c,

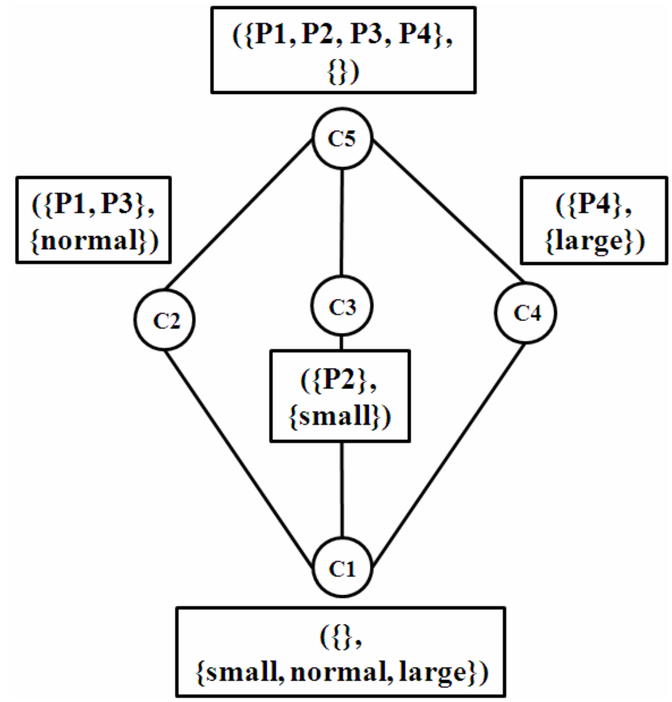


Fig. □ Formal concept lattice of Table IV(c)

d}).

Usually a data set would be expressed by each attribute with a value in range of the attribute which is similar to a table of relational database. In that case, many-valued context can be used, but formal context can not express these kinds of data.

**Definition 5.** A many-valued context (G, M, V, *I*) is composed of a set G of objects, a set M of attributes, a set V of attribute values and a ternary-relation *I* between G, M and V(i.g., $I \subseteq G \times M \times V$). An element of *I*, $(g, m, w) \in I$ indicates the attribute m has the value w for the objet g.

Table. IV Example of conceptual scaling

| | height (cm) |
|---|---|
| P1 | 173 |
| P2 | 150 |
| P3 | 171 |
| P4 | 190 |

(a) Many-valued context

| height(H) | small | normal | large |
|---|---|---|---|
| H < 155 | X | | |
| 155 ≤ H < 180 | | X | |
| 180 ≤ H | | | X |

(b) Scale context

| | small | normal | Large |
|---|---|---|---|
| P1 | | X | |
| P2 | X | | |
| P3 | | X | |
| P4 | | | X |

(c) Derived context

Table IV(a) shows a simple example of many-valued context that consists of object set G = {P1, P2, P3, P4}, attribute set M = {height} and the ternary-relation *I* is represented as numeric values "173, 150, 171, 190". In order to extract formal concepts from the many-valued context, each attribute of the many-valued context should be transformed into a formal context(is called derived context (Table IV(c)) based on scale context. This procedure is called conceptual scaling. Table IV shows some contexts related to conceptual scaling. Fig 3 shows concept lattice for Table IV(c).

**Definition 6.** A scale context $S_m := (G_m, M_m, I_m)$ for the attribute m of a many-valued context is a formal context with $m(G) \subseteq G_m$. The objects of a scale context are called scale values and the attributes are called scale attributes.

*B. Fuzzy Formal Concept Analysis*

We introduce some definitions of fuzzy formal concept analysis based on [3].

**Definition 7.** A fuzzy formal context is a triple K := (G, M, *I* = φ(G X M)) where G is a finite set of objects, M is a finite set of attributes, and *I* is a fuzzy set on domain G X M. Each relation (g, m) ∈ *I* has a membership value μ(g, m) in [0,1].■

A fuzzy formal context can also be represented as a cross-table as presented in Table V. The context has objects as G = {O1, O2, O3, O4}. It also has attributes as M = {a, b, c, d}. Each relation between objects and attributes is represented by a membership value.

Table. V Fuzzy formal context

| | a | b | C | d |
|---|---|---|---|---|
| O1 | 0.8 | 0.12 | 0.61 | 0.6 |
| O2 | 0.9 | 0.85 | 0.13 | 0.1 |
| O3 | 0.1 | 0.14 | 0.87 | 0.1 |
| O4 | 0.6 | 0.12 | 0.13 | 0.3 |

Table. VI Fuzzy formal context with T = [0.5, 1.0]

| | a | b | c | d |
|---|---|---|---|---|
| O1 | 0.8 | — | 0.61 | 0.6 |
| O2 | 0.9 | 0.85 | — | — |
| O3 | — | — | 0.87 | — |
| O4 | 0.6 | — | — | — |

A confidence threshold T has an interval $[t_1, t_2]$, where $0 \le t_1 < t_2 \le 1$. By using the confidence threshold T, we can eliminate some relations that are out of the interval values from a given fuzzy context. The confidence threshold T can be set by user according to the application or the domain knowledge. For instance, Table VI shows a fuzzy formal context with T = [0.5, 1.0].

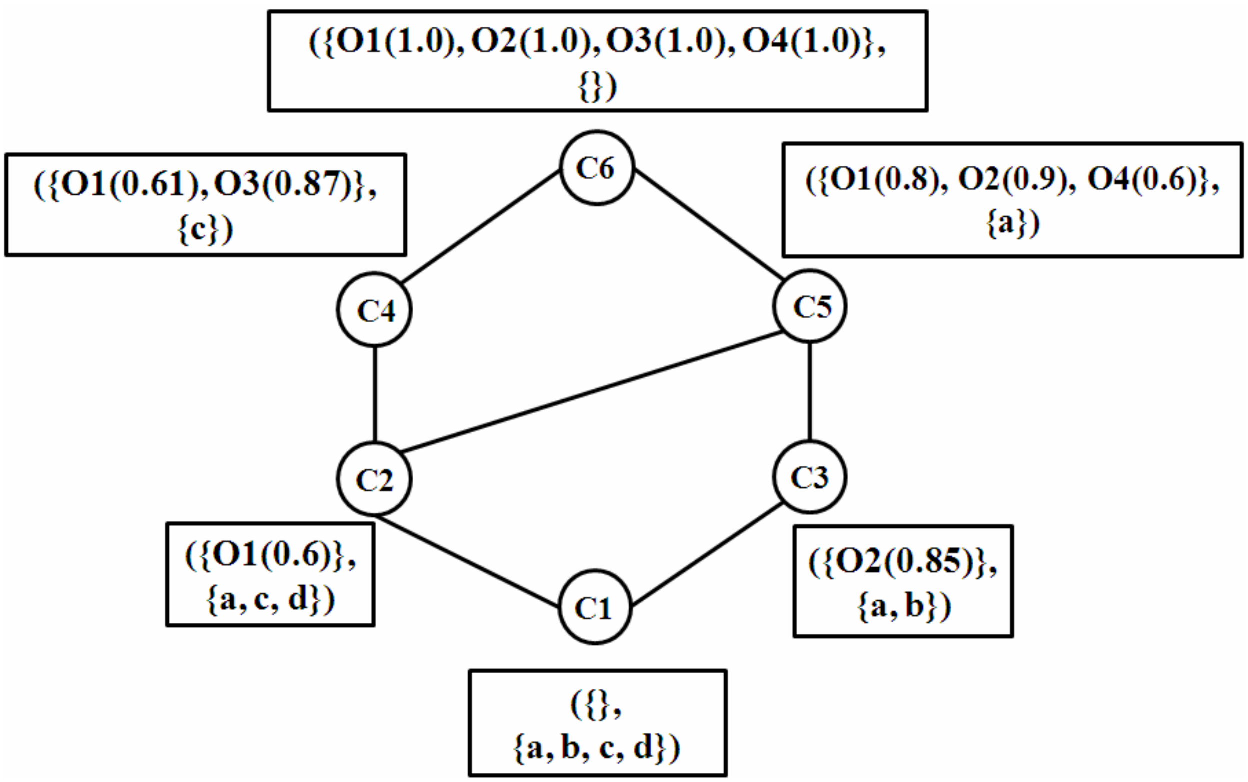


Fig. □ Fuzzy concept lattice of the Table VI

Table. VII Fuzzy formal concepts of Table VI

| | Extents with membership value | Intents |
|---|---|---|
| C1 | {} | {a, b, c, d} |
| C2 | {O1(0.6) } | {a, c, d} |
| C3 | {O2(0.85)} | {a, b} |
| C4 | {O1(0.61), O3(0.87)} | {c} |
| C5 | {O1(0.8), O2(0.9),O4(0.6)} | {a} |
| C6 | {O1(1.0),O2(1.0),O3(1.0),O4(1.0)} | {} |

**Definition 8.** Given a fuzzy formal context K := (G, M, I) and a confidence threshold $T=[t_1, t_2]$, we define $FI(A) = \{m \in M \mid \forall g \in A : t_1 \le \mu(g, m) \le t_2\}$ for $A \subseteq G$ and $FE(B) = \{g \in G \mid \forall m \in B : t_1 \le \mu(g, m) \le t_2\}$ for $B \subseteq M$. A fuzzy formal concept (or fuzzy concept) of a fuzzy formal context (G, M, I) with a confidence threshold T is a pair $(A_f = \varphi(A), B)$ where $A \subseteq G$, $B \subseteq M$, $FI(A) = B$ and $FE(B) = A$. A and B are extents and intents of the fuzzy formal concept, respectively. Each object $g \in \varphi(A)$ has a membership $\mu_g$ defined as;

$$\mu_g = \min_{m \in B} \mu(g, m) \qquad (1)$$

where $\mu(g, m)$ is a membership value between object g and attribute m, which is defined in *I*. Note that if B = {} then $\mu_g = 1$

for every g.■

The memberships between attributes and objects of a fuzzy formal concept indicate the grade of relationships between the objects and the fuzzy formal concept. According to the fuzzy set theory[15], the intersection of membership values between an object and all attributes of the fuzzy formal concept should be the minimum of these membership values.

According to the definition 8, all fuzzy formal concepts are extracted from Table VI and listed in Table VII. As shown from the Table VII, in C3, an object O2 has "0.85" as a membership value that is calculated by the equation (1) of definition 6. In C5, O2 has "0.9" as a membership value, because, the C5 has only one attribute named "a".

**Definition 9.** Let $(\varphi(A_1), B_1)$ and $(\varphi(A_2), B_2)$ be two fuzzy concepts of a fuzzy formal context (G, M, I). $(\varphi(A_1), B_1)$ is the sub-concept of $(\varphi(A_2), B_2)$, denoted as $(\varphi(A_1), B_1) \leq (\varphi(A_2), B_2)$, if and only if, $\varphi(A_1) \subseteq \varphi(A_2)$ ($\Leftrightarrow B_2 \subseteq B_1$). ■

**Definition 10.** A fuzzy concept lattice of a fuzzy formal context K with a confidence threshold T is a set F(K) of all fuzzy concepts of K with the partial order ≤ with the confidence threshold T.■

Based on definition 3 and 4, we can construct a fuzzy concept lattice(Fig. 4) for Table VI.

We extended the concept covered algorithm and the interaction algorithm based on the above definitions [16]. Algorithm 1 accomplish extracting all fuzzy concepts from given context, and calculating membership values between the objects and the attribute of the fuzzy concepts. The fuzzy formal concepts extracted from Algorithm 1 are constructed by using Algorithm 2.

**Algorithm 1. Extracting fuzzy concepts with membership values between the objects and the attributes of the fuzzy concepts**

**Input:** Fuzzy Formal Context K := (G, M, $I = \varphi(G \times M)$) with a confidence threshold T
**Output:** The set of C of all fuzzy concepts of K

```
1. C := {(FE(M), M)}
2. setMembershipValue((FE(M), M))
3. for each g ∈ G
4.     for each (X, Y) ∈ C
5.         Inters:=Y ∩ FI({g})
6.         if Inters different from any concept intent in C then
7.             C:= C∪{(FI(Inters), Inters)}
8.             setMemebershipValue((FI(Inters), Inters))
9.         end if
10.    end for
11. end for
```

**function setMemebershipValue((X, Y))**
**Input:** A fuzzy concept (X, Y)

```
1. for each x ∈ X
2.     min = 0.0
3.     for each y ∈ Y
4.         if( (x, y) < min ) then
5.             min = (x, y)
6.             x.membershipValue = min;
7.         end if
8.     end for
9. end for
```

**Algorithm 2. Constructing fuzzy concept lattice**

**Input:** Fuzzy Formal context K=(G, M, $I = \varphi(G \times M)$) and C = F(K) that is a set of all fuzzy concepts in K
**Output:** Fuzzy Concept Lattice L = (C, E)

1. Find C with the algorithm 1
2. XCovering Edges (C, K)

**function XCoveringEdges (C, K)**
**<u>Input:</u>** A set of fuzzy concepts C and a fuzzy formal context K

```
for each (X, Y) ∈ C
    Set count of any concept in C to 0
    for each m ∈ M \ Y
        inters := X ∩ FE({m})
        Find (X1, Y1) ∈ C such that X1 = inters
        count(X1, Y1):= count(X1, Y1) + 1
        if(|Y1|-|Y|) = count(X1, Y1) then
          Add edge (X1, Y1) □ (X, Y) to E
        end if
    end for
end for
```

## III. Fuzzy FCA-wizard and experiment

In this section, we explain our tool, "Fuzzy FCA-Wizard" supporting fuzzy formal concept analysis approach, and some experiments show usefulness and potentiality of our tool.

### *A. Fuzzy FCA-Wizard*

As shown in Fig 5, Fuzzy FCA-Wizard is composed of two main components which are Core Component and UI Components.

Core Components have modules to handle three internal data model such as fuzzy formal context, fuzzy formal concept and fuzzy concept lattice. Fuzzy Context Handler Component handles fuzzy formal context like a way of importing and exporting raw data(as csv format file) and transforming the file into the fuzzy formal context. Fuzzy Concept Extractor Component extracts all formal concepts from imported fuzzy formal context with a threshold. Fuzzy Lattice Constructor Component constructs a fuzzy concept lattice by a way of extracting the sub-super relations between the fuzzy formal concepts which are provided in the fuzzy concept extractor component.

UI Components provide some functionality for browsing the fuzzy concepts and the fuzzy concept lattice, and editing the fuzzy context. The UI Components have three sub-views. Fuzzy

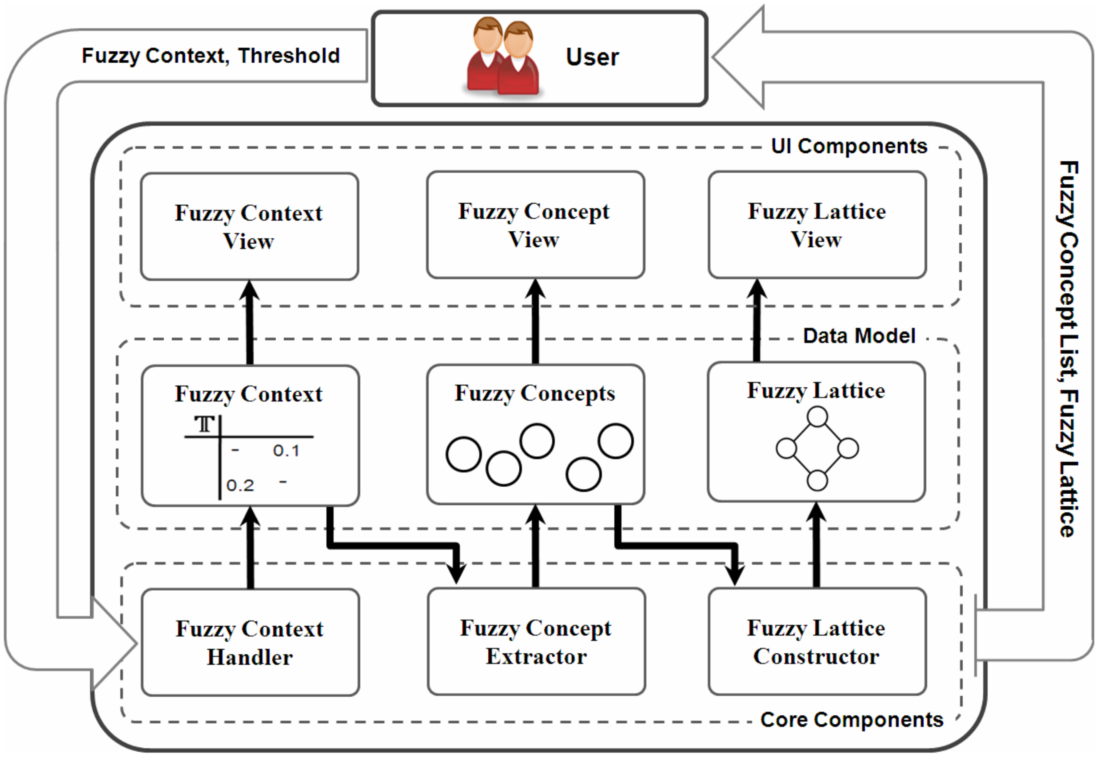


Fig. □ Architecture of Fuzzy Formal Concept Analysis Wizard

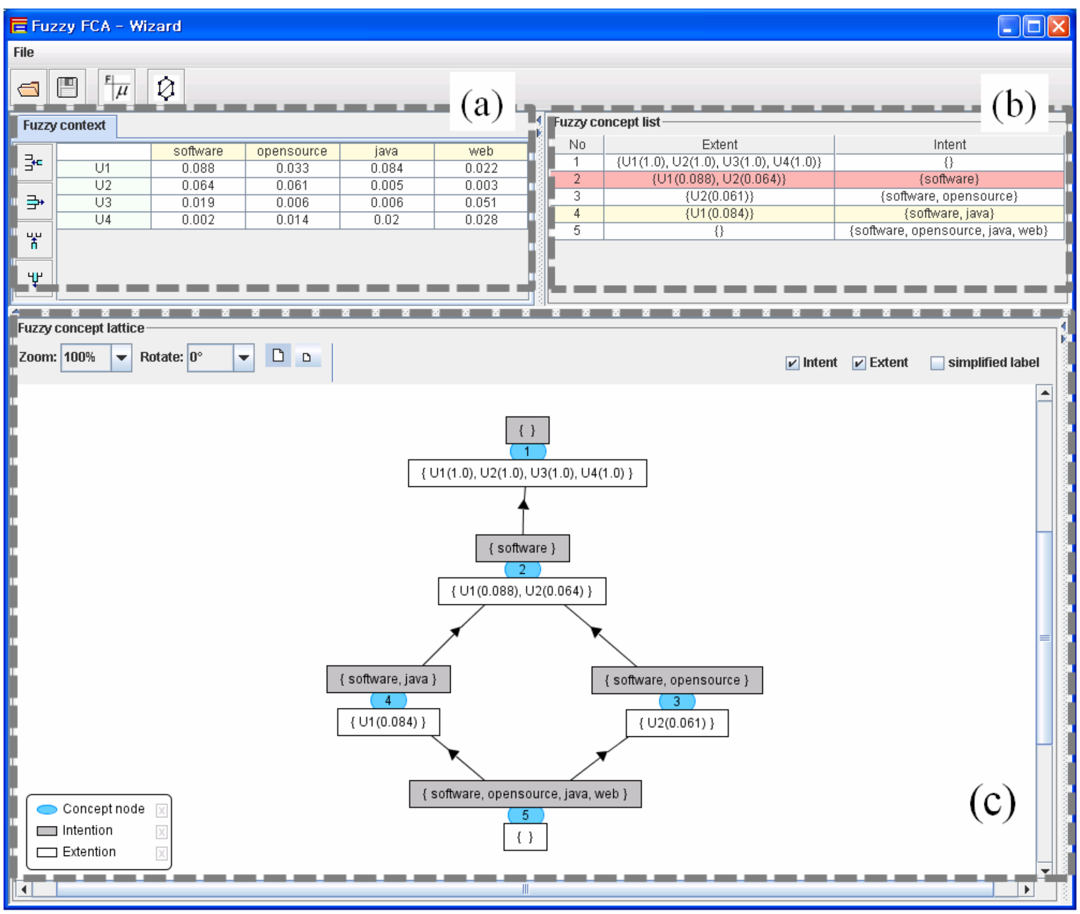


Fig. □ Screenshot of Fuzzy Formal Concept Analysis Wizard

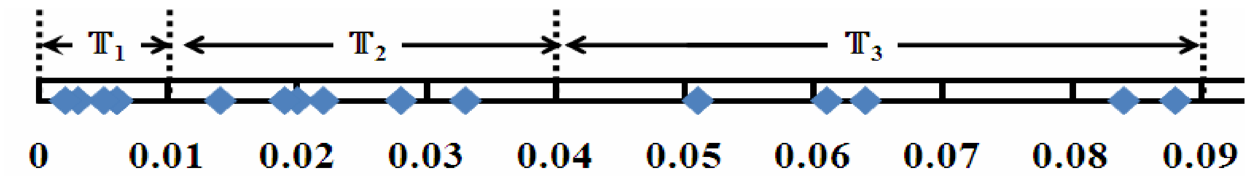


Fig. □ Density of membership values and corresponding confidence thresholds

Context View represents a fuzzy formal context model converted from Fuzzy Context Component. Also it can allow editing table, like a MS excel(Fig 6(a)). Fuzzy Concept List View represents all fuzzy formal concepts which are provided in the fuzzy concept component(Fig 6(b)). Fuzzy Lattice View shows the fuzzy concept lattice model graphically by using JPower Graph API [17] (Fig 6(c)).

In the Fuzzy FCA-Wizard, there are three data models such as fuzzy context, fuzzy concept and fuzzy lattice. The fuzzy context model is composed of two finite nonempty sets of objects and attributes and relationship between them and it is implemented by bitset for the improvement in performance. According to the definition 8, the fuzzy concept model indicates a pair of a set of objects with corresponding memberships and a set of attributes. The fuzzy lattice model is a line diagram that is implemented by graph API.

### B. *Experiment*

In this section, we make an experiment with Fuzzy FCA-Wizard based on user and tag data sets that are extracted from BibSonomy, social bookmarking system. Table VIII and IX show the data sets for some arbitrary users and tags that are gathered in BibSonomy.

In order to construct fuzzy lattice, we first transform these data sets into a fuzzy formal context as shown in Table X. In Table X, relations between user and tag are represented by membership values, which are calculated by a membership function. The membership function is derived by tag frequency as;

$$\mu(g,m)=\frac{freq(g,m)}{total\_freq(g)} \qquad (2)$$

The frequency of a tag m for a given user g (i.e., freq(g, m)) is the number of time the given tag uses by the user. The total frequency of a user g(i.e., total_freq(g)) is a summation of all tags frequency used by the user g. For example, a membership value of U1 and opeonsource is real number “0.033”(Table X).

Table. VIII Total frequency of tags

| | Total frequency of tags |
|---|---|
| U1 | 3,469 |
| U2 | 14,613 |
| U3 | 6,362 |
| U4 | 2,540 |

Table. IX Frequencies of tags

| | software | opensource | java | web |
|---|---|---|---|---|
| U1[1] | 305 | 290 | 115 | 75 |
| U2[2] | 941 | 77 | 891 | 44 |
| U3[3] | 118 | 40 | 36 | 326 |
| U4[4] | 6 | 50 | 35 | 70 |

Table. X Fuzzy formal context

| | software | opensource | java | web |
|---|---|---|---|---|
| U1 | 0.088 | 0.033 | 0.084 | 0.022 |
| U2 | 0.064 | 0.061 | 0.005 | 0.003 |
| U3 | 0.019 | 0.006 | 0.006 | 0.051 |
| U4 | 0.002 | 0.014 | 0.020 | 0.028 |

Table. XI Fuzzy formal contexts with thresholds

| | software | opensource | java | web |
|---|---|---|---|---|
| U1 | — | — | — | — |
| U2 | — | — | 0.005 | 0.003 |
| U3 | — | 0.006 | 0.006 | — |
| U4 | 0.002 | — | — | — |

(a) A fuzzy context with T = [0.0, 0.01]

| | software | opensource | java | web |
|---|---|---|---|---|
| U1 | — | 0.033 | — | 0.022 |
| U2 | — | — | — | — |
| U3 | 0.019 | — | — | — |
| U4 | — | 0.014 | 0.020 | 0.028 |

(b) A fuzzy context with T = [0.01, 0.04]

| | software | opensource | java | web |
|---|---|---|---|---|
| U1 | 0.088 | — | 0.084 | — |
| U2 | 0.064 | 0.061 | — | — |
| U3 | — | — | — | 0.051 |
| U4 | — | — | — | — |

(c) A fuzzy context with T = [0.04, 0.09]

[1] U1 : http://www.bibsonomy.org/user/timo
[2] U2 : http://www.bibsonomy.org/user/gresch
[3] U3 : http://www.bibsonomy.org/user/walterra
[4] U4 : http://www.bibsonomy.org/user/ewomant

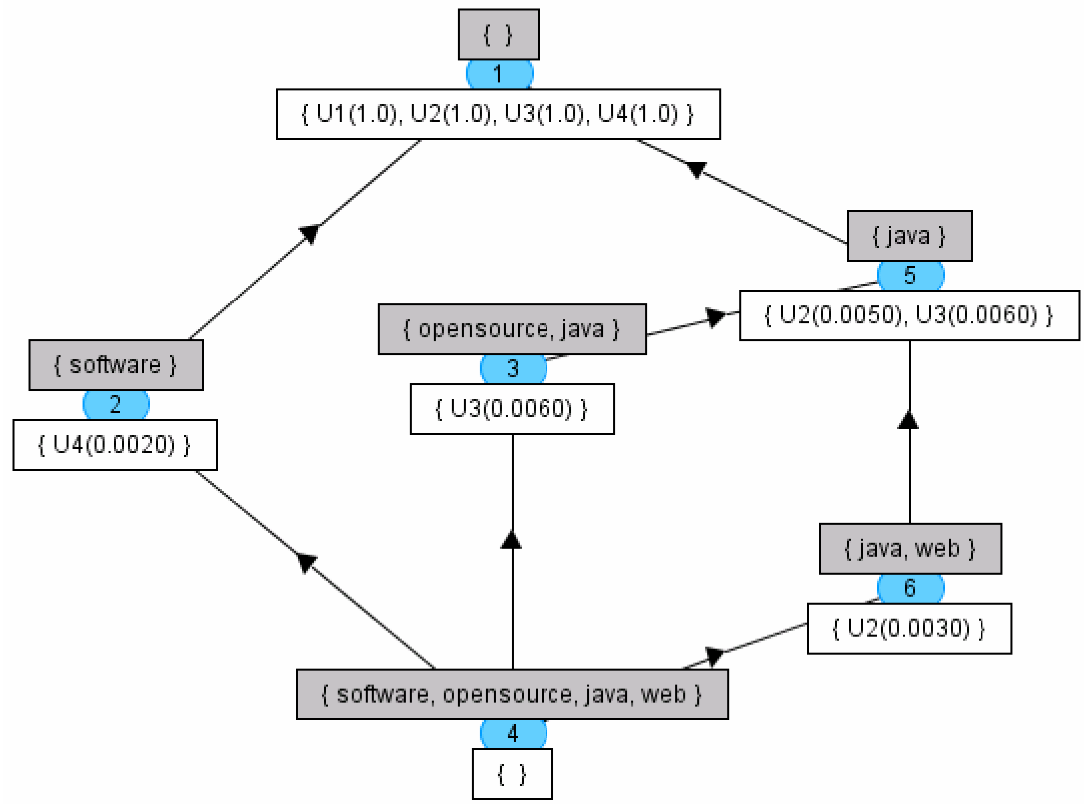


Fig. Fuzzy formal lattice of Table XI(a)

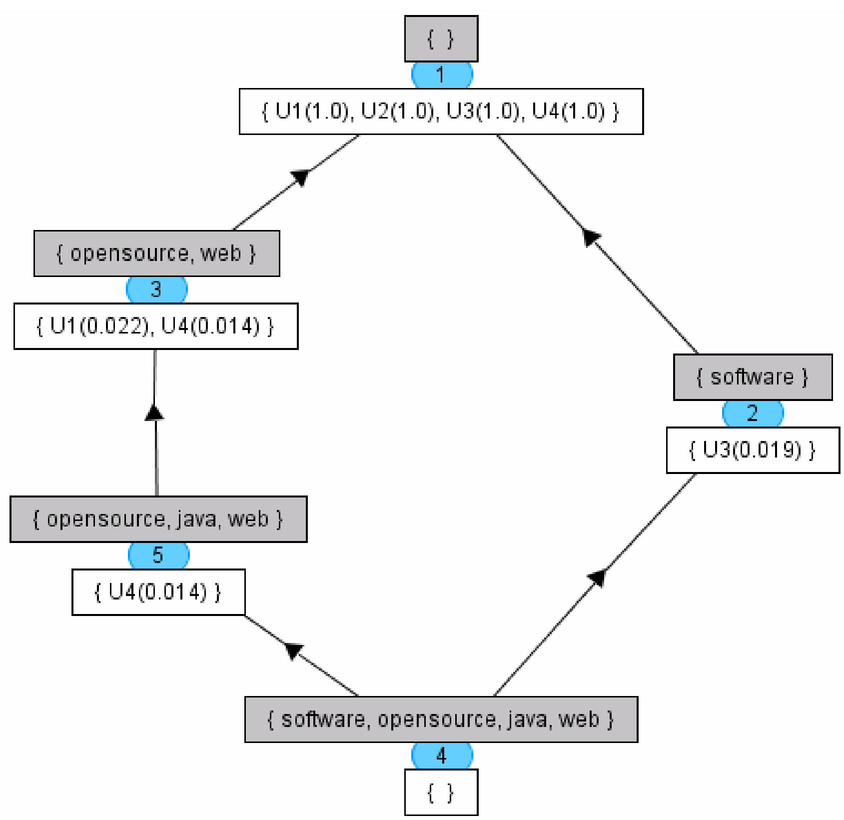


Fig. Fuzzy formal lattice of Table XI(b)

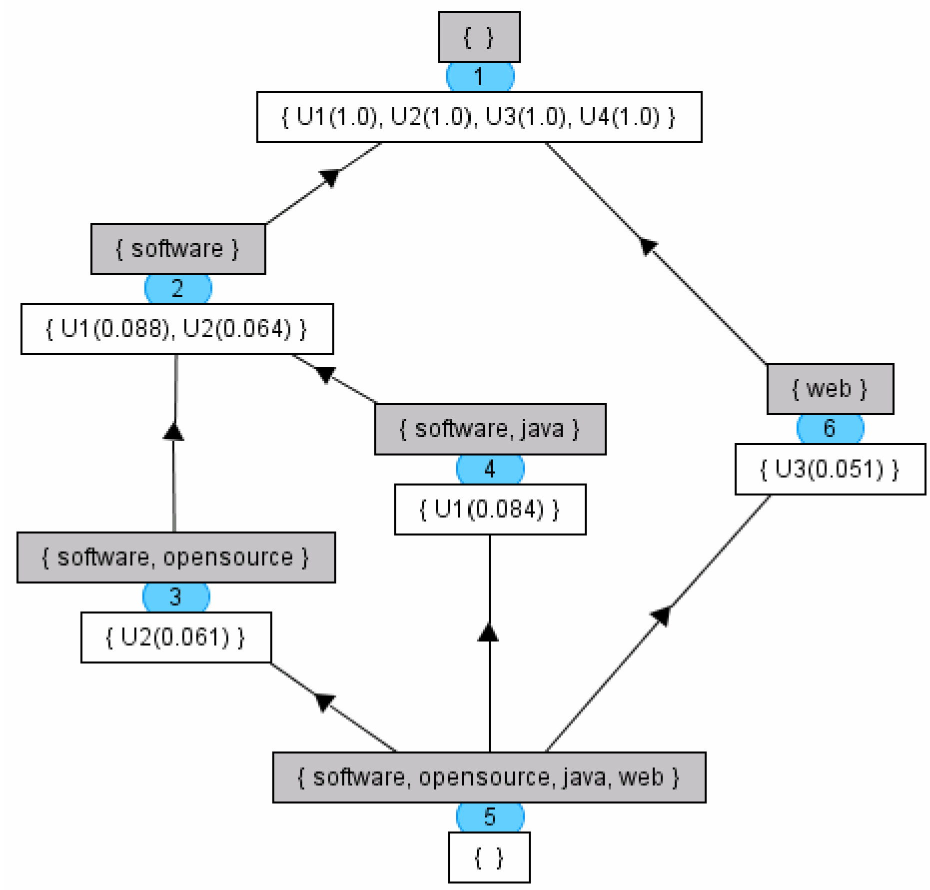


Fig. □□ Fuzzy formal lattice of Table XI(C)

In this experiment, we constructed fuzzy concept lattices per confidence threshold. The steps are described as follows :

1) Preprocessing; in this step, we transformed Table VIII and Table IX into a fuzzy formal context as a table X with the membership function(2). For example, a membership value of between a user U2 and a tag software is a real number "0.064" that is calculated by 941 divided by 14,613(such as freq(U2, software)/total_freq(U2)). The membership values of the fuzzy formal context are in the range from "0.002" to "0.088".
2) Selecting Confidence Threshold T; we selected out three confidence thresholds as $T_1$=[0.0,0.01], $T_2$=[0.01,0.04] and $T_3$=[0.04,0.09] according to the density of membership values(Fig 7).
3) Classifying given data into Fuzzy Formal Concept Lattices; The confidence thresholds are applied to the fuzzy formal context(Table X) and we constructed three fuzzy concept lattices from the applied fuzzy formal contexts(Fig 8-10).

## IV. Conclusion

Data Mining(also known as Knowledge Discovery) is defined as "the non-trivial extraction of implicit, previously unknown, and potentially useful information from data"[18]. It includes not only methods for extracting information from the given data, but also visualizing the information. In these days, mechanism for interacting between human and machinery is also considered to a part of data mining[19, 20]. However, it becomes almost impossible to manually analyze huge amounts and various kinds of data that can be crisp or exist fuzzy/vagueness for extracting and discovering valuable information and knowledge.

Therefore, we need assistances in the well defined Fuzzy FCA tool's functionalities. In this paper, we introduce basic notions of FCA and Fuzzy FCA, and develop the Fuzzy Formal Concept Analysis Wizard(Fuzzy FCA-Wizard). In order to evaluate usefulness of Fuzzy FCA-Wizard, we experiment on our tool with users and tags data extracted from the online social tagging system such as BibSonomy.

In this paper, we introduced a Formal Concept Analysis and a Fuzzy Formal Concept Analysis, and developed Fuzzy Formal Concept Analysis Wizard to analysis and extract implicit information from given vague data. We have an experiment which has demonstrated how Fuzzy FCA-Wizard can discover implicit knowledge as classified Fuzzy Concept Lattices. This paper describes the process and supporting tool of knowledge discovery in extracting fuzzy concepts from fuzzy context. It can be applied to some interesting areas such as traditional data mining, semantic web mining and so on.

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